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Solutions of Fuzzy Linear Systems using Ranking function

M. A. Jahantigh*, S. Khezerloo, A. A. Hosseinzadeh, M. Khezerloo

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Abstract In this work, we propose an approach for computing the compromised solution of an LR fuzzy linear system by using of a ranking function when the coefficient matrix is a crisp $m \times n$ matrix. To do this, we use expected interval to find an LR fuzzy vector, $\tilde{X}$, such that the vector $\Re(A\tilde{X})$ has the least distance from $\Re(\tilde{b})$ in $1-$norm and the $1-$cut of $\tilde{X}$ satisfies the crisp linear system $AX = b$. Then, we solve the constrained LP-problem by using LINGO 11 software package and obtain optimal fuzzy number solution. Finally, we illustrate the proposed method by solving two numerical examples.

Keywords Fuzzy Linear System, Fuzzy LR Number, Expected Interval, Ranking Function, Fuzzy Number Solution.

1 Introduction

The concept of fuzzy numbers and arithmetic operations with these numbers were first introduced and investigated by Zadeh [1]. Additional related material can be found in [2, 3, 4]. Many researchers have developed methods to compare and to rank fuzzy numbers, too [5, 6, 7, 8]. Cheng [9] has proposed some methods, and recently many ranking methods have been presented by Ma, Kandel and Friedman [10] and Chu and Tsao [11] . In [12] a fuzzy distance measure for Gaussian type fuzzy numbers is defined. In this study a new metric distance on fuzzy numbers is introduced and then it is used for ranking fuzzy numbers by comparison with two crisp numbers max(M) and min(m).

This paper is organized as follows:In Section 2, the basic concept of fuzzy number operation is brought. In Section 3, the main section of the paper, a new method based on ranking functions for solving fuzzy linear system, is suggested. The proposed idea is illustrated by two numerical examples in the Section 4. Finally conclusion is drawn in Section 5.

*** Corresponding Author.** (✉)
E-mail: mohamadjahantig@yahoo.com (M. A. Jahantigh)

M. A. Jahantigh
Department of Mathematics, Zahedan Branch, Islamic Azad University, Zahedan, Iran.

S. Khezerloo
Department of Mathematics, Karaj Branch, Islamic Azad University, karaj, Iran.

A. A. Hosseinzadeh, M. Khezerloo
Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.

2 Preliminaries

In this section, we give some basic definitions of fuzzy numbers.

Definition 1. A fuzzy number is a fuzzy set $\mu_{\tilde{A}} : R \to I = [0,1]$ which satisfies:

- $\mu_{\tilde{A}}$ is upper semi continuous.
- $\mu_{\tilde{A}}(x) = 0$ outside some interval $[c,d]$.
- There are real numbers $a,b : c \le a \le b \le d$ for which

a. $\mu_{\tilde{A}}(x)$ is monotonic increasing on $[c,a]$,

b. $\mu_{\tilde{A}}(x)$ is monotonic decreasing on $[b,d]$,

c. $\mu_{\tilde{A}}(x) = 1, \quad a \le x \le b$.

The set of all fuzzy numbers (as given by Definition (2.1)) is denoted by E^1. An alternative definition or parametric form of a fuzzy number which yields the same E^1 is given by Kaleva [13].

Definition 2. A fuzzy number $\tilde{A}$ is $LR-$type if there exit L (for left) and R (for right) and scalars $\alpha > 0$, $\beta > 0$ with

$$\mu_{\tilde{A}}(x) = \begin{cases} L(\frac{a-x}{\alpha}), & x \le \alpha, \\ R(\frac{x-a}{\beta}), & x \ge a \end{cases}$$

where L and R are strictly decreasing functions defined on $[0,1]$ and satisfy the conditions:

$$\begin{aligned} &L(0) = R(0) = 1 \\ &L(1) = R(1) = 0 \\ &0 < L(x) < 1, \qquad 0 < R(x) < 1, \quad x \ne 0,1 \end{aligned} \tag{1}$$

The mean value of $\tilde{A}$, a , is a real number, and α, β are called the left and right spreads, respectively. $\tilde{A}$ is denoted by $(a,\alpha,\beta)_{LR}$.

Definition 3. A fuzzy number $\tilde{A} = (m,\alpha,\alpha)_{LL}$ is a symmetric fuzzy number.

Remark 1. According to Definition (2.1), throughout the paper, we assume that all the supports of $LR-$type fuzzy numbers are bounded.

Definition 4. $\tilde{M} = (m,\alpha,\beta)_{LR}$ is a triangular fuzzy number if $L = R = max(0,1-x)$.

Theorem 1. [14] Let $\tilde{M} = (m,\alpha,\beta)_{LR}$, $\tilde{N} = (n,\gamma,\delta)_{LR}$ and $\lambda > 0$. Then

- $\lambda\tilde{M} = (\lambda m,\lambda\alpha,\lambda\beta)_{LR}$.
- $-\tilde{M} = (-m,-\beta,-\alpha)_{LR}$.
- $\tilde{M} \oplus \tilde{N} = (m+n,\alpha+\gamma,\beta+\delta)_{LR}$.

Cheng`s ranking function is based on the centroid point [15] (other ranking function based on centroid point can be found in [16, 17]). Suppose that the fuzzy number $\tilde{x} = (x, \alpha, \beta)_{LR}$ is defined by the membership function,

$$f_{\tilde{x}} = \begin{cases} f_{\tilde{x}}^{L}(y), & x - \alpha \le y \le x \\ f_{\tilde{x}}^{R}(y), & x \le y \le x + \beta \\ 0, & OW. \end{cases} \tag{2}$$

where $f_{\tilde{x}}^{L}(y) = L\left(\frac{x-y}{\alpha}\right)$, $f_{\tilde{x}}^{R}(y) = L\left(\frac{y-x}{\beta}\right)$, L and R are defined in Eq. (1). Ghanbari and Mahdavi-Amiri [18] solved LR fuzzy linear system by using the ranking function introduced by cheng [15] which is based on centroid point and they didn`t attend that centroid formula provided him are incorrect. In this paper, we use the expected interval proposed by Grzegorzewski [19] and compare our results with the results obtained in [12].
We denote the set of LR fuzzy numbers by $F(\Re)_{LR}^{1}$.

Definition 5. [18] A vector $\tilde{X} = (\tilde{x}_1, \ldots, \tilde{x}_n)^T$, denoted by $\tilde{X} \in F(\Re)_{LR}^{n}$, is called an LR fuzzy vector, where $\tilde{x}_i \in F(\Re)_{LR}^{1}$, $i = 1, \ldots, n$.

Suppose A is a fuzzy number and $[A_L(r), A_U(r)]$ is its $r-$cut. Let us also recall that an expected interval $EI(A)$ of a fuzzy number A is given by

$$EI(A) = \left[\int_0^1 A_L(r)dr, \int_0^1 A_U(r)dr\right]$$

which is the nearest interval approximation of A (see [20, 19, 21]).

3 LR Fuzzy Linear System

Consider the fuzzy linear system

$$A\tilde{X} = \tilde{b} \tag{3}$$

where $A \in \Re^{m \times n}$, $m \le n$ is a crisp matrix and $\tilde{X}_{n\times 1}$ and $\tilde{b}_{m\times 1}$ are LR fuzzy vectors.

We use ranking function to define a compromised solution for system (3).
Consider

$$A\tilde{X} =_{\Re} \tilde{b} \tag{4}$$

where $\Re$ is a ranking function. Using the following definition, we define the solution of (4) as a compromised solution for (3).

Definition 6. [18] We call $\tilde{X} \in F(\Re)^n_{LR}$ a solution of $A\tilde{X} =_{\Re} \tilde{b}$ with respect to the ranking function $\Re$ if and only if we have,

$$\begin{cases} AX = b \, {*}3mm \\ a_i^T \tilde{X} =_{\Re} \tilde{b}_i, \quad i = 1,\ldots,n \end{cases}$$

where the crisp linear system $AX = b$ is the $1-$cut or mean value of fuzzy number system $A\tilde{X} = \tilde{b}$ and a_i^T is the $i-$th row of A.

Proposition 1. [18] If $\tilde{X}$ is a solution of $A\tilde{X} = \tilde{b}$, then $\tilde{X}$ is a solution of $A\tilde{X} =_{\Re} \tilde{b}$, where $\Re$ is a ranking function. Equivalently, if the system $A\tilde{X} =_{\Re} \tilde{b}$ lacks a solution, then $A\tilde{X} = \tilde{b}$ lacks a solution.

Throughout the paper, the triangular form of $LR-$fuzzy numbers is applied.

Suppose $\left[(a_i^T\tilde{X})_L(r), (a_i^T\tilde{X})_U(r)\right]$ and $\left[(\tilde{b}_i)_L(r), (\tilde{b}_i)_U(r)\right]$ are the $r-$cut of $a_i^T\tilde{X}$ and $\tilde{b}_i$, respectively. Then,

$$EI(a_i^T\tilde{X}) = [p_i, q_i],$$
$$EI(b_i) = [m_i, n_i]$$

where

$$p_i = \int_0^1 (a_i^T\tilde{X})_L(r)dr, \quad q_i = \int_0^1 (a_i^T\tilde{X})_U(r)dr$$

and

$$m_i = \int_0^1 (\tilde{b}_i)_L(r)dr, \quad n_i = \int_0^1 (\tilde{b}_i)_U(r)dr$$

We try to find an LR fuzzy vector, $\tilde{X}$, such that the vector $\Re(A\tilde{X})$ has the least distance from $\Re(\tilde{b})$ in $1-$norm and the $1-$cut of $\tilde{X}$ satisfies the crisp linear system $AX = b$. For this idea, we assume the ranking function $\Re$ as follows:

$$\Re(a_i^T\tilde{X}, \tilde{b}_i) = \frac{1}{2}\left(|p_i - m_i| + |q_i - n_i|\right) \tag{5}$$

Therefore, from Definition (2.2) and using the above notations, we define the following constrained LP-problem:

$$
\begin{aligned}
&Min \quad \sum_{i=1}^{m} |\Re(a_i^T \tilde{X}, \tilde{b}_i)| \\
&s.t. \\
&\qquad AX = b, \\
&\qquad \alpha = (\alpha_1, \ldots, \alpha_n)^T > 0, \quad \beta = (\beta_1, \ldots, \beta_n)^T > 0, \\
&\qquad \alpha, \beta \in C.
\end{aligned} \tag{6}
$$

Remark 2. [18] For the above optimization problem, we may consider the numbers α_j , β_j for $j = 1, \ldots, n$ to have small positive values (e.g., 10^{20}). In this case, the fuzzy numbers $(x_j, \alpha_j, \beta_j)_{LR}^T$ are very close to the crisp numbers x_j . Thus, the user can define a positive parameter, ε , such that $\alpha_j, \beta_j \geq \varepsilon$ for all j . By this setting, it is guaranteed that each fuzzy number has left and right spreads as much as ε . Also, suppose the user wants to impose $\alpha_j, \beta_j \leq M_j$ or decides to have some LR fuzzy numbers to be approximately symmetric, then the user adjoins appropriate constraints to the constrained least squares problem (6); e.g., $|\alpha_j - \beta_j| \leq 10^{-k}$ for some j , $j \in \{1, \ldots, n\}$. Other constraints can also be interesting. Assume the user wants the measure of fuzziness of some fuzzy numbers to be smaller than certain predefined positive numbers Nj. In this case, the user adds $\alpha_j + \beta_j \leq N_j$ to the constrained least squares problem (6).
So, using Remark (3.1), we have

$$
\begin{aligned}
&Min \quad \sum_{i=1}^{m} |\Re(a_i^T \tilde{X}, \tilde{b}_i)| \\
&s.t. \\
&\qquad AX = b, \\
&\qquad \alpha = (\alpha_1, \ldots, \alpha_n)^T > \varepsilon e, \quad \beta = (\beta_1, \ldots, \beta_n)^T > \varepsilon e, \\
&\qquad \alpha, \beta \in C.
\end{aligned} \tag{7}
$$

where $e = (1, \ldots, 1)^T$ and C is the feasible space defined by user's specified conditions.
Let z^* be the optimal value of (6) or (7). If $z^* \neq 0$, then it is apparent that the system (4) lacks a solution. In this case, let,

$$S_w = \{\tilde{y} \mid \tilde{y} \in F(\Re^n)_{LR}, \tilde{y} \ \textit{is a solution of} \ (6) \, or \ (7)\} \tag{8}$$

Then, we define a compromised solution, namely approximate solution, for the system (4) as follows.

Definition 7. [18] We say that $\tilde{y}$ is an approximate solution of (4) if $\tilde{y} \in S_w$, which S_w was defined by (8).

If $z^* = 0$, then the system (4) has a solution. We call this solution as a strong solution. We redefine a solution of (4) as follow.

Definition 8. [18] We say that $\tilde{y}$ is a strong solution of (4) if $\tilde{y} \in S_w$, where,

$$S_w = \{\tilde{y} \mid \tilde{y} \in F(\Re^n)_{LR}, \tilde{y} \ \ is\ a\ solution\ of\ \ (6)\, or\ (7)\, with\ z^* = 0\}$$

It is obvious that Definition (3.3) and Definition (3.1) are equivalent and the solution of (6) or (7) depends on the choice of $\Re$.
Substituting (5) into (7), we have

$$\begin{aligned} & Min \quad \frac{1}{2}\sum_{i=1}^{m}\left(|p_i - m_i| + |q_i - n_i|\right) \\ & s.t. \\ & \qquad AX = b, \\ & \qquad \alpha = (\alpha_1,\ldots,\alpha_n)^T > \varepsilon e, \quad \beta = (\beta_1,\ldots,\beta_n)^T > \varepsilon e, \\ & \qquad \alpha, \beta \in C, \\ & \qquad p_i, q_i, m_i, n_i \quad free \qquad i = 1,\ldots,m. \end{aligned} \tag{9}$$

Now, redefining $t_i = p_i - m_i$ and $s_i = q_i - n_i$ and by substituting them into the (9),

$$\begin{aligned} & Min \quad \frac{1}{2}\sum_{i=1}^{m}\left(|t_i| + |s_i|\right) \\ & s.t. \\ & \qquad AX = b, \\ & \qquad t_i = p_i - m_i, \\ & \qquad s_i = q_i - n_i, \\ & \qquad \alpha = (\alpha_1,\ldots,\alpha_n)^T > \varepsilon e, \quad \beta = (\beta_1,\ldots,\beta_n)^T > \varepsilon e, \\ & \qquad \alpha, \beta \in C, \\ & \qquad p_i, q_i, m_i, n_i \quad free \qquad i = 1,\ldots,m. \end{aligned} \tag{10}$$

We can suppose $t_i = t_i^{'} - t_i^{''}$ and $s_i = s_i^{'} - s_i^{''}$ where $t_i^{'} \geq 0$, $t_i^{''} \geq 0$, $s_i^{'} \geq 0$ and $s_i^{''} \geq 0$, so, we can rewrite (10) as follows:

$$
\begin{aligned}
& Min \quad \frac{1}{2}\sum_{i=1}^{m}\left(t_i^{'}+t_i^{''}+s_i^{'}+s_i^{''}\right) \\
& s.t. \\
& \qquad AX=b, \\
& \qquad t_i^{'}-t_i^{''}=p_i-m_i, \\
& \qquad s_i^{'}-s_i^{''}=q_i-n_i, \\
& \qquad \alpha=(\alpha_1,\ldots,\alpha_n)^T>\varepsilon e, \quad \beta=(\beta_1,\ldots,\beta_n)^T>\varepsilon e, \\
& \qquad \alpha,\beta\in C, \\
& \qquad t_i^{'}\geq 0,\ t_i^{''}\geq 0,\ s_i^{'}\geq 0,\ s_i^{''}\geq 0.
\end{aligned} \tag{11}
$$

where is a LP problem and it is easily solved by using LINGO 11 software package. By solving (11), we can obtain the fuzzy number solutions $\tilde{x}_j=(x_j,\alpha_j,\beta_j)$.

4 Example

Example 1. (Example 2.2. [18]). Consider,

$$
\begin{cases}
\tilde{x}_1-\tilde{x}_2=(1,1,1) \\
\tilde{x}_1+3\tilde{x}_2=(5,1,2)
\end{cases}
$$

Using (11), we have

$$
\begin{aligned}
& Min \quad \frac{1}{2}\sum_{i=1}^{2}\left(t_i^{'}+t_i^{''}+s_i^{'}+s_i^{''}\right) \\
& s.t. \\
& \qquad x_1-x_2=1, \\
& \qquad x_1+3x_2=5, \\
& \qquad t_1^{'}-t_1^{''}=c_1-\alpha_1-c_2-\beta_2+0.5\,\alpha_1+0.5\,\beta_2-0.5, \\
& \qquad s_1^{'}-s_1^{''}=c_1+\beta_1-c_2+\alpha_2-0.5\,\beta_1-0.5\,\alpha_2-1.5, \\
& \qquad t_2^{'}-t_2^{''}=c_1-\alpha_1+3c_2-3\,\alpha_2+0.5\,\alpha_1+1.5\,\alpha_2-4.5, \\
& \qquad s_2^{'}-s_2^{''}=c_1+\beta_1+3c_2+3\,\beta_2-0.5\,\beta_1-1.5\,\beta_2-6, \\
& \qquad \alpha=(\alpha_1,\alpha_2)^T>0.0001e, \quad \beta=(\beta_1,\beta_2)^T>0.0001e, \\
& \qquad \alpha,\beta\in C, \\
& \qquad t_i^{'}\geq 0,\ t_i^{''}\geq 0,\ s_i^{'}\geq 0,\ s_i^{''}\geq 0, \quad i=1,2.
\end{aligned}
$$

then, using the LINGO 11 software package, we can obtain

$$\tilde{x}_1=(2.0000,0.6250,0.8750), \quad \tilde{x}_2=(1.0000,0.1250,0.3750)$$

It is clear that $\tilde{X} = (\tilde{x}_1, \tilde{x}_2)$ is a fuzzy number solution.

Example 2. (Example 2. [22]). Consider,

$$\begin{cases} \tilde{x}_1 + \tilde{x}_2 - \tilde{x}_3 = (1,1,1) \\ \tilde{x}_1 - 2\tilde{x}_2 + \tilde{x}_3 = (3,1,0) \\ 2\tilde{x}_1 + \tilde{x}_2 + 3\tilde{x}_3 = (-2,0,1) \end{cases} \tag{12}$$

Using (11), we have

$$\begin{aligned}
& Min \quad \frac{1}{2}\sum_{i=1}^{3}\left(t_i^{'} + t_i^{''} + s_i^{'} + s_i^{''}\right) \\
& s.t. \\
& \quad x_1 + x_2 - x_3 = 1, \\
& \quad x_1 - 2x_2 + x_3 = 3, \\
& \quad 2x1 + x_2 + 3x_3 = -2, \\
& \quad t_1^{'} - t_1^{''} = c_1 - \alpha_1 + c_2 - \alpha_2 - c_3 - \beta_3 + 0.5\,\alpha_1 + 0.5\,\alpha_2 + 0.5\,\beta_3 - 0.5, \\
& \quad s_1^{'} - s_1^{''} = c_1 + \beta_1 + c_2 + \beta_2 - c_3 + \alpha_3 - 0.5\,\beta_1 - 0.5\,\beta_2 - 0.5\,\alpha_3 - 1.5, \\
& \quad t_2^{'} - t_2^{''} = c_1 - \alpha_1 - 2c_2 - 2\,\beta_2 + c_3 - \alpha_3 + 0.5\,\alpha_1 + \beta_2 + 0.5\,\alpha_3 - 2.5, \\
& \quad s_2^{'} - s_2^{''} = c_1 + \beta_1 - 2c_2 + 2\,\alpha_2 + c_3 + \beta_3 - 0.5\,\beta_1 - \alpha_2 - 0.5\,\beta_3 - 3, \\
& \quad t_3^{'} - t_3^{''} = 2c_1 - 2\,\alpha_1 + c_2 - \alpha_2 + 3c_3 - 3\,\alpha_3 + 0.5\,\alpha_1 + \alpha_2 + 1.5\,\alpha_3 + 2, \\
& \quad s_3^{'} - s_3^{''} = 2c_1 + 2\,\beta_1 + c_2 + \beta_2 + 3c_3 + 3\,\beta_3 - 0.5\,\beta_1 - \beta_2 - 1.5\,\beta_3 + 1.5, \\
& \quad \alpha = (\alpha_1, \alpha_2, \alpha_3)^T > 0.0001e, \quad \beta = (\beta_1, \beta_2, \beta_3)^T > 0.0001e, \\
& \quad \alpha, \beta \in C, \\
& \quad t_i^{'} \ge 0,\ t_i^{''} \ge 0,\ s_i^{'} \ge 0,\ s_i^{''} \ge 0, \quad i = 1,2,3.
\end{aligned}$$

then, using the LINGO 11 software package, we can obtain

$$\tilde{x}_1 = (1.3077, 0, 0), \quad \tilde{x}_2 = (-1.3846, 0, 0.4999), \quad \tilde{x}_3 = (-1.0769, 0, 0.3332)$$

It is clear that $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ is a fuzzy number solution but in [22] the system (12) has not a fuzzy number solution.

5 Conclusion

The presented approach in this paper in fact is a model that appoints weights based on. In this paper, an approach for computing the compromised solution of an LR fuzzy linear system by use of a ranking function was proposed when the coefficient matrix is a crisp $m \times n$ matrix.

To this end, expected interval was used to find an LR fuzzy vector, $\tilde{X}$, such that the vector $\Re(A\tilde{X})$ has the least distance from $\Re(\tilde{b})$ in $1-$norm and the $1-$cut of $\tilde{X}$ satisfies the crisp linear system $AX = b$. Then, the constrained LP-problem was solved and optimal fuzzy number solution was obtained. Finally, For showing the application of this method we used two numerical examples.

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