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Bi-criteria Three Stage Fuzzy Flowshop Scheduling with Transportation Time and Job Block Criteria

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Abstract Scheduling is an enduring process where the existence of real time information frequently forces the review and modification of pre-established schedules. The real world is complex and complexity generally arises from uncertainty. From this prospective the concept of fuzziness is introduced in the field of scheduling. The present paper pertains to a bi-criterion in n-jobs, three machines flowshop scheduling to minimize the makespan and the rental cost of the machines taken on rent under a specified rental policy in which processing time, transportation time are in fuzzy environment and are represented by triangular fuzzy membership function. Further, the concept of job block to represent the relative importance of some jobs over other is also introduced. A heuristic algorithm to find optimal or near optimal sequence optimizing the bi-criteria is discussed. A numerical illustration is given to demonstrate the computational efficiency of proposed algorithm.

Keywords Flowshop Scheduling, Fuzzy Processing Time, Fuzzy Transportation Time, Equivalent Job, Rental Cost, Utilization Time.

1 Introduction

Multicriteria scheduling stems from the need to address real-world problems that often involve conflicting objective. A schedule that optimize one criteria may infact perform quite poorly for another. Decision makers must carefully evaluate the trade-offs involved in considering several criteria. Bicriteria scheduling problems are motivated by the fact that they are more meaningful from practical point of view. Over the last decades several theories such as fuzzy set theory, Probability theory, D – S theory and approaches based on certainity factors have been developed to account for uncertainity. Among them, fuzzy set theory is more and more frequently used in intelligent control because of its simplicity and similarity to human reasoning. Moreover, the fuzzy approach seems a natural extension of its crisp counterpart so that we need to know how the fuzziness of processing times and transportation times affects the job sequence itself. In most manufacturing systems, finished and semi-finished jobs are transferred from one machine to another for further processing. In most of the published literature explicitly or implicitly assumes that either there is an infinite number

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of jobs are transported instantaneously from one machine to another without transportation time involved. However, there are many situations where the transportation times are quite significant and cannot be simply neglected. Johnson (1954) whose work is one of the earliest, developed an algorithm to minimize the makespan in two stage flowshop scheduling problem. Dileepan and T.Sen [1988] extensively surveyed the bicriterion static scheduling research for a single machine. MacCahon and Lee [1990] discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee [1996] addressed the formulation of fuzzy flow shop scheduling problem with fuzzy processing time. Some of the noteworthy approaches are due to Zadeh (1965), Gupta J.N.D (1975), Maggu and Das (1977), Yager (1981), Marin and Roberto (2001), Yao and Lin (2002), Singh and Gupta (2005), Singh, Sunita and Allawalia (2008).

Gupta D. and Sharma S. [2011] studied bicriteria in n×3 flow shop scheduling under specified rental policy, processing time associated with probabilities including transportation time and job block criteria. As the fuzzy approach seems much more natural to us, we investigate its potential by solving the flowshop problem in real life situations. Here our study recommends the use of triangular fuzzy membership functions to represent the uncertain processing times and transportation times.

2 Practical Situation

Fuzzy set theory has emerged as a profitable tool for controlling and steering of systems and complex industrial processes, as well as for household and entertainment electronics. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. Medical science can save the patient's life but proper care leads to a faster recovery. Care giving techniques often require hi-tech, expensive medical equipment. Many of these equipments can even help in saving the life of critical patients. Most of these equipments are expensive & they are often needed for a few days or weeks thus buying them do not make much sense even if one can afford them. Many patients even lose their lives just because they cannot afford to buy these products. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allow up gradation to new technology. When the machines on which jobs are to be processed are planted at different places, the transportation time (which includes loading time, moving time and unloading time etc.) has a significant role in production concern.

3 Fuzzy Membership Function

All information contained in a fuzzy set is described by its membership function. The triangular membership functions are used to represent fuzzy processing times and fuzzy setup times in our algorithm. The membership value of the x denoted by $\mu_x, x \in R^+$, can be calculated according to the formula

$$\mu_x = \begin{cases} 0; x \le a \\ \frac{x-a}{b-a}; a \le x \le b \\ \frac{c-x}{c-b}; b < x < c \\ o; x \ge c \end{cases}$$

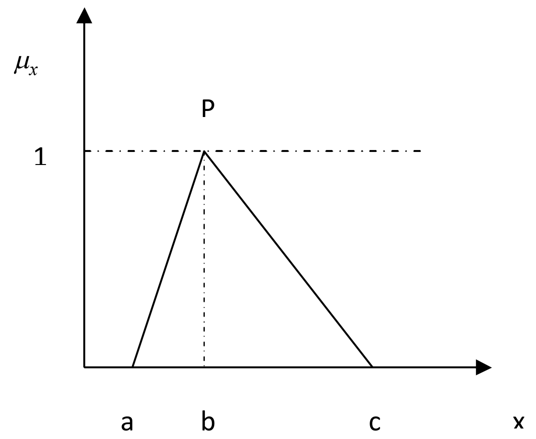

Fig. 1 Triangular membership function

Figure 1 shows the triangular membership function of a fuzzy set $\tilde{P}$, $\tilde{P}$=(a, b, c). The membership value reaches the highest point at '*b*', while '*a*' and '*c*' denote the lower bound and upper bound of the set $\tilde{P}$ respectively.

3.1 Average High Ranking (A.H.R.)

To find the optimal sequence, the expected processing time of the jobs are calculated by using Yager's(1981) average high ranking formula (AHR) = $h(A) = \frac{3b+c-a}{3}$.

3.2 Fuzzy Arithmetic Operations

The following are the four operations that can be performed on triangular fuzzy numbers:
Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ then

1.Addition: $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

2.Subtraction: $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

3.Multiplication: $A \times B = (\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$

4.Division: $A / B = (\min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), \max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3))$

A new operation for Subtraction on triangular fuzzy numbers
Let $A=(a_1,a_2,a_3)$ and $B=(b_1,b_2,b_3)$ then
$A-B=(a_1-b_1,a_2-b_2,a_3-b_3)$ .This subtraction operation exist only if the following condition is satisfied DP(A) ≥ DP(B), where $DP(A)=(a_3-a_1)/2$ and $DP(B)=(b_3-b_1)/2$
where DP denotes difference point of a triangular fuzzy number.

4 Notations & Various Definition's

- S : Sequence of jobs 1,2,3,….,n
- S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, -----
- M_j : Machine j, j = 1,2,3
- M : Minimum makespan
- a_{ij} : Fuzzy processing time of i^{th} job on machine M_j
- A_{ij} : AHR of processing time of i^{th} job on machine M_j
- $t_{i,j\to k}$: Fuzzy transportation time of i^{th} job from j^{th} machine to k^{th} machine
- $T_{i,j\to k}$: AHR of transportation time of i^{th} job from j^{th} machine to k^{th} machine
- β : Equivalent job for job – block
- C_i : Rental cost of i^{th} machine
- $L_j(S_k)$: The latest time when machine M_j is taken on rent for sequence S_k
- $t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
- $t'_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j when machine M_j start processing jobs at time $L_j(S_k)$
- $I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k
- $U_j(S_k)$:Utilization time for which machine M_j is required, when M_j starts processing jobs at time $L_j(S_k)$
- $R(S_k)$:Total rental cost for the sequence S_k of all machine
- $CT(S_i)$:Total completion time of the jobs for sequence S_i

4.1 Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

$t_{ij} = max\ (t_{i-1,j},\ t_{i,j-1}) + T_{i,(j-1)\to j} + a_{ij}$ for $j \geq 2$.

where

$a_{i,,j}$ = Fuzzy processing time of i^{th} job on j^{th} machine

4.2 Definition

Completion time of i^{th} job on machine M_j when M_j starts processing jobs at time L_j is denoted by t'_{ij} and is defined as

$$t'_{i,j} = L_j + \sum_{k=1}^{i} a_{k,j} = \sum_{k=1}^{i} I_{k,j} + \sum_{k=1}^{i} a_{k,j}$$

Also $t'_{i,j} = \max(t_{i,j-1}, t'_{i-1,j}) + a_{i,j}$.

4.3 Rental Policy (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on 1st machine and transported to 2nd machine, 3rd machine will be taken on rent at time when 1st job is completed on the 2nd machine and transported.

5 Problem Formulation & Assumptions

Let some job i (i = 1,2,……..,n) are to be processed on three machines M_j (j = 1,2,3) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine described by triangular fuzzy numbers. Let A_{ij} be the average high ranking (AHR) of processing time of i^{th} job on j^{th} machine. Let $t_{i,j\to k}$ be the fuzzy transportation time of i^{th} job from j^{th} machine to k^{th} machine. Let $T_{i,j\to k}$ be the AHR of transportation time of i^{th} job from j^{th} machine to k^{th} machine. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of all the three machines while minimizing total elapsed time.

Table 1 The mathematical model of the problem in matrix form

Jobs	Machine M_1		Machine M_2		Machine M_3
i	a_{i1}	$T_{i,1\to 2}$	a_{i2}	$T_{i,2\to 3}$	a_{i3}
1	a_{11}	$T_{1,1\to 2}$	a_{12}	$T_{1,2\to 3}$	a_{13}
2	a_{21}	$T_{2,1\to 2}$	a_{22}	$T_{2,2\to 3}$	a_{23}
3	a_{31}	$T_{3,1\to 2}$	a_{32}	$T_{3,2\to 3}$	a_{33}
4	a_{41}	$T_{4,1\to 2}$	a_{42}	$T_{4,2\to 3}$	a_{43}
-	-	-	-	-	-
n	a_{n1}	$T_{n,1\to 2}$	a_{n2}	$T_{n,2\to 3}$	a_{n3}

Minimize $U_j(S_k)$ and

Minimize $R(S_k) = \sum_{i=1}^{n} A_{i1}(S_k) \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing total elapsed time.

5.1 Assumptions

1. n jobs be processed through three machines M_1, M_2 & M_3 in the order $M_1M_2M_3$ i.e. no passing is allowed.
2. A sequence of k jobs $i_1, i_2 \ldots\ldots i_k$ as a block or group-job in the *order* ($i_1, i_2 \ldots\ldots i_k$) shows priority of job i_1 over i_2, etc.
3. Jobs may be held in inventory before going to a machine.
4. The storage space is available and the cost of holding inventory for each job is either same or negligible.
5. Time intervals for processing are independent of the order in which operations are performed.

6 Theorems

The various theorems have been given to find the latest time at which machines should be taken on rent so as to optimize the rental cost of the machines with minimum makespan.

6.1 Theorem

The processing of jobs on M_3 at time $L_3 = \sum_{i=1}^{n} I_{i,3}$ keeps $t_{n,3}$ unaltered.

Proof. Let $t'_{i,3}$ be the competition time of i^{th} job on machine M_3 when M_3 starts processing of jobs at time L_3. We shall prove the theorem with the help of Mathematical Induction.

Let $P(n): t'_{n,3} = t_{n,3}$

Basic Step: For n = 1

$$t'_{1,3} = L_3 + A_{1,3} = I_{1,3} + A_{1,3}$$
$$= (A_{1,1} + (T_{1,1\to 2} + A_{1,2}) + T_{1,2\to 3}) + A_{1,3} = t_{1,3}.$$

Therefore P(1) is true.

Induction Step: Let P (k) be true.

i.e. $t'_{k,3} = t_{k,3}$.

Now, we shall show that P(k+1) is also true.

i.e. $t'_{k+1,3} = t_{k+1,3}$

But $\quad t'_{k+1,3} = \max(t_{k+1,2}, t'_{k,3}) + T_{k+1,2\to 3} + A_{k+1,3}$ (As per Definition 2)

$$\therefore t'_{k+1,3} = \max(t_{k+1,2}, L_3 + \sum_{i=1}^{k} A_{i,3}) + T_{k+1,2\to 3} + A_{k+1,3}$$

$$= \max(t_{k+1,2}, \sum_{i=1}^{k+1} I_{1,3} + \sum_{i=1}^{k} A_{i,3}) + T_{k+1,2\to 3} + A_{k+1,3}$$
$$= \max(t_{k+1,2}, \sum_{i=1}^{k} I_{1,3} + \sum_{i=1}^{k} A_{i,3} + I_{k+1,3}) + T_{k+1,2\to 3} + A_{k+1,3}$$
$$= \max(t_{k+1,2}, t_{k,3} + I_{k+1,3}) + T_{k+1,2\to 3} + A_{k+1,3}$$
$$= \max(t_{k+1,2}, t'_{k,3} + I_{k+1,3}) + T_{k+1,2\to 3} + A_{k+1,3} \qquad \text{(by assumption)}$$
$$= \max\left(t_{k+1,2}, t'_{k,3} + \max\left(\left(t_{k+1,2} - t_{k,3}\right), 0\right)\right) + T_{k+1,2\to 3} + A_{k+1,3}$$
$$= \max\left(t_{k+1,2}, t_{k,3}\right) + T_{k+1,2\to 3} + A_{k+1,3}$$
$$= t_{k+1,3}$$

$\Rightarrow$ *P(k+1)* is true .

Hence by principle of mathematical induction P(n) is true for all n, .i.e. $t'_{n,3} = t_{n,3}$.

Remarks.

If M_3 starts processing jobs for minimum $L_3(S_r) = t_{n3}(S_r) - \sum_{i=1}^{n} A_{i3}$ then the total elapsed time $L_3(S_r) = t_{n3}(S_r) - \sum_{i=1}^{n} A_{i3}$ is not altered and M_3 is engaged for minimum time equal to sum of the processing times of all the jobs on M_3. Also, if M_3 starts processing jobs at time L_3, then it can be easily shown that

$$t_{n,3} = L_3 + \sum_{i=1}^{n} A_{i,3}.$$

Lemma. If M_3 starts processing jobs at $L_3 = \sum_{i=1}^{n} I_{i,3}$ then

(i). $L_3 > t_{1,2}$

(ii). $t'_{k+1,3} \geq t_{k,2}$, $k > 1$.

6.2 Theorem

The processing of jobs on M_2 at time $L_2 = \min_{i\leq k\leq n}\{Y_k\}$ keeps total elapsed time unaltered where $Y_1 = L_3 - A_{1,2} - T_{1,2\to 3}$ and $Y_k = t'_{k-1,3} - \sum_{i=1}^{k} A_{i,2} - \sum_{i=1}^{k} T_{i,2\to 3}; k > 1.$

Proof. We have

$$L_2 = \min_{i\leq k\leq n}\{Y_k\} = Y_r \text{ (say)}$$

In particular for k =1

$$Y_r \leq Y_1$$
$$\Rightarrow Y_r + A_{1,2} + T_{1,2\to 3} \leq Y_1 + A_{1,2} + T_{1,2\to 3}$$
$$\Rightarrow Y_r + A_{1,2} + T_{1,2\to 3} \leq L_3 \qquad (1)$$
$$\left(\because Y_1 = L_3 - A_{1,2} - T_{1,2\to 3}\right)$$

By Lemma 1; we have

$$t_{1,2} \le L_3 \tag{2}$$

Also, $t'_{1,2} = \max\left(Y_r + A_{1,2} + T_{1,2\to 3}, t_{1,2}\right)$

On combining, we get

$$t'_{1,2} \le L_3$$

For k >1, As $Y_r = \min_{i \le k \le n}\{Y_k\}$

$$\Rightarrow Y_r \le Y_k; \qquad k = 2,3\ldots\ldots\ldots,n$$

$$\Rightarrow Y_r + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to 3} \le Y_k + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to 3}$$

$$\Rightarrow Y_r + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to 3} \le t'_{k-1,3} \tag{3}$$

By Lemma 1; we have

$$t_{k,2} \le t'_{k-1,3} \tag{4}$$

Also, $t'_{k,2} = \max\left(Y_r + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to 3}, t_{k,2}\right)$

Using (3) and (4) , we get

$$t'_{k,2} \le t'_{k-1,3}$$

Taking $k = n$, we have

$$t'_{n,2} \le t'_{n-1,3} \tag{5}$$

Total time elapsed = $t_{n,3}$

$= \max\left(t'_{n,2}, t'_{n-1,3}\right) + A_{n,3} + T_{n,2\to 3}$

$= t'_{n-1,3} + A_{n,3} + T_{n,2\to 3}$ (using 5)

$= t'_{n,3}$.

Hence, the total time elapsed remains unaltered if M_2 starts processing jobs at time $L_2 = \min_{i \le k \le n}\{Y_k\}$.

6.3 Theorem

The processing time of jobs on M_2 at time $L_2 > \min_{i \le k \le n}\{Y_k\}$ increase the total time elapsed, where $Y_1 = L_3 - A_{1,2} - T_{1,2\to3}$ and $Y_k = t'_{k-1,3} - \sum_{i=1}^{k} A_{i,2} - \sum_{i=1}^{k} T_{i,2\to3}; k > 1$.

The proof of the theorem can be obtained on the same lines as of the previous Theorem 2.

By Theorem 1, if M_3 starts processing jobs at time $L_3 = t_{n,3} - \sum_{i=1}^{n} A_{i,3}$ then the total elapsed time $t_{n,3}$ is not altered and M_3 is engaged for minimum time equal to sum of processing times of all the jobs on M_3, .i.e. reducing the idle time of M_3 to zero. Moreover total elapsed time/rental cost of M_1 is always least as idle time of M_1 is always zero. Therefore the objective remains to minimize the elapsed time and hence the rental cost of M_2.

The following algorithm provides the procedure to determine the times at which machines should be taken on rent to minimize the total rental cost without altering the total elapsed time in three machine flow shop problem under rental policy (P).

7 Algorithm

Step 1. Find the average high ranking A_{ij}, $T_{i,j\to k}$ of the processing times and transportation time respectively for all the jobs on three machines M_1, M_2 and M_3.

Step 2. Check the condition

Either $Min\{A_{i1} + T_{i,1\to2}\} \geq Max\{A_{i2} + T_{i,1\to2}\}$

or $Min\{A_{i3} + T_{i,2\to3}\} \geq Max\{A_{i2} + T_{i,2\to3}\}$ or both for all i

If the conditions are satisfied then go to step 3, else the data is not in the standard form.

Step 3. Introduce the two fictitious machines G and H with processing times G_i and H_i as

$$G_i = A_{i1} + T_{i,1\to2} + A_{i2} + T_{i,2\to3},\quad H_i = A_{i2} + T_{i,1\to2} + A_{i3} + T_{i,2\to3}$$

Step 4. Find the expected processing time of job block $\beta = (k,m)$ on fictitious machines G & H using equivalent job block criterion given by Maggu & Das [1977]. Find G_β and H_β using

$$G_\beta = G_k + G_m - min\,(G_m\,,\, H_k),\quad H_\beta = H_k + H_m - min\,(G_m\,,\, H_k)$$

Step 5. Define new reduced problem with processing time G_i & H_i as defined in step 3 and replace job block (k,m) by a single equivalent job β with processing times G_β & H_β as defined in step 4.

Step 6. Using Johnson's procedure, obtain all sequences S_k having minimum elapsed time. Let these be $S_1, S_2, \ldots, S_r$

Step 7. Prepare In – Out tables for S_k and compute total elapsed time $t_{n3}(S_k)$

Step 8. Compute latest time L_3 for machine M_3 for sequence S_k as $L_3(S_k) = t_{n3}(S_k) - \sum_{i=1}^{n} A_{i3}$

Step 9. For the sequence S_k (k = 1,2,……...,r), compute

I. $t_{n2}(S_k)$

II. $Y_1(S_k) = L_3(S_k) - A_{1,2}(S_k) - T_{1,2\to3}$

III. $Y_q(S_k) = L_3(S_k) - \sum_{i=1}^{q} A_{i2}(S_k) - \sum_{i=1}^{q} T_{i,2\to3} + \sum_{i=1}^{q-1} A_{i,3} + \sum_{i=1}^{q-1} T_{i,1\to2};\ q = 2,3,\ldots\ldots,n$

IV. $L_2(S_k) = \min_{1\le q\le n}\left\{Y_q(S_k)\right\}$

V. $U_2(S_k) = t_{n2}(S_k) - L_2(S_k)$

Step 10. Find min $\{U_2(S_k)\}; k = 1,2,..........,r$

Let it be for the sequence S_p and then sequence S_p will be the optimal sequence.

Step 11. Compute total rental cost of all the three machines for sequence S_p as:

$$R(S_p) = \sum_{i=1}^{n} a_{i1}\times C_1 + U_2(S_p)\times C_2 + U_3(S_p)\times C_3.$$

8 Numerical Illustration

Consider 5 jobs, 3 machine flow shop problem with processing time and transportation time described by triangular fuzzy numbers as given in table and jobs 2 and 4 are processed as group job (2,4). The rental cost per unit time for machines M_1, M_2 and M_3 are 4 units, 2 units and 3 units respectively, under the rental policy P. Our objective is to obtain an optimal schedule to minimize the total rental cost of machines.

Table 2 The machines with processing time and transportation time

Jobs	Machine M_1	$T_{i,1\to2}$	Machine M_2	$T_{i,2\to3}$	Machine M_3
i	a_{i1}		a_{i2}		a_{i3}
1	(7,8,9)	(2,3,4)	(6,7,8)	(1,2,3)	(3,4,5)
2	(12,13,14)	(4,5,6)	(5,6,7)	(2,3,4)	(4,5,6)
3	(8,10,12)	(5,6,7)	(4,5,6)	(3,4,5)	(6,7,8)
4	(10,11,12)	(2,3,4)	(5,6,7)	(1,2,3)	(11,12,13)
5	(9,10,11)	(5,6,7)	(6,7,8)	(3,4,5)	(8,9,10)

Solution: As per step 1: The A.H.R of processing time and transportation time of jobs is as follows:

Table 3 Machines with AHR processing time and transportation time

Jobs	Machine M_1	$T_{i,1\to2}$	Machine M_2	$T_{i,2\to3}$	Machine M_3
i	A_{i1}		A_{i2}		A_{i3}
1	26/3	11/3	23/3	8/3	14/3
2	41/3	17/3	20/3	11/3	17/3
3	34/3	20/3	17/3	14/3	23/3
4	35/3	11/3	20/3	8/3	38/3
5	32/3	20/3	23/3	14/3	29/3

As per step 3: The expected processing time for two fictitious machine *G* & *H* is as shown in table 4.

Table 4 Two fictitious machines *G* & *H*

Jobs	G_i	H_i
1	68/3	56/3
2	89/3	65/3
3	85/3	74/3
4	74/3	77/3
5	89/3	86/3

As per step 4: Here β = (2,4)

$$G_\beta = 89/3 + 74/3 - 65/3 = 98/3$$

$$H_\beta = 65/3 + 77/3 - 65/3 = 77/3$$

As per step 5: The reduced problem is

Table 5 Reduced problem with fictitious machines *G* & *H*

Jobs	G_i	H_i
1	68/3	56/3
β	98/3	77/3
3	85/3	74/3
5	89/3	86/3

As per step 6: Using Johnson's method optimal sequence is

S = 5 – β – 3 – 1 i.e. 5 – 2 – 4 – 3 – 1

As per step 7: The In – Out table for the sequence S is

Table 6 In-Out flow table

Jobs	Machine M_1	$T_{i,1\to 2}$	Machine M_2	$T_{i,2\to 3}$	Machine M_3
i	In - Out		In - Out		In - Out
5	(0,0,0) – (9,10,11)	(5,6,7)	(14,16,18) – (20,23,26)	(3,4,5)	(23,27,31) – (31,36,41)
2	(9,10,11) – (21,23,25)	(4,5,6)	(25,28,31) – (30,34,38)	(2,3,4)	(32,37,42) – (36,42,48)
4	(21,23,25) – (31,34,37)	(2,3,4)	(33,37,41) – (38,43,48)	(1,2,3)	(39,45,51) – (50,57,64)
3	(31,34,37) – (39,44,49)	(5,6,7)	(44,50,56) – (48,55,62)	(3,4,5)	(51,59,67) – (57,66,75)
1	(39,44,49) – (46,52,58)	(2,3,4)	(48,55,62) – (54,62,70)	(1,2,3)	(57,66,75) – (60,70,80)

(Tableau 6)

Total elapsed time $t_{n,3}(S) = (60,70,80)$

As per Step 8: $L_3(S) = t_{n3}(S) - \sum_{i=1}^{n} a_{i,3}$

$= (60,70,80) - (32,37,42) = (28,33,38)$

As per Step 9: For sequence S, we have

$t_{n2}(S) = (54,62,70)$

$Y_1(S) = (19,22,25), Y'_1(S) = 24$

$Y_2(S) = (25,28,31), Y'_2(S) = 30$

$Y_3(S) = (27,30,33), Y'_3(S) = 32$

$Y_4(S) = (33,36,39), Y'_4(S) = 38$

$Y_5(S) = (37,40,43), Y'_5(S) = 42$

$L'_2(S) = Min\{Y'_k\} = 24$

where $Y'_k =$ A.H.R of Y_k, $L'_2(S) =$ A.H.R of $L_2(S)$

$U_2(S) = t_{n2}(S) - L_2(S)$

$= (35,40,45)$

The new reduced Bi – objective In – Out table is

Table 7 The Bi-objective In-Out flow table

Jobs	Machine M_1	$T_{i,1\to 2}$	Machine M_2	$T_{i,2\to 3}$	Machine M_3
i	In - Out		In - Out		In - Out
5	(0,0,0) – (9,10,11)	(5,6,7)	(19,22,25) – (25,29,33)	(3,4,5)	(28,33,38) – (36,42,48)
2	(9,10,11) – (21,23,25)	(4,5,6)	(25,29,33) – (30,35,40)	(2,3,4)	(36,42,48) – (40,47,54)
4	(21,23,25) – (31,34,37)	(2,3,4)	(33,37,41) – (38,43,48)	(1,2,3)	(40,47,54) – (51,59,67)
3	(31,34,37) – (39,44,49)	(5,6,7)	(44,50,56) – (48,55,62)	(3,4,5)	(51,59,67) – (57,66,75)
1	(39,44,49) – (46,52,58)	(2,3,4)	(48,55,62) – (54,62,70)	(1,2,3)	(57,66,75) – (60,70,80)

The latest possible time at which machine M_2 should be taken on rent $= L_2(S) = (19,22,25)$

Also, utilization time of machine $M_2 = U_2(S) = (35,40,45)$

Total minimum rental cost = $R(S) = \sum_{i=1}^{n} a_{i1}(S) \times C_1 + U_2(S) \times C_2 + U_3(S) \times C_3$

$= (350,399,448)$

9 Conclusions

If machine M_3 starts processing the jobs at latest time $L_3 = t_{n3} - \sum_{i=1}^{n} a_{i3}$, then the total elapsed time t_{n3} is not altered and M_3 is engaged for minimum time equal to sum of processing of all the jobs on M_3, i.e. reducing the idle time of M_3 to zero. If the machine M_2 is taken on rent when it is required and is returned as soon as it completes the last job, the starting of processing of jobs at the latest time $L_2(S_k) = \min_{1 \le q \le n} \{Y_q(S_k)\}$ on M_2 will , reduce the idle time of all jobs on it. Therefore, the utilization time and hence total rental cost of machine M_2 will be minimum. Also the rental cost of M_1 will always be minimum as the idle time of machine M_1 is always zero.

References

1. Johnson, S. M., (1954). Optimal two and three stage production schedule with set up times included. Naval Research Logistics Quart, 1(1), 61-68.
2. Zadeh, L. A., (1965). Fuzzy sets. Information and control, 8, 338-353.
3. Smith, R. D., Dudek, R. A., (1967). A general algorithm for solution of the N-job, M-machine scheduling problem. Operations Research, 15(1), 71-82.
4. Gupta, J. N. D., (1975). Optimal Schedule for specially structured flow shop. Naval Research Logistic, 22(2), 255-269.
5. Maggu, P. L., Das, G., (1977). Equivalent jobs for job block in job scheduling. Opsearch, 14(4), 277-281.
6. Dileepan, P., Sen, T., (1988). Bicriteria static scheduling research for a single machine. OMEGA, 16, 53-59.
7. Van, L. N., Wassenhove & Gelders, L. F., (1980). Solving a bicriteria scheduling problem. AIIE Tran, 15s., 84-88.
8. Yager, R. R., (1981). A procedure for ordering fuzzy subsets of the unit interval. Information Sciences, 24, 143-161.
9. McCahon, S., Lee, E. S., (1990). Job sequencing with fuzzy processing times. Computer and mathematics with applications, 19(7), 31-41.
10. Ishibuchi, H., Lee, K. H., (1996). Formulation of fuzzy flow shop scheduling problem with fuzzy processing time. Proceeding of IEEE international conference on Fuzzy system, 199-205.
11. Martin, L., Roberto, T., (2001). Fuzzy scheduling with application to real time system. Fuzzy sets and Systems, 121(3), 523-535.
12. Yao, J. S., Lin, F. T., (2002). Constructing a fuzzy flowshop sequencing model based on statistical data. International Journal of Appropriate Reasoning, 29(3), 215-234.
13. Singh, T. P, Gupta, D., (2005). Minimizing rental cost in two stage flow shop, the processing time associated with probabilies including job block. Reflections de ERA, 1(2), 107-120.
14. Gupta, D., Singh, T. P., Kumar, R., (2007). Bicriteria in scheduling under specified rental policy, processing time associated with probabilities including job block concept. Proceedings of VIII Annual Conference of Indian Society of Information Theory and Application (ISITA), 22-28.
15. Singh, T. P., Sunita and Allawalia, P., (2008). Reformation of non fuzzy scheduling using the concept of fuzzy processing time under blocking. International Conference on intelligence system & Networks, 322-324.
16. Gupta, D., Shefali and Sharma, S., (2012). A Fuzzy logic based Approach to Minimize Rental Cost of Machines for Specially Structured Three Stages Flow Shop Scheduling. Advances in Applied Science Research, 3(2), 1071-1076
17. Gupta, D., Sharma, S. and Seema, (2011). Bicriteria in n×3 flowshop scheduling under specified rental policy, processing time associated with probabilities. Industrial Engineering Letters, 1(2).
18. Gupta, D., Sharma, S. and Shefali, (2012). A Fuzzy logic based approach for bicriteria in n-jobs, 2 machine flow shop production scheduling. International Journal of Mathematical Archieve, 3(9), 3437–3444.