Multi-Objective Mathematical Model as a Decision Support for Customer Service Marketing

R. Esmaeilpour, H. Fazlollahtabar, E. Aghasi*

Received: 13 September 2012; Accepted: 17 January 2013

Abstract In this paper we propose a multi-objective mathematical model to aid the marketing team of a company in customer service marketing. Customer reflects to the services provided by a company, and the reflections affect the profit of the company. Thus, the services can be evaluated by the customers to imply the company’s performances. First, the services are purified based on the opinions of the customers conducting a survey study by a questionnaire. The service purification is carried out using statistical hypothesis testing. The remained services are then assessed regarding time, cost and quality objectives constructing a multi-objective mathematical model. Then, a multi-objective mathematical model is utilized to determine the services with more profits. Analytic hierarchy process (AHP) is applied to solve the multi-objective model. The applicability and validity of the proposed mechanism is illustrated in a case study.

Keywords Decision Support, Multi-Objective Mathematical Model, Analytic Hierarchy Process (AHP).

1 Introduction

Marketing has received extensive attention from both managers and experts in recent years. From a managerial viewpoint, top management increasingly calls for “marketing accountability” pressuring marketers to produce metrics that document marketing activities [1]. From an academic perspective, the growing interest in marketing metrics can be attributable to five theoretical angles [2]. First, according to control theory suggesting the need for the past information on marketing programs as an essential segment of the cycle of analysis, planning, implementation and control [3,4], marketing metrics were utilized to evaluate past performance to improve future strategy and execution.[5] Second, with respect to agency theory focusing on contract between a principle and an agent and the need for past data on the extent to which the principal’s objectives have been met [6], marketing metrics could be used to facilitate the contract between corporate and marketing management[7]. Third, reinforcing the broader quest for a balanced scorecard of [8] which puts emphasis on such

* Corresponding Author. (✉) E-mail: Ermia.aghasi@gmail.com (E. Aghasi)

R. Esmaeilpour
Assistant Professor, Department of Executive Management, Faculty of Human Sciences, University of Guilan, Rasht, Iran

H. Fazlollahtabar
PhD Student, Faculty of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

E. Aghasi
MSc Student, Department of Executive Management, Faculty of Human Sciences, University of Guilan, Rasht, Iran
intangible assets as brand equity that account for a large and increasing proportion of shareholder value, marketing metrics are used to measure its various dimensions. Fourth, consistent with the literature on market orientation [9,10] that argues for the need of market sensing and appropriate cross-functional responsiveness to the resulting data, marketing metrics are part of ‘marketing sensing’. Finally, as marketing metrics become more widespread among firms, institutional theory [11] suggests that their use will become an institutional norm [12]. Operationally, the present study focuses on how firms use marketing metrics as tools for customer relationship management (CRM).

Gitomer [13] points out that the most important factor affecting the success of CRM implementations is top management’s participation. Further, Fitzgerald and Brown [14] suggest that the implementation of CRM needs to be managed by “executive committees” rather than a single executive [15]. Although researchers have proposed that CEO involvement is critical to CRM implementation, they haven’t provided a recommended way for management to help the CRM implementation [15,16]. Thus, the purpose of this article is to identify some success factors contributing to CRM implementation. The results from the present study could provide some recommended ways for executives to participate and support their CRM implementation projects [17].

The dynamism was categorized into three subcategories: incompleteness, imprecision, and uncertainty [18]. Incomplete information is that a value is missing. Imprecise information is that we have a value for the variable but not with the required precision. Uncertainty, instead, is a form of dynamism appearing when the observer is taken into account [19]. It means that the observer gives complete and precise information, but is unreliable itself. For information and references on approaches dealing with dynamism, see Stewart. In some models, the decision makers (DMs) did not want to reveal their preference model, and therefore exact parameter values could not be obtained in others, the alternatives had uncertain or imprecise values for criteria measurements. Therefore, new advances seem necessary to preserve the usefulness of the approach.

The development of marketing decision support systems (MDSS) inaugurated a new era in marketing activities [20,21]. They are especially designed to allow marketing managers to benefit from advances in marketing theories underlying models [22,23]. As marketing models are created, MDSS makes them easy to function. For example, Lilien and Rangaswamy [24] present a set of marketing models embedded in software. Yet, Little [25] stated that “marketing managers in companies are not so eager to use marketing models”. Many authors still agree on this assertion [26,27]. Several studies have examined the effectiveness of MDSS models. In a field experiment, Fudge and Lodish [28] considered that salespersons using CALLPLAN outperform their counterparts who did not use it. In contrast, Chakravarti et al. [11] showed that ADBUDG, Little's original decision calculus model does not help the users to make better decisions and is even worse than decisions based on intuition. However, McIntyre [29] showed that using CALLPLAN in an experimental setting improves decision-makers' performance, at least for problems involving constrained budget allocations in simple and stationary environments. In a marketing strategy game, Van Bruggen et al. [30] findings reveal that the availability and the quality of the MDSS improve decision-makers' performance with no negative effect on user confidence, whatever the level of time pressure. Van Bruggen et al. [30] showed that decision-makers using MDSS are better able to set the values of decision variables to increase performance. Yet, Barr and Sharda [31] proposed two potential explanations of the performance improvement: reliance and development effects. The former effect suggests that DSS usage leads to deferring the decision process to “let the computer do it” whilst the latter refers to an increased understanding of relationships between
relevant variables (i.e., the decision model). The question arises as to whether decision-makers are willing to use tools that do not lead them to better evaluate their own decision making. Similarly, Eisenstein and Lodish [32] mention that the use of a model that users do not understand might affect the likelihood of adoption and usage. Consequently, we propose to enhance the transparency of MDSS. Indeed, some studies look into MDSS characteristics. Van Bruggen et al. [33] studied the impact of MDSS quality on managers' performance. Yet, researchers in the DSS field [34] believe that DSS specific parameters may influence the understanding of decisions that managers gain from using systems [35].

The remainder of our work is organized as follows. Next, we define the problem. Section 3 models the problem in two stages. Section 4 proposes a weighing method to integrate the objectives for the proposed mathematical model. The efficiency and validity of the proposed model is illustrated in a case study in Section 5. We conclude in Section 6.

2 The proposed problem

Here, a company is considered providing several services to customers. The services are assessable depending on customers' satisfaction. Customers can express their views about the services. It is significant for the marketing team of the company to evaluate the customers' opinions on the received services to analyze them for obtaining the maximum profit. Therefore, consider $s_1, \ldots, s_n$, for $j=1, \ldots, n$, as services. We propose a mechanism to assess customers' satisfaction about the received services. The aim is to purify and determine the services providing the maximum profit for the company. All the services should be analyzed with respect to the customers’ opinion. Thus, a questionnaire is conducted considering the statistical population and sample. Then, using the corresponding hypothesis tests and some analysis some of the services are chosen by the customers. Next, the obtained services by the aid of marketing team are being used as inputs of the decision support. The proposed decision support is mathematical-based. That is, three objective functions considering their related constraints are optimized in the decision support and the results are reported to the management. The objectives are minimizing the service cost, minimizing the service delivery time and maximizing the delivered service quality. To optimize the proposed multi-objective model, AHP is employed. The flowchart of our mechanism is shown in Fig. 1.
The significances of this research include the followings:
- Providing a methodology as a decision aid for company's service marketing team,
- Considering and including customers' opinions to improve serviceability of the company,
- Considering multiple objectives for better service provision,
- Obtaining the services provide the maximum profit to the company.

3 Modeling the problem
3.1 Survey study

Here, we determine the population to be studied and also construct the questionnaire. The questions of the questionnaire are the services and the answers ranging from very good to very bad (5 items likert spectrum). The questions are also the hypothesis to be tested by statistical tests. If a hypothesis is rejected then the corresponding service in realized to be not important in customers’ viewpoint. The output is inserted to the mathematical models given next.
3.2 Multi-objective optimization

This way, a purification is performed on the services as alternatives and those attract customers' satisfaction more are determined. Here, the marketing team explores the more beneficial services optimizing the objectives of the company. The objectives are minimizing the service cost, minimizing the service delivery time and maximizing the delivered service quality.

Cost of service is the cost of providing a service. It can also be used as an adjective (cost-of-service or simply COS) to denote rate structures, analysis and expenses among other things. Cost-of-service pricing is the setting of a price for a service based on the costs incurred in providing it. COS pricing can be applied to an individual customer based on the costs of serving that customer, or as an average cost of service for a group of similar customers. Since we want to minimize the service cost, therefore we consider,

Mathematical notations:

\[ j \] Counter for services; \[ j=1,2,...,n. \]
\[ e_j \] Amount of investment for the \( j^{th} \) service.
\[ s_j \] 1, if service \( j \) is selected, 0, otherwise.
\[ F_j \] Service transformation function of \( s_1,...,s_n \).
\[ b \] Fixed cost

Then, the mathematical model for cost is:

\[
\text{Min} \quad Z_1 = \sum_{j=1}^{n} s_j \cdot e_j + b. \quad (1)
\]

\[
\text{s.t.,} \quad s^0 = F_i(s_1,...,s_n). \quad (2)
\]

where \( F_i(s_1,...,s_n) \) is a mathematical function for relating the \( s_j \) and \( s^0 \) is an iso-service level being the locus of the services having same service level.

This model is a constrained nonlinear one. To solve the cost minimization problem, we set the Lagrangian integrated function \( L \) as equation (3) and set the partial derivatives equations (4) and (5) equal to zero (first order conditions):

\[
L = \sum_{j=1}^{n} s_j \cdot e_j + b + \lambda \left[ s^0 - F_i(s_1,...,s_n) \right], \quad (3)
\]

\[
\frac{\partial L}{\partial s_j} = e_j - \lambda . f_j = 0, \quad (4)
\]

\[
\frac{\partial L}{\partial \lambda} = s^0 - F_i(s_1,...,s_n) = 0. \quad (5)
\]

where \( f_j \) is the marginal cost for each service, \( f_j = \frac{\partial F_i}{\partial s_j} \).
To investigate whether the obtained service is optimal or not, we check the second order conditions. The second order conditions for the minimization of cost require that the relevant Hessian of $L$ be positive semi-definite (if the Hessian of $L$ is not positive semi-definite, then we are sure that the obtained service is not a minimizer). Here, the Hessian of $L$ is:

$$
\nabla^2 L = \frac{\partial^2 L}{\partial s_i \partial s_j},
$$

(6)

To be positive semi-definite, the eigenvalues of $\nabla^2 L$, the $\lambda_i$, must satisfy:

$$
\lambda_i \geq 0, \quad \forall i,
$$

(7)

If the solution point is so that $\nabla^2 L$ is positive definite, that is,

$$
\lambda_i > 0, \quad \forall i,
$$

(8)

then the point is a local minimizer of $L$.

**Service delivery time** is the length of time between the preparation of a service and the delivery of the service to the end consumer. It is also sometimes referred to as the delivery period. Companies keep track of their delivery times for the purpose of being able to provide accurate estimates when orders are placed so that consumers know when to expect a delivery. This tracking is also used internally to monitor efficiency. When customers place an order, they are usually provided with information about the estimated delivery time. Here, we make use of earliness and tardiness being penalized for the service delivery time minimization. We consider,

Mathematical notations:

- $\alpha$: weight for total earliness; $\alpha \geq 0$
- $\beta$: weight for total tardiness; $\beta \geq 0$
- $E_j$: earliness of service $j$; $j = 1,2,...,n$
- $T_j$: tardiness of service $j$; $j = 1,2,...,n$
- $P_j$: Processing time of service $j$; $j = 1,2,...,n$
- $C_j$: completion time for service $j$; $j = 1,2,...,n$
- $d_j$: due date of service $j$; $j = 1,2,...,n$

Objective function:

$$
\text{Min} \quad Z_2 = \sum_{j=1}^{n} \alpha \cdot E_j \cdot s_j + \sum_{j=1}^{n} \beta \cdot T_j \cdot s_j
$$

(9)

s.t.

$$
\alpha = \left| P_j - d_j \right|^2, \quad j = 1,...,n,
$$

(10)

$$
\beta = \left| P_j - C_j \right|^2, \quad j = 1,...,n,
$$

(11)

$$
T_j = \max \left\{ 0, C_j - d_j \right\}, \quad j = 1,...,n,
$$

(12)

$$
E_j = \max \left\{ 0, d_j - C_j \right\}, \quad j = 1,...,n,
$$

(13)

Since formulae (12) and (13) are nonlinear, we linearize them as follows:
Service quality involves a comparison of expectations with performance. Service quality is a measure of how well a delivered service matches the customers' expectations. Generally the customer is requesting a service at the service interface where the service encounter is being realized, and then the service is being provided by the provider and in the same time delivered to or consumed by the customer. The main reason to focus on quality is to meet customer needs while remaining economically competitive in the same time. This means satisfying customer needs is very important for the enterprises survive. The outcome of using quality practices is:

- Understanding and improving of operational processes
- Identifying problems quickly and systematically
- Establishing valid and reliable service performance measures
- Measuring customer satisfaction and other performance outcomes

To control the quality of various services, the concept of six sigma is considered as service weight. The six sigma approach is one of the most widely known best practices in providing a tolerance for a parameter. The concept of six sigma originates from statistical terminology, wherein sigma (σ) represents standard deviation. In the recent years, a few researchers have focused on the application of six sigma methodology in balancing process.

This approach assumes that the ideal value of the process mean is between specification intervals, i.e., $\varphi^l < \varphi < \varphi^u$ (with $1.5\sigma$ shift from the mean). It is implied that six sigma concept with a $1.5\sigma$ shift from the mean holds and the probability of conformance can be shown to be $0.9999966$ (or $3.44$ppm). The level of assurance is targeted, but the terminology is also used to evaluate current level of $\varphi$ with the following sigma level (Table 1).

<table>
<thead>
<tr>
<th>Sigma Level (SL)</th>
<th>PPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $\sigma$</td>
<td>691,462</td>
</tr>
<tr>
<td>2 $\sigma$</td>
<td>308,538</td>
</tr>
<tr>
<td>3 $\sigma$</td>
<td>66,807</td>
</tr>
<tr>
<td>4 $\sigma$</td>
<td>6,210</td>
</tr>
<tr>
<td>5 $\sigma$</td>
<td>233</td>
</tr>
<tr>
<td>6 $\sigma$</td>
<td>3.44</td>
</tr>
</tbody>
</table>

Analysis of the six sigma approach makes use of process capability indices $C_p$ and $C_{pk}$. Process capability index $C_p$ is defined as,
\[ C_p = \frac{\phi^U - \phi^L}{SL}, \]  \tag{18}

The difference \( \phi^U - \phi^L \) represents specification width. When \( C_p = 2 \) and service presence mean is centered at \( \frac{\phi^U - \phi^L}{2} \) without any shift, then the probability of conformance is 99.9999998%.

Here, we make use of six sigma concept and equation (18) to control the variation of the services offered by the company. Thus, the simplified version of equation (18) is,

\[ \rho_j = C_p = \frac{\phi_{s_j}^U - \phi_{s_j}^L}{SL}. \]  \tag{19}

Therefore, as we explore maximizing the delivered service quality and considering process capability index as a weight coefficient for any service, we obtain,

\[
\text{Max} \quad Z_3 = \sum_{j=1}^{n} s_j \cdot \rho_j. \tag{20}
\]

The complete multi-objective mathematical model is presented below:

\[
\begin{align*}
\text{Min} \quad & Z_1 = \sum_{j=1}^{n} s_j \cdot e_j + b. \tag{21} \\
\text{Min} \quad & Z_2 = \sum_{j=1}^{n} \alpha \cdot E_j \cdot s_j + \sum_{j=1}^{n} \beta \cdot T_j \cdot s_j. \tag{22} \\
\text{Max} \quad & Z_3 = \sum_{j=1}^{n} s_j \cdot \rho_j. \tag{23}
\end{align*}
\]

\[
\text{s.t.}
\begin{align*}
s^0 & = F_j(s_1, \ldots, s_n), & j = 1, \ldots, n, \tag{24} \\
\alpha & = |P_j - d_j|^2, & j = 1, \ldots, n, \tag{25} \\
\beta & = |P_j - C_j|^2, & j = 1, \ldots, n, \tag{26} \\
E_j & \geq 0, & j = 1, \ldots, n, \tag{27} \\
E_j & \geq d_j - C_j, & j = 1, \ldots, n, \tag{28} \\
T_j & \geq 0, & j = 1, \ldots, n, \tag{29} \\
T_j & \geq C_j - d_j, & j = 1, \ldots, n, \tag{30} \\
\rho_j & = C_p = \frac{\phi_{s_j}^U - \phi_{s_j}^L}{SL}, & j = 1, \ldots, n, \tag{31} \\
s_j & \in \{0, 1\}, & j = 1, \ldots, n. \tag{32}
\end{align*}
\]

To solve the proposed multi-objective model, AHP is employed to integrate the objectives.
4 Weighting the objectives

To weight the objectives, we take a multi-criteria decision-making approach. Multi-criteria decision-making (MCDM), dealing primarily with problems of evaluation or selection [35], is a rapidly developing area in operations research and management science. AHP, developed by Saaty [35], is a technique of considering data or information for a decision in a systematic manner [36]. It is mainly concerned with a way of solving decision problems with uncertainties in multiple-criteria characterization. It is based on three principles: constructing the hierarchy, priority setting, and logical consistency. We apply AHP to weight the objectives.

Construction of the hierarchy
A complicated decision problem, composed of various attributes of an objective, is structured and decomposed into sub-problems (sub-objectives, criteria, alternatives, etc.), within a hierarchy.

Priority setting
The relative “priority” given to each element in the hierarchy is determined by pair-wise comparisons of the contributions of elements at a lower level in terms of the criteria (or elements) with a causal relationship. In AHP, multiple paired comparisons are based on a standardized comparison scale of nine levels (see table 2, from Saaty, [33]).

Table 2 Scale of relative importance

<table>
<thead>
<tr>
<th>Intensity of importance</th>
<th>Definition of importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal</td>
</tr>
<tr>
<td>2</td>
<td>Weak</td>
</tr>
<tr>
<td>3</td>
<td>Moderate</td>
</tr>
<tr>
<td>4</td>
<td>Moderate plus</td>
</tr>
<tr>
<td>5</td>
<td>Strong</td>
</tr>
<tr>
<td>6</td>
<td>Strong plus</td>
</tr>
<tr>
<td>7</td>
<td>Very strong or demonstrated</td>
</tr>
<tr>
<td>8</td>
<td>Very, very strong</td>
</tr>
<tr>
<td>9</td>
<td>Extreme</td>
</tr>
</tbody>
</table>

Let \( C = \{c_1, \ldots, c_n\} \) be the set of criteria. The result of the pair-wise comparisons on \( n \) criteria can be summarized in an \( n \times n \) evaluation matrix \( A \) in which every element \( a_{ij} \) is the quotient of weights of the criteria, as shown below:

\[
A = (a_{ij}), \quad i, j = 1, \ldots, n.
\] (33)

The relative priorities are given by the eigenvector \((w)\) corresponding to the largest eigenvalue \((\lambda_{\text{max}})\) as:

\[
Aw = \lambda_{\text{max}} w.
\] (34)
When pair-wise comparisons are completely consistent, the matrix $A$ has rank 1 and $\lambda_{\text{max}} = n$. In that case, weights can be obtained by normalizing any of the rows or columns of $A$.

The procedure described above is repeated for all subsystems in the hierarchy. In order to synthesize the various priority vectors, these vectors are weighted with the global priority of the parent criteria and synthesized. This process starts at the top of the hierarchy. As a result, the overall relative priorities to be given to the lowest level elements are obtained. These overall, relative priorities indicate the degree to which the alternatives contribute to the objective. These priorities represent a synthesis of the local priorities, and reflect an evaluation process that permits integration of the perspectives of the various stakeholders involved.

**Consistency check**

A measure of consistency of the given pair-wise comparison is needed. The consistency is defined by the relation between the entries of $A$; that is, we say $A$ is consistent if $a_{ik} = a_{ij} \cdot a_{jk}$, for all $i,j,k$. The consistency index ($CI$) is:

$$CI = \frac{\lambda_{\text{max}} - n}{(n-1)}.$$  \hspace{1cm} (35)

The final consistency ratio ($CR$), on the basis of which one can conclude whether the evaluations are sufficiently consistent, is calculated to be the ratio of the $CI$ and the random consistency index ($RI$):

$$CR = \frac{CI}{RI}.$$  \hspace{1cm} (36)

The value $0.1$ is the accepted upper limit for $CR$. If the final consistency ratio exceeds this value, the evaluation procedure needs to be repeated to improve consistency. The measurement of consistency can be used to evaluate the consistency of decision-makers as well as the consistency of all the hierarchies.

We are now ready to give an algorithm for computing objective weights using the AHP. The following notations and definitions are used.

- $n$: number of criteria
- $i$: number of objectives
- $p$: index for objectives, $p=1$ or $2$
- $d$: index for criteria, $1 \leq d \leq D$
- $R_{pd}$: the weight of $p$th item with respect to $d$th criterion
- $w_d$: the weight of $d$th criterion

**Algorithm 1: OWAHP (compute objective weights using the AHP)**

**Step 1:** Define the decision problem and the goal.
**Step 2:** Structure the hierarchy from the top through the intermediate to the lowest level.
**Step 3:** Construct the objective-criteria matrix using steps 4 to 8 using the AHP.
(Steps 4 to 6 are performed for all levels in the hierarchy.)

**Step 4:** Construct pair-wise comparison matrices for each of the lower levels for each element in the level immediately above by using a relative scale measurement. The decision-maker has the option of expressing his or her intensity of preference on a nine-point scale. If two criteria are of equal importance, a value of 1 is set for the corresponding component in the comparison matrix, while a 9 indicates an absolute importance of one criterion over the other (table 1 shows the measurement scale defined by Saaty, [33]).

**Step 5:** Compute the largest eigenvalue by the relative weights of the criteria and the sum taken over all weighted eigenvector entries corresponding to those in the next lower level of the hierarchy.

Analyze pair-wise comparison data using the eigenvalue technique. Using these pair-wise comparisons, estimate the objectives. The eigenvector of the largest eigenvalue of matrix $A$ constitutes the estimation of relative importance of the attributes.

**Step 6:** Construct the consistency check and perform consequence weights analysis as follows:

$$A = \left( a_{ij} \right) = \begin{bmatrix}
1 & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\
\frac{w_2}{w_1} & 1 & \cdots & \frac{w_2}{w_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & 1
\end{bmatrix}.$$  

Note that if the matrix $A$ is consistent (that is, $a_{ik} = a_{ij} \cdot a_{jk}$, for all $i, j, k = 1, 2, ..., n$), then we have (the weights are already known),

$$a_{ij} = \frac{w_i}{w_j}, \quad i, j = 1, 2, ..., n.$$  

(37)

If the pair-wise comparisons do not include any inconsistencies, then $\lambda_{max} = n$. The more consistent the comparisons are, the closer the value of computed $\lambda_{max}$ is to $n$. Set the consistency index ($CI$), which measures the inconsistencies of pair-wise comparisons, to be:

$$CI = \frac{\lambda_{max} - n}{(n-1)},$$

and let the consistency ratio ($CR$) be:

$$CR = 100 \left( \frac{CI}{RI} \right),$$

where $n$ is the number of columns in $A$ and $RI$ is the random index, being the average of the CI obtained from a large number of randomly generated matrices.

Note that $RI$ depends on the order of the matrix, and a $CR$ value of 10% or less is considered acceptable [33].
Step 7: Form the objective-criteria matrix as specified in table 3:

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>...</th>
<th>$C_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>objective 1</td>
<td>$R_{11}$</td>
<td>$R_{12}$</td>
<td>...</td>
<td>$R_{1d}$</td>
</tr>
<tr>
<td>objective 2</td>
<td>$R_{21}$</td>
<td>$R_{22}$</td>
<td>...</td>
<td>$R_{2d}$</td>
</tr>
<tr>
<td>objective 3</td>
<td>$R_{31}$</td>
<td>$R_{32}$</td>
<td>...</td>
<td>$R_{3d}$</td>
</tr>
</tbody>
</table>

Step 8: As a result, configure the pair-wise comparison for criteria-criteria matrix as in table 4:

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>...</th>
<th>$C_d$</th>
<th>$W_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria 1</td>
<td>1</td>
<td>$a_{12}$</td>
<td>...</td>
<td>$a_{1d}$</td>
<td>$w_1$</td>
</tr>
<tr>
<td>Criteria 2</td>
<td>$1/a_{12}$</td>
<td>1</td>
<td>...</td>
<td>$a_{2d}$</td>
<td>$w_2$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Criteria d</td>
<td>$1/a_{1d}$</td>
<td>$1/a_{2d}$</td>
<td>...</td>
<td>1</td>
<td>$w_d$</td>
</tr>
</tbody>
</table>

The $w_j$ are gained by a normalization process. The $w_j$ are the weights for criteria.

Step 9: Compute the overall weights for the objectives, using tables 3 and 4, as follows:

$\psi = \text{Total weight for objective } 1 = R_{11} \times w_1 + R_{12} \times w_2 + ... + R_{1d} \times w_d,$

$\psi' = \text{Total weight for objective } 2 = R_{21} \times w_1 + R_{22} \times w_2 + ... + R_{2d} \times w_d,$

$\psi'' = \text{Total weight for objective } 3 = R_{31} \times w_1 + R_{32} \times w_2 + ... + R_{3d} \times w_d,$

where $\psi + \psi' + \psi'' = 1$. As a result the aggregated objective function is,

$$\min \ Z = \left( \psi \left[ \sum_{j=1}^{n} s_j \cdot e_j + b \right] \right) + \left( \psi' \left[ \sum_{j=1}^{n} \alpha \cdot E_j \cdot s_j + \sum_{j=1}^{n} \beta \cdot T_j \cdot s_j \right] \right) - \left( \psi'' \left[ Z_3 = \sum_{j=1}^{n} s_j \cdot \rho_j \right] \right).$$

Therefore, we obtain the weights for the objectives. We illustrate the applicability and validity of our mechanism in a case study.

5 Case study

Here, a case study is conducted in a car company in Iran to illustrate the applicability and effectiveness of the proposed mechanism. In this car company several services are offered to
the customers. These services support customers before, during and after purchase process. The services are listed in Table 5, chronologically.

Table 5 Different services offered to the customers

<table>
<thead>
<tr>
<th>Before purchase</th>
<th>During purchase</th>
<th>After purchase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product information provision</td>
<td>Monitoring purchase process</td>
<td>Parts provision</td>
</tr>
<tr>
<td>News teller</td>
<td>Online purchase process</td>
<td>Periodic maintenance</td>
</tr>
<tr>
<td>Product information update</td>
<td>Online user error correction</td>
<td>Free checkups</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Guarantee of repair</td>
</tr>
<tr>
<td></td>
<td></td>
<td>After sale services</td>
</tr>
</tbody>
</table>

As shown in Table 2, there are eleven services that the company offers to the customers. Therefore a survey study to determine the effective services from customers’ viewpoint is possible. In this study we collected 384 samples according to the following hypotheses. Also, note that the significance level is 1% and pearson correlation test (due to Normal distribution of data resulted from Kolmogorov-Smirnof test) is employed for accept/reject purpose.

H1: Product information provision increase customers’ satisfaction. (accept)
H2: News teller increase customers’ satisfaction. (accept)
H3: Product information update increase customers’ satisfaction. (accept)
H4: Monitoring purchase process increase customers’ satisfaction. (reject)
H5: Online purchase process increase customers’ satisfaction. (accept)
H6: Online user error correction increase customers’ satisfaction. (reject)
H7: Parts provision increase customers’ satisfaction. (accept)
H8: Periodic maintenance increase customers’ satisfaction. (accept)
H9: Free checkups increase customers’ satisfaction. (accept)
H10: Guarantee of repair increase customers’ satisfaction. (reject)
H11: After sale services increase customers’ satisfaction. (accept)

As shown in the tests hypotheses 1,2,3,5,7,8,9 and 11 are accepted and therefore are inserted to the multi-objective mathematical model to find the most optimal services from company’s viewpoint.

Now, using the selected services, we configure the proposed multi-objective mathematical model. As stated, eight services are qualified to be considered for profit optimization. The amount of investment for the eight services are 75, 68, 34, 82, 56, 49, 63, 76, respectively. Also the service transformation function is

\[ S^0 = s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2 + s_6 + \frac{1}{5} s_7 \]

The fixed cost is 1500 unit of money. The processing times for the services are 25, 32, 41, 17, 28, 37, 43, 31, respectively. Also, the completion times are 37, 59, 63, 48, 52, 45, 67 and 46. And the due dates are 29, 49, 57, 34, 33, 51, 68, 38. The process capability indices for services are computed to be 2.33, 2.59, 1.9, 3.3, 2.25, 3.14, 2.78 and 2.45.

Next, we weight the cost, time and quality objectives with respect to three criteria: economic viewpoint, demand fluctuation, and competitiveness. Algorithm 1 is applied. The following weights are gained for our proposed objectives:
weight for cost objective $\psi = 0.27$

weight for time objective $\psi' = 0.42$

weight for quality objective $\psi'' = 0.31$

Now these weights are used in equation (39). Optimizing the proposed single objective linear mathematical program in MATLAB 7.0, we obtain $s_2$, $s_5$, $s_6$, and $s_8$ as the services maximizing profit.

### 6 Conclusions

We proposed a marketing decision model to aid the marketing team of a company based on customers’ satisfaction. The company’s services performance was evaluated conducting a questionnaire as a purification tool of services. Then a multi-objective mathematical model was utilized to determine the services with more profits considering the objectives of minimizing the service cost, minimizing the service delivery time and maximizing the delivered service quality. To optimize the proposed mathematical model, AHP was applied. We illustrated all aspects of our model in a case study conducted at a car company.

### References


