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A New Method for Solving Linear Bilevel Multi-Objective Multi-Follower Programming Problem

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Abstract Linear bilevel programming is a decision making problem with a two-level decentralized organization. The leader is in the upper level and the follower, in the lower level. This study addresses linear bilevel multi-objective multi-follower programming (LB-MOMFP) problem, a special case of linear bilevel programming problems with one leader and multiple followers where each decision maker has several objective functions conflicting with each other. We propose a simple and efficient method for solving these problems. In our method, objectives of multi-objective programming problem of the each level decision maker are transformed into fuzzy goals (membership functions) by assigning an aspiration level to each of them, and a max-min decision model is generated for each level problem. Then, we transform obtained linear bilevel multi-follower problem into equivalent single-level problem by extended Karush-Kuhn-Tucker approach. Finally, numerical examples are given to demonstrate the feasibility and efficiency of the proposed method. This paper aims to present a simple technique to obtain better compromise solution of LB-MOMFP problem than earlier techniques. A comparative analysis based on numerical examples is carried out to show preference of the proposed method.

Keywords: Linear Bilevel Programming, Multi-Objective Programming, Multi-Follower Programming, Fuzzy Goal Programming, Karush-Kuhn-Tuker Approach.

1 Introduction

Bi-level mathematical programming (BLMP) is identified as mathematical programming that solves decentralized planning problems with two decision makers (DMs) in a two-level or hierarchical organization. The basic concept of the BLMP technique is that an upper level decision maker (ULDM) (the leader) sets his goals and/or decisions and then asks each subordinate level of the organization for their optima which are calculated separately; the lower level DM's (LLDM) (the follower's) decisions are then submitted and modified by the ULDM considering the overall benefit for the organization; the process continued until a satisfactory solution is reached. In other words, although the ULDM independently optimizes its own benefits, the decision may be affected by the reaction of the LLDM. As a consequence, decision deadlock arises frequently and the problem of distribution of proper decision power is encountered in most of the practical decision situations.

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Bilevel programs are intrinsically hard to solve, being typically non-convex and non-differentiable [1]. It has been proved that solving the linear bilevel programming (LBP) problem is an NP-hard problem and finding local optimal solution of the LBP problem is an NP-hard problem too [2]. Many papers have been published investigating results, applications and solution methods for bilevel optimization [3,4], however, there are only very few dealing with the bilevel multi-objective programming problem, where the upper level or the lower level or both of a bilevel decision have multiple conflicting objectives [5].

The use of the concept of membership function of fuzzy set theory to BLMP programming problems to obtain satisfactory decisions was first introduced by Lai [6] in 1996. Thereafter, Lai's satisfactory solution concept was extended by to multi-level programming problems Shih et al.[7]. The basic concept of these fuzzy programming (FP) approaches implies that each lower level DM optimizes his/her objective function, taking a goal or preference of the first level DMs into consideration. In the decision process, a FP problem with the set of constraints on an overall satisfactory degree of any upper levels is solved considering the membership functions of the fuzzy goals for the decision variables of all the DMs . If the proposed solution is not satisfactory for any upper level, the solution search is continued by redefining the elicited membership functions until a satisfactory solution is reached [8]. The main difficulty which arise with the FP approach of Shih et al.[7] is that there is the possibility of rejecting the solution again and again by the ULDM and re-evaluation of the problem is repeatedly needed to reach the satisfactory decision, where the objectives of the DMs are over-conflicting. Even inconsistency between the fuzzy goals of the objectives and the decision variables may arise.

The fuzzy goal programming (FGP) technique introduced by Mohamed [9] for proper distribution of decision powers to the DMs to arrive at a satisfying decision for the overall benefit of the organization was developed to overcome the above undesirable situation.

In this paper, we consider linear bilevel multi-objective programming (LB-MOMFP) problems where there are a single DM at the upper level and two or more DMs at the lower level, and objective functions of the DMs and constraint functions are linear functions.

Although solving the bilevel multi-objective multi-followers problem is not an easy task, some researchers have presented feasible approaches for this problem. Zhang, Lu and Dillon [10] solved decentralized multi-objective bilevel decision making with fuzzy demands. Taran and Roghanian [11] propose a method for a fuzzy LB-MOMFP problem related to supply chain optimization. Zhang and Lu [12] considered a LB-MOMFP problem with fuzzy uncertainty in parameters and cooperative relationship between followers. They solved the problem using kth-best method. Among other studies in this area, we can mention Ansari and Zhiani Rezai [13] where they extended kth- best method for LB-MOMFP problems with uncooperative relationship between followers. Baky [14] used FGP introduced by Mohamed [9] to achieve compromise solution of the LB-MOMFP problem by minimizing the sum of negative and positive deviational variables from the aspired levels and the lower and upper deviational of decision variables provided by the DMs. In the former studies such as Baky [14], Baky did not mention the bounds on the maximum negative and positive tolerance values and appropriate method to determine these values. Also, satisfactory solution of LB-MOMFP problem using algorithm found by Baky depends on the choice of these tolerance values which very often leads to the possibility of rejecting the solution again and again by upper level decision makers and so the solution process becomes very time consuming [15].

This paper aims to present a simple method to obtain more efficient solution for LB-MOMFP problem compared to other techniques suggested. We use fuzzy goal programming and max-min solution approach to convert LB-MOMFP problem to a linear bilevel problem with multiple followers in the second level. Then, we extend Kuhn-Tucker approach to convert the achievement BLP problem with multiple followers into equivalent single-level problem. The proposed method can be applied for LB-MOMFP problems with cooperative or uncooperative relationship between followers. Also, there is not difficulties of earlier methods such as FGP approach proposed by Baky [14] and the possibility of rejecting the solution again and again by upper level decision makers does not arise in our method. To compare the efficiency of our proposed approach, distance function [16] is used. The paper is organized as follows: In next Section, the problem formulation and solution concept is introduced. In Section 3, we convert our problem to a linear bilevel multi-follower programming (LB-MFP) problem. LB-MFP problem is transformed into single-level programming problem in section 4. In Section 5, the solution algorithm for solving LB-MOMFP problem is given. The numerical examples are shown for illustrating the proposed approach, in section 6. Finally, the conclusion is presented in the last section.

2 Problem formulation

Consider there are two levels in a hierarchy stracture with ULDM or DM_0 at the LLDM_i or DM_i, $i=1,2,...,p$. Let the vector of decision variables $x=(x_0,x_1,...,x_p)\in R^n$ be partitioned between the upper and lower DM_i. The ULDM has control over the decision vector $x_0\in R^{n_0}$, and $\mathrm{LLDM}_k, k=1,2,...,p$, has control over the decision vector $x_k\in R^{n_k}$, where $n=n_0+n_1+...+n_p$, $x_k=(x_{k1},x_{k2},...,x_{kn_k})$, $k=0,1,...,p$. Furthermore assume that

$F_i(x_0,x_1,...,x_p)\equiv F_i(x): R^{n_0}\times R^{n_1}\times...\times R^{n_p}\to R^{m_i}, i=0,1,...,p$

are the vector of objective functions to the $\mathrm{DM}_i, i=0,1,...,p$. So the BL-MOMFP problem of maximization type may be formulated as follows [14]:

[upper Level]

$$\max_{x_0} F_0(x)=\max_{x_0}(f_{01}(x),f_{02}(x),...,f_{0m_0}(x)) \tag{1}$$

where $x_1,x_2,...,x_p$ solve

[lower Level]

$$\max_{x_1} F_1(x)=\max_{x_1}(f_{11}(x),f_{12}(x),...,f_{1m_1}(x))$$
$$\max_{x_2} F_2(x)=\max_{x_2}(f_{21}(x),f_{22}(x),...,f_{2m_2}(x))$$
$$\vdots$$
$$\max_{x_p} F_p(x)=\max_{x_p}(f_{p1}(x),f_{p2}(x),...,f_{pm_p}(x))$$

s.t.

$$x\in G=\{x\in R^n \mid \boldsymbol{A}_0x_0+\boldsymbol{A}_1x_1+...+\boldsymbol{A}_px_p\le b,\ x\ge 0, b\in R^m\}\ne\varnothing$$

where

$$F_i(x)=\boldsymbol{C}_ix_0+\sum_{k=1}^{p}\boldsymbol{B}_{ik}x_k,\ i=0,1,...,p$$

$$f_{ij}(x)=c^{ij}x_0+\sum_{k=1}^{p}b_k^{ij}x_k,\ i=0,1,...,p,\ j=1,...,m_i$$

and where $m_i, i=0,1,..., p$ are the number of DM_i 's objective functions, m is the number of constraints, $c^{ij}=(c_1^{ij},c_2^{ij},...,c_{n_0}^{ij}),\ b_k^{ij}=(b_1^{ij},b_2^{ij},...,b_{n_k}^{ij}),\ k=1,2,...,p$ and $c_k^{ij}, b_{n_k}^{ij}, k=1,2,....,n_k$ are constants, and A_i are the coefficients matrices of size $m\times n_i, i=0,1,..., p$. We assume that the costraint set G is nonempty and compact. In a multi-objective programming problem, several objective functions have to be maximized simultaneously. Usually, there is no single point which can maximize all objective functions given at once. Therefore, we use the concept of efficiency or Pareto optimality. Thus, we introduce the following concepts of optimal solutions to the LB-MOMFP problems.

Definition 1. A point $x^*=(x_0^*,x_1^*,...,x_p^*)$ is said to be a complete optimal solution for the LB-MOMFP problem if it holds that

$F_i(x^*)\geq F_i(x),\ i=0,1,...,p$

for all $x\in G$.

Definition 2. A point $x^*=(x_0^*,x_1^*,...,x_p^*)$ is said to be a Pareto optimal solution for the LB-MOMFP problem if there is no other $x\in G$ such that

$F_i(x)\geq F_i(x^*),\ i=0,1,...,p$

with strict inequality holding for at least i.

3 Converting the LB-MOMFP problem to a linear bilevel multi-follower problem based on fuzzy goal programming

In LB-MOMFP problems, if an imprecise aspiration level is assigned to each of the objectives in each level of the LB-MOMFP, then these fuzzy objectives are called as fuzzy goals. They are characterized by their associated membership functions by defining the tolerance limits for achievement of their aspired levels.

3.1 Construction of membership functions

Since all the DMs are interested to maximizing their own objective functions over the same feasible region defined by the system of constraints, the optimal solutions of both of them calculated separately can be taken as the aspiration levels of their associated fuzzy goals.

Let $x^{ij}=(x_0^{ij},x_1^{ij},...,x_p^{ij}); f_{ij}^{max}, i=0,1,...,p,\ j=1,2,...,m_i$ be the optimal solutions of DMs objective functions at both levels, calculated separately. Let $g_{ij}\leq f_{ij}^{max}$ be the aspiration level assigned to the ij th objective f_{ij} (the subscript ij means that $j=1,2,...,m_0$ when $i=0$ for ULDM problem, and $j=1,2,...,m_1$ when $i=1$ for DM_1 problem, and $j=1,2,...,m_p$ when $i=p$ for DM_p problem). Then, the fuzzy goals of the decision makers' objective functions at both levels appear as:

$f_{ij}(x)\succ g_{ij}, i=0,1,...,p,\ j=1,...,m_i$

where "$\succ$" indicate the fuzziness of the aspiration levels, which is described as "essentially more than" [9].

Then, fuzzy goal programming (FGP) problem of BL-MOMFP can be written as follows:

Find $x=(x_0,x_1,...,x_p)$ so as to satisfy
$f_{0j}(x)\succ g_{0j},\ j=1,...,m_0$
where $x_1,x_2,...,x_p$ solve
$f_{1j}(x)\succ g_{1j},\ j=1,...,m_1$
$f_{2j}(x)\succ g_{2j},\ j=1,...,m_2$
$\vdots$
$f_{pj}(x)\succ g_{pj},\ j=1,...,m_p$
$s.t.$
$x\in G$

To solve the above problem, we should first choose an appropriate memebership function for each fuzzy inquality and use a max-min operator proposed by Bellman and Zadeh [17] to drive the equivalent crisp problem of the given fuzzy goal programming problem at each level. To build membership functions, fuzzy goals and tolerance values should be determined. The minimum value of each objective function $f_{ij},\ i=0,1,...,p,\ j=1,2,...,m_i$ give lower tolerance limit or aspired level of achievement for the ij th objective function i.e.

$$f_{ij}^l=\min_{x\in G} f_{ij}(x),\ i=0,1,...,p,\ j=1,2,...,m_i \tag{2}$$

The maximum value of each objective function gives the upper tolerance limit or aspired level of achievement for the ij th objective function i.e.

$$f_{ij}^u=\max_{x\in G} f_{ij}(x),\ i=0,1,...,p,\ j=1,2,...,m_i \tag{3}$$

Then, membership functions $\mu_{ij}(f_{ij})$ for the ij th fuzzy goal can be formulated as:

$$\mu_{ij}(f_{ij}(x))=\begin{cases}0 & \text{if } f_{ij}(x)\le f_{ij}^l\\ \dfrac{f_{ij}(x)-f_{ij}^l}{f_{ij}^u-f_{ij}^l} & \text{if } f_{ij}^l\le f_{ij}(x)\le f_{ij}^u,\quad i=0,1,...,p,\ j=1,2,...,m_i\\ 1 & \text{if } f_{ij}(x)\ge f_{ij}^u\end{cases} \tag{4}$$

Here, linear membership functions are considered because these are more suitable than nonlinear functions as less computational difficulties arise in models due to it.

3.2 Max-Min solution approach

In a fuzzy programming, the highest possible value of membership function is always 1 and the aim of each DM is to achieve highest membership value (unity) of the associated fuzzy goal in order to obtain the absolute satisfactory solution. Therefore, we use max-min solution approach to determine the highest degree of membership for each of the goals at each level. Consider the following ULDM problem of the LB-MOMFP problem:

$$\max_{x_0} F_0(x)=\max_{x_0}(f_{01}(x),f_{02}(x),...,f_{0m_0}(x))$$
$$s.t. \tag{5}$$
$$x\in G$$

To obtain the satisfactory solution of this problem, we define a satisfactory degree of the ULDM level as:

$\lambda_0 = \min\{\mu_{0j}(f_{0j}(x)),\ j = 1,2,...,m_0\}$

Then, we can get the solution of the ULDM problem by solving the following equivalent crisp linear programming problem:

$$\begin{aligned}
&\max_{x_0,\lambda_0} \lambda_0\\
&s.t.\\
&\quad \mu_{0j}(f_{0j}(x) \geq \lambda_0,\ j = 1,2,...,m_0\\
&\quad \lambda_0 \in [0,1]\\
&\quad x \in G
\end{aligned} \tag{6}$$

In the same way, consider the each LLDM_i problem of the LB-MOMFP problem:

$$\begin{aligned}
&\max_{x_i} F_i(x) = \max_{x_i}(f_{i1}(x), f_{i2}(x),..., f_{im_i}(x)) \qquad (7)\\
&s.t.\\
&\quad x \in G
\end{aligned}$$

We define a satisfactory degree of the i th LLDM level as:

$\lambda_i = \min\{\mu_{ij}(f_{ij}(x)), i = 1,2,..., p,\ j = 1,2,..., m_i\}$

Then, each LLDM_i , $i = 1,2,...,p$ problem can be written as the following equivalent crisp linear programming problem:

$$\begin{aligned}
&\max_{x_i,\lambda_i} \lambda_i\\
&s.t.\\
&\quad \mu_{ij}(f_{ij}(x) \geq \lambda_i,\ j = 1,2,...,m_i\\
&\quad \lambda_i \in [0,1]\\
&\quad x \in G
\end{aligned} \tag{8}$$

Now, by substituting the single objective function problems (6) to ULDM problem and (8) to aech LLDM_i problem, the LB-MOMFP problem can be formulated to the following equivalent linear bilevel multi-follower programming (LB-MFP) problem:

[upper level]

$$\begin{aligned}
&\max_{x_0,\lambda_0} \lambda_0\\
&s.t.\\
&\quad \boldsymbol{C}_0 x_0 + \sum_{k=1}^{p} \boldsymbol{B}_{0k} x_k - (F_0^u - F_0^l)\lambda_0 \geq F_0^l\\
&\quad 0 \leq \lambda_0 \leq 1\\
&\quad x \in G
\end{aligned} \tag{9}$$

[lower level]

$$\max_{x_1,\lambda_1} \lambda_1$$
$$s.t.$$
$$\mathbf{C}_1 x_0 + \sum_{k=1}^{p} \mathbf{B}_{1k} x_k - (F_1^u - F_1^l)\lambda_1 \geq F_1^l$$
$$0 \leq \lambda_1 \leq 1$$
$$x \in G$$
$$\vdots$$
$$\max_{x_p,\lambda_p} \lambda_p$$
$$s.t.$$
$$\boldsymbol{C}_p x_0 + \sum_{k=1}^{p} \boldsymbol{B}_{pk} x_k - (F_p^u - F_p^l)\lambda_p \geq F_p^l$$
$$0 \leq \lambda_p \leq 1$$
$$x \in G$$

It is noted that, in this reformulation problem (9) the bilevel multi-objective problem is converted to the bilevel problem with single objective function in the upper and i th lower level problems ($i = 1,2,...,p$), in which, x_0 and λ_0 are the decision variables for the upper level, and x_i and $\lambda_i, i = 1,2,..., p$ are the decision variables for the i th lower level. To ensure that the LB-MFP problem (9) has an optimal solution, we assume that the feasible region including all of the constraints, is nonempty and compact [18].

4 Converting the LB-MFP problem (9) into a single-level problem

Karush-Kuhn-Tucker (KKT) approach is one of the popular approachs to deal with programming problems with hierarchical forms. We develop the KKT approach for driving an optimal solution from the LB-MFP decision model (9). The fundamental idea to deal with the LB-MF decision problems is that it replaces each follower's problem with its KKT optimality conditions and appends the resultant system to the leader's problem. The reformulation of the LB-MFP problem is a standard mathematical program and relatively easy to solve because all but complementary constraints are linear. Therefore, we obtain the following reformulation of the LB-MFP problem by replacing the each $\mathrm{LLDM}_i, i = 1,2,..., p$ problem by its KKT optimality conditions:

$$\max_{x_0,\lambda_0} \lambda_0 \tag{10}$$
$$s.t.$$
$$\boldsymbol{C}_i x_0 + \sum_{k=1}^{p} \boldsymbol{B}_{ik} x_k - (F_i^u - F_i^l)\lambda_i \geq F_i^l, i = 0,1,...,p \tag{11}$$
$$\boldsymbol{A}_0 x_0 + \boldsymbol{A}_1 x_1 + ... + \boldsymbol{A}_p x_p \leq b \tag{12}$$
$$\lambda_i \leq 1, \ \ i = 0,1,..., p \tag{13}$$
$$x_i \geq 0, \ \ i = 0,1,..., p \tag{14}$$

$$\lambda_i \geq 0,\ i = 0,1,\dots,p \tag{15}$$

$$-\boldsymbol{B}_i^T u_i + \boldsymbol{A}_i^T v_i - q_i = 0,\ i = 1,\dots,p \tag{16}$$

$$(F_i^u - F_i^l)^T u_i + w_i - q'_i = 1,\ i = 1,\dots,p \tag{17}$$

$$u_i(C_i x_0 + \sum_{k=1}^{p} \boldsymbol{B}_{ik} x_k - (F_i^u - F_i^l)\lambda_i - F_i^l) + v_i(b - \boldsymbol{A}_0 x_0 - \boldsymbol{A}_1 x_1 - \dots - \boldsymbol{A}_p x_p) + w_i(1-\lambda_i) +$$

$$q_i x_i + q'_i \lambda_i = 0,\ i = 1,\dots,p \tag{18}$$

$$u_i, v_i, w_i, q_i, q'_i \geq 0,\ i = 1,\dots,p \tag{19}$$

where u_i, v_i, w_i, q_i and q'_i are the dual variables associated with the constraints of (11)- (15). The branch-and-bound algorithm has been successfully used to solve both linear and nonlinear optimization problems. The basic idea of this algorithm is to suppress the complementarity term (18) and solve the resulting linear program. At each iteration, a check is made to see if (18) is satisfied. If yes, the corresponding point is in the feasible region, and hence, is feasible solution to problem (10); if not, a branch and bound scheme is used to examine implicitly all the combinations of the complementary slackness. Details of this algorithm are explained in [19].

4.1 Performance analysis

To compare the solution with other methods, the following family of distance functions [16] is defined

$$L_p(\tau,k) = (\sum_{k=1}^{T} \tau^p (1-d_k)^p)^{\frac{1}{p}} \tag{20}$$

Here, $d_k, k = 1,2,\dots,T$ represents the degrees of closeness of the preferred compromise solution to the optimal solution vector with respect to the k-th objective function. Here, $\tau = (\tau_1, \tau_2, \dots, \tau_T)$ represents vector of attribute attention levels τ_k. We assume that $\sum_{k=1}^{T} \tau_k = 1$. If all the attributes are equal, then $\tau_k = \frac{1}{T}, k = 1,2,\dots,T$. The power p represents the distance parameter ($1 \leq p \leq \infty$). Now, for $p = 2$, the distance functions become:

$$L_2(\tau,k) = (\sum_{k=1}^{T} \tau^2 (1-d_k)^2)^{\frac{1}{2}}. \tag{21}$$

For maximization problem, d_k is denoted by $d_k =$ (the preferred compromise solution) / (the individual best solution). For minimization problem, d_k is denoted by $d_k =$ (the individual best solution) / (the preferred compromise solution). The solution for which $L_2(\tau,k)$ will be minimal would be the most satisfying solution for ULDM and LLDM. Therefore, by comparing the distance $L_2(\tau,k)$, one can compare the performance of the solutions obtained by different approaches.

5 The suggested algorithm to solve LB-MOMFP

Consider the LB-MOMFP problem.

Step 1. Calculate the individual minimum and maximum values of all the objective functions at the two levels under the given constraints.

Step 2. Set the goals and the lower tolerance limits f_{ij}^u, $f_{ij}^l, i = 0,1,..., p, j = 1,2,..., m_i$ for all the objective functions at the two levels.

Step 3. Construct the linear membership functions $\mu_{ij}(f_{ij}), i = 0,1,..., p, j = 1,2,..., m_i$ for each objective at each level.

Step 4. Formulate the fuzzy goal programming model (9) to obtain the LB-MFP problem.

Step 5. Formulate model (10) for the LB-MFP problem.

Step 6. Solve the model (10) to get the Pareto solution of the BL-MOMFP problem by using the branch-and-bound algorithm described in [19].

6 Numerical examples

In this section, two examples will be considered to illustrate the efficiency of the proposed algorithm.

Example 1 [13], Consider the following LB-MOMFP problem with $x_0, x_1, x_2 \in R$

[upper level]

$$\max_{x_0} F_0(x_0, x_1, x_2) = (f_{01} = x_0 + 2x_1 + 3x_2,\ f_{02} = x_1 - x_2)$$

where x_1 and x_2 solve

[lower level]

$$\max_{x_1} F_1(x_0, x_1, x_2) = (f_{11} = x_0 + x_1,\ f_{12} = x_1)$$

$$\max_{x_2} F_2(x_0, x_1, x_2) = (f_{21} = x_0 + x_2,\ f_{22} = x_2)$$

$$\begin{aligned} \text{subject to}\quad & x_0 + 2x_1 + 3x_2 \le 6, && x_0 + x_2 \le 4 \\ & x_0 + x_1 \le 3, && x_0 + x_2 \le 4 \\ & x_1 \le 1, && x_2 \le 2 \\ & x_0, x_1, x_2 \ge 0 \end{aligned}$$

(Step 1 and Step 2) The following table summarizes minimum and maximum individual optimal solutions, of all objective functions for the two levels of the LB-MOMFP problem, subjected to the given constraints. To demonstrate the proposed algorithm, the aspiration levels and upper tolerance limits to the objective functions can be taken as the minimum and maximum individual optimal solutions.

Table 1 The individual minimum and maximum values

	f_{01}	f_{02}	f_{11}	f_{12}	f_{21}	f_{22}
$\max f_{ij} = f_{ij}^u$	6	1	3	1	4	2
$\min f_{ij} = f_{ij}^l$	0	-2	0	0	0	0

(Step 3) Thus the linear membership functions $\mu_{ij}(f_{ij}),\ i=0,1,2,\ j=1,2$ at each level are constructed as:

$$\mu_{01}(f_{01})=\frac{1}{6}x_0+\frac{1}{3}x_1+\frac{1}{2}x_2,\quad \mu_{02}(f_{02})=\frac{1}{3}x_1-\frac{1}{3}x_2+\frac{2}{3}$$

$$\mu_{11}(f_{11})=\frac{1}{3}x_0+\frac{1}{3}x_1\quad,\quad \mu_{12}(f_{12})=x_1$$

$$\mu_{21}(f_{21})=\frac{1}{4}x_0+\frac{1}{4}x_2\quad,\quad \mu_{22}(f_{22})=\frac{1}{2}x_2$$

(Step 4) The fuzzy goal programming mdel (9) for this numerical example can be written as

$$\begin{aligned}
&\max_{x_0,\lambda_0} \lambda_0\\
&s.t.\\
&\quad -x_0-2x_1-3x_2+6\lambda_0\geq 0,\quad -x_1+x_2+3\lambda_0\geq 0,\quad \lambda_0\leq 1,\\
&\quad x_0+2x_1+3x_2\leq 6,\quad x_0+x_1\leq 3,\quad x_0+x_2\leq 4,\\
&\quad x_1\leq 1,\quad x_2\leq 2,\quad x_0,\lambda_0\geq 0
\end{aligned}$$

$$\begin{aligned}
&\max_{x_1,\lambda_1} \lambda_1\\
&s.t.\\
&\quad -x_0-x_1+3\lambda_1\geq 0,\quad -x_1+\lambda_1\geq 0,\quad \lambda_1\leq 1,\\
&\quad x_0+2x_1+3x_2\leq 6,\quad x_0+x_1\leq 3,\quad x_0+x_2\leq 4,\\
&\quad x_1\leq 1,\quad x_2\leq 2,\quad x_1,\lambda_1\geq 0
\end{aligned}$$

$$\begin{aligned}
&\max_{x_2,\lambda_2} \lambda_2\\
&s.t.\\
&\quad -x_0-x_2+4\lambda_2\leq 0,\quad -x_2+2\lambda_2\leq 0,\quad \lambda_2\leq 1\\
&\quad x_0+2x_1+3x_2\leq 6,\quad x_0+x_1\leq 3,\quad x_0+x_2\leq 4,\\
&\quad x_1\leq 1,\quad x_2\leq 2,\quad x_2,\lambda_2\geq 0
\end{aligned}$$

(Step 5) By using KKT optimality conditions for each $LLDM_i$, $i=1,2$, we obtained the following single-level problem:

$$
\begin{aligned}
&\max_{x_0,\lambda_0} \lambda_0 \\
&s.t. \\
&\quad -x_0 - x_1 - 3x_2 + 6\lambda_0 + s_{01} = 0, \qquad -x_1 + x_2 + 3\lambda_0 + s_{02} = 0 \\
&\quad \lambda_0 + s_{03} = 1, \qquad -x_0 - x_1 + 3\lambda_1 + s_{11} = 0 \\
&\quad -x_1 + \lambda_1 + s_{12} = 0, \qquad \lambda_1 + s_{13} = 1 \\
&\quad -x_0 - x_2 + 4\lambda_2 + s_{21} = 0, \qquad -x_2 + 2\lambda_2 + s_{22} = 0 \\
&\quad \lambda_2 + s_{23} = 1, \qquad x_0 + 2x_1 + 3x_2 + s_1^{'} = 6 \\
&\quad x_0 + x_1 + s_2^{'} = 3, \qquad x_1 + s_3^{'} = 1 \\
&\quad x_0 + x_2 + s_4^{'} = 4, \qquad x_2 + s_5^{'} = 2 \\
&\quad -u_{11} - u_{12} + 2v_{11} + v_{12} + v_{13} - w_{11} = 0, \qquad 3u_{11} + u_{12} + u_{13} - w_{12} = 1 \\
&\quad -u_{21} - u_{22} + 3v_{21} + v_{24} + v_{25} - w_{21} = 0, \qquad 3u_{21} + u_{22} + u_{23} - w_{22} = 1 \\
&\quad \sum_{j=1}^{3} u_{ij} s_{ij} + \sum_{j=1}^{5} v_{ij} s_j^{'} + w_{i1} x_i + w_{i2} \lambda_i = 0, \quad i = 1,2 \\
&\quad x_0, x_1, x_2, \lambda_0, \lambda_1, \lambda_2 \geq 0 \\
&\quad u_{ij}, v_{ik}, w_{ij} \geq 0, \quad i = 1,2, \quad j = 1,2,3, \quad k = 1,2...,5
\end{aligned}
$$

(Step 6) We solve the above problem by using the branch-and-bound algorithm. The optimal solution is obtained as:
$(x_0^*, x_1^*, x_2^*, \lambda_0^*, \lambda_1^*, \lambda_2^*) = (2,1,0.67,0.78,1,0.33)$
Then the Pareto optimal solution of this Example is $(x_0^*, x_1^*, x_2^*) = (2,1,0.67)$ with upper level objective value $F_0 = (6,0.33)$ and lower level objective values $F_1 = (3,1)$, $F_2 = (2.67,0.67)$, and membership functions values are $\mu_{01} = 1$,$\mu_{02} = 0.78$, $\mu_{11} = 1$, $\mu_{12} = 1$, $\mu_{12} = 1$, $\mu_{21} = 0.67$ $\mu_{22} = 0.34$.

Table 2 Comparison of solutions obtained by different methods

	Optimal solution	Objective values	Membership values	Distance values
Proposed Method	$x_0 = 2$	$f_{01} = 6, f_{02} = 0.33$	$\mu_{01} = 1, \mu_{02} = 0.78$	0.16680671
	$x_1 = 1$	$f_{11} = 3, f_{12} = 1$	$\mu_{11} = 1, \mu_{12} = 1$	
	$x_2 = 0.67$	$f_{21} = 2.67, f_{22} = 0.67$	$\mu_{21} = 0.67, \mu_{22} = 0.33$	
Method in Ansari, Rezai[13]	$x_0 = 2$	$f_{01} = 4.75, f_{02} = 0.75$	$\mu_{01} = 0.375, \mu_{02} = 0.375$	0.171813115
	$x_1 = 1$	$f_{11} = 3, f_{12} = 1$	$\mu_{11} = 1, \mu_{12} = 1$	
	$x_2 = 0.25$	$f_{21} = 2.25, f_{22} = 0.25$	$\mu_{21} = 0.375, \mu_{22} = 0.125$	

In Table 2, we compare the optimal solution obtained in this paper with that in the corresponding reference. From the above comparison, it is shown that the optimal solution obtained in this paper is the Pareto optimal solution for the example. Then, the proposed method is feasible for the LB-MOMFP problem. Moreover, our proposed method offers better compromise optimal solution than the solution obtained by Ansari and Zhiani Rezai [13], because all of the sums of the membership values produced by the proposed method are greater than the produced solution method in [13], and distance value produced by the proposed method is smaller than the distance value of the method given in [13].

Example 2

[1st level]

$$\max_{x_0} F_0(x) = (f_{01} = -x_0 + x_1 + 4x_2, f_{02} = x_0 - 3x_1 + 4x_2)$$

where x_1 and x_2 solve

[2nd level]

$$\max_{x_1} F_1(x) = (f_{11} = -2x_0 + x_1 - 2x_2, f_{12} = -2x_0 - x_1 + 3x_2, f_{13} = -3x_0 + x_1 - x_2)$$

$$\max_{x_2} F_2(x) = (f_{21} = -7x_0 - 3x_1 + 4x_2, f_{22} = -x_0 - x_2)$$

$s.t.$

$$(x_0, x_1, x_2) \in G = \begin{cases} x_0 + x_1 + x_2 \le 3, & x_0 + x_1 - x_2 \le 1 \\ x_0 + x_1 + x_2 \ge 1, & -x_0 + x_1 + x_2 \le 1 \\ x_2 \le 0.5, & x_0, x_1, x_2 \ge 0 \end{cases}$$

The individual optimal solutions are $f_{01} = 2.5$, $f_{02} = 3.5$, $f_{11} = 1$, $f_{12} = 1$, $f_{13} = 1$, $f_{21} = 0.5$, $f_{22} = 0$. We find that the Pareto optimal solution to this problem is $(x_0, x_1, x_2) = (0.38, 0.12, 0.5)$ with objective values $f_{01} = 1.74$, $f_{02} = 2.02$, $f_{11} = -1.64$, $f_{12} = 0.62$, $f_{13} = -1.52$, $f_{21} = -1.02$, $f_{22} = -0.88$, and membership functions values are $\mu_{01} = 0.78$, $\mu_{02} = 0.77$, $\mu_{11} = 0.47$, $\mu_{12} = 0.88$, $\mu_{13} = 0.57$, $\mu_{21} = 0.82$, $\mu_{22} = 0.56$.

We compare the optimal solution obtained in this paper with that the FGP approach proposed by Baky [14]. Comparative results are given in the following Table.

Table 3 Comparison of solutions obtained by different methods

	Optimal solution	Objective values	Membership values	Distance values
Proposed Method	$x_0 = 0.38$ $x_1 = 0.12$ $x_2 = 0.5$	$f_{01} = 1.74, f_{02} = 2.02,$ $f_{11} = -1.64, f_{12} = 0.62$ $f_{13} = -1.58$ $f_{21} = -1.02, f_{22} = -0.88$	$\mu_{01} = 0.78,\ \mu_{02} = 0.77$ $\mu_{11} = 0.46, \mu_{12} = 0.88$ $\mu_{13} = 0.57$ $\mu_{21} = 0.82, \mu_{22} = 0.56$	1.53226006
FGP approach by Bkay [14]	$x_0 = 0.5$ $x_1 = 1$ $x_2 = 0.5$	$f_{01} = 2.5, f_{02} = -0.5,$ $f_{11} = -1, f_{12} = -0.5$ $f_{13} = -1,$ $f_{21} = -4.5, f_{22} = -1$	$\mu_{01} = 1,\ \mu_{02} = 0.39,$ $\mu_{11} = 0.6,\ \mu_{12} = 0.5$ $\mu_{13} = 0.67,$ $\mu_{21} = 0.45, \mu_{22} = 0.5$	2.17858599

On comparing the distance function and the membership function values (see the Table 3), we observe that our proposed method offers better compromise optimal solution than the solution obtained by Baky [14]. Also, the solution suggested by Baky [14] is obtained using tolerance values repeatedly according to algorithm in order to obtain satisfactory solution of problem. However, in our proposed technique the solution preference by the decision maker is not considered. Then, the approach presented here to the LB-MOMFP problem shows usefulness and viability.

7 Conclusion

This paper present a new method to find a Pareto optimal solution to the linear bilevel multi-objective multi-follower programming problem, by using fuzzy goal programming and Karush-Kuhn-Tucker approach. The main advantage of the proposed methodology is that it yields an efficient solution, reduces the complexity of the solution of the LB-MOMFP problem, which is an NP-hard problem and requires less computational efforts than earlier techniques suggested because the Pareto optimal solution of the propoaed appproach is calculated without considering any inference of any decision variable at any level. Also, the possibility of rejecting the solution again and again by the upper DMs and re-evaluation of the problem repeatedly, by redefining the elicited membership functions, needed to reach the satisfactory decision does not arise. Finally, application of the proposed solution procedure is handled with two numerical examples and then the effectiveness of the solutions obtained by the proposed method is proved. Distance function is utilized to identify optimal compromise solution. On comparing the distance function and the membership function values, we observe that the proposed solution procedure in this paper provides more efficient solutions compared to the solutions procedure of Bkay [14] and Ansari and Rezai [13].

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