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Grey Prediction Model for Forecasting Electricity Consumption

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Abstract Accurate prediction of the future electricity consumption is crucial for production electricity management. Since the storage of electrical energy is very difficult, reliable and accurate prediction of power consumption is important. Different approaches for this purpose were used. In this paper, Grey model (1,1) based on grey system theory has been used for forecasting results. Annual electricity consumption and forecasting data in Mazandaran were used as our case study. Root mean squared error, Mean absolute error and Mean of average percentage error accuracy testing results show that GM(1,1) is outperformed compared with model fitting and model forecasting.

Keywords: Accuracy, Grey system theory, Forecasting, GM(1,1) model, DGM(2,1) model.

1 Introduction

Forecasting the further system development directly from previous observation (data) seems to be a practicable alternative. For this purpose, building a predictor capable of approximating system evolution is necessary. Usually, the used approach is either linear, for example the autoregressive moving average model or nonlinear, and the bilinear model and the nonlinear autoregressive model. Because of the limitation of information and knowledge, only part of the system structure could be fully realized. To overcome this problem, Deng proposed grey systems theory to catch the system development tendency [4]. In recent years, grey systems theory has been successfully employed in agriculture, industry, ecology, meteorology, earthquake, science and technology, medical care and other fields and grey theory of prediction is an important embracement of grey systems theory. GM(1,1) model is the main model in grey theory of prediction. It is one of the most widely used techniques in the grey systems. In recent years, it has also been successfully utilized in many fields and has demonstrated satisfactory results [2]. Numerous studies have shown that the GM(1,1) model has extremely high forecast accuracy with small data samples [24]. The prediction methods include predicting grey models GM(1,1) and DGM(2,1), which, in addition to predicting the accuracy of the methods, are also evaluated using precision measurement indicators. The

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GM(1,1) uses a first order differential equation to characterize an unknown system. At present, grey forecast has been more widely used because it has advantages of a small sample data, computing convenience and short time forecast of high accuracy [23]. DGM(2,1) model is a kind of new grey model which is constructed by grey derivative and second order grey derivative[13].

Nowadays energy requirements continue to rising with increasing population. The development and growth of a country is almost related to the energy utilization rate. For developing governments, making a long term electricity consumption forecasting is a vital for increasing energy productivity. Electricity as one of the most important energy sectors of the country while having an effective role in production and consumption has a special importance in the economic decision making process. Therefore, modeling electricity consumption with good accuracy becomes vital in order to avoid costly mistakes [10]. Many studies have been done on electricity production in the world. Different type of quantitative techniques such as modified Yang's model of granger causality test, box Jenkins models, unit root test for co integration, econometric models with regressions and robust statistical models has been wildly used. In recent studies neural networks [1, 20], neural-network-based grey prediction for electricity consumption prediction [7, 8], It is interesting that prediction accuracy obtained by the original GM(1,1) model can be effectively improved using the Markov chain to realize the residual model [14,22], an optimized nonlinear grey Bernoulli model [15], econometric models [6, 11, 26], regression models [5, 27], and grey models [12, 28] are the most commonly used techniques in energy forecasting studies for different countries [21].

The forecasts will produce new information that will be happen in the future; this information is certainly beneficial for the business or policy makers and decision makers. But in terms of data collection is not any data is easily obtained so, the data are limited. The forecasts using grey forecasting model as one of the approaches that can be used to build a model with limited data sample, with forecasts of short-term problems, to generate forecasting models are valid and does not require consideration of the statistical distribution [19]. Grey modeling methods had been successfully applied to solve time series problems by prof. Deng [3]. This novel concept has become a very effective approach to solve incomplete, poor and uncertain data. Many grey forecasting models have been developed rapidly and successfully applied to multidisciplinary systems such as; financial, economic, energy consumption, military, geological and agricultural systems[18]. Also, there are scholars tried to compare and combine models each other. For example, Yao and Chi [25] compared taguchi method with grey model to optimize electricity demand settings. Electricity demand predictor system with PC based was expected to decrease the usage of electricity. Lu et al. [16] used grey model with time series model (ARIMA) for correction. Three statistical measures namely Mean Absolute Error (MAE), Root Mean Squared Error (RMSE) and Mean of Average Percentage Error (MAPE) were used to analyses and evaluate prediction accuracy. The basic idea of the grey theory is to combine the known time series according to specific rules to form either a dynamic or non-dynamic combination, and then to solve the law of future development according to specific criterion and solution.

In this study, we used the GM(1,1) and DGM(2,1) models to forecasting electricity consumption in Mazandaran province. Also, the forecast accuracy of these models was compared. The rest of the paper is organized as follows: In Section 2, we introduced the basic concepts of the grey systems theory, GM(1,1) and DGM(2,1) models. The some statistical measurements, including relative percentage error, root mean squared error and mean of average percentage error are presented in Section 3. In Section 4, Whit electricity consumption statistics in Mazandaran province by using GM(1,1) and DGM(2,1) models are forecasted for the future. Finally, conclusions are given in Section 5 briefly.

2 Grey prediction models

Grey prediction System is the important part of the grey system theory [4]. In this section, some definitions and concepts which are useful in our further consideration about grey prediction and forecasting are introduced. Grey systems theory is one of the approaches used to study uncertainty, being superior in the mathematical analysis of systems with uncertain information [2]. Grey system theory, by which an information system can be classified into three categories, had been introduced for the first time by prof. Deng [3]. This theory includes white system, grey system and black system. If the system is completely unknown, it is called black while a system that is fully known is called white and a grey system is the system between black and white. Grey refers to incomplete information; in other words, information that is partially clear and partially unclear [24]. As an advantage of statistical experimental model, grey system requires only limited data to estimate the behavior of an unknown system [4]. In summary, the main aim of grey system theory is to focus on the relation between analysis model structure and conditions such as uncertainty, multi data input, discrete data and lack of data for forecasting and decision making. Grey prediction uses GM(1,1) as a foundation for predicting existing data. In reality, this model seeks the future dynamic conditions of elements within a series. The primary advantage of grey prediction is that it does not require substantial amounts of data and has a simple mathematical foundation [24].

2.1 GM(1,1) model

GM(1,1) type of grey model is the most widely used in the literature, pronounced as Grey Model First Order One Variable. This model is a time series forecasting model. The differential equations of the GM(1,1) model have time varying coefficients. In other words, the model is renewed as the new data become available to the prediction model. The GM(1,1) model can only be used in positive data sequences [4]. Although it is not necessary to employ all the data from the original series to construct the GM(1,1) model, the potency of the series data must be more than four[2]. In this paper, since all the primitive data points are positive, grey models can be used to forecast the future values of the primitive data points. In order to smooth the randomness, the primitive data obtained from the system to form the GM(1,1) is subjected to an operator, named Accumulating Generation Operator (AGO). The differential equation (i.e. GM(1,1)) is solved to obtain the n-step ahead predicted value of the system. Finally, using the predicted value, the Inverse Accumulating Generation Operator (IAGO) is applied to find the predicted values of original data. Grey prediction has the following advantages:

- Grey prediction does not require large amounts of historical data. The size of the data is selected based only on actual circumstances and needs. In general, as long as at least four pieces of data are used, a prediction model can be established and forecasting can be performed.
- Grey prediction does not require numerous related factors. Data is easy to acquire, substantially reducing the time and cost of collecting data.
- The accuracy of grey prediction is high [24].

The GM(1,1) model constructing process is described below:
The generation of the initial sequences. By observing certain equal time interval segments of the system, a set of numerical sequences are obtained, named as original numerical sequences

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right\} \tag{1}$$

where $X^{(0)}$ is a non-negative sequence and n is the sample size of the data. When this sequence is subjected to the Accumulating Generation Operation (AGO), the following sequence $X^{(1)}$ is obtained.

$$X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right\}, \tag{2}$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \ k = 1, 2, \ldots, n. \tag{3}$$

The generated mean sequence $Z^{(1)}$ of $X^{(1)}$ is defined as:

$$Z^{(1)} = \left\{z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)\right\}, \tag{4}$$

where $z^{(1)}(k)$ is the mean value of adjacent data, i.e.

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2, 3, \ldots, n \tag{5}$$

The least square estimate sequence of the grey difference equation of GM (1, 1) is defined as follows:

$$x^{(0)}(k) + az^{(1)}(k) = b. \tag{6}$$

The whitening equation is therefore, as follows:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b. \tag{7}$$

In above, $[a, b]^T$ is a sequence of parameters that can be found as follows:

$$[a, b]^T = \left[B^T B\right]^{-1} B^T Y, \tag{8}$$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right]^T \tag{9}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}. \tag{10}$$

According to Eq. (7), the solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-at} + \frac{b}{a}. \tag{11}$$

To obtain the predicted value of the primitive data at time (k + 1), the IAGO is used to establish the following grey model.

$$x_p^{(0)}(k+1) = x_p^{(1)}(k+1) - x_p^{(1)}(k) = \left[x^{(0)}(1) - \frac{b}{a} \right] \left(1 - e^{a}\right) e^{-ak}, k = 1, 2, ..., n-1. \quad (12)$$

GM(1,1) is a model used to make predictions in grey theory. It is expressed as a first differential and has a single input variable. Grey prediction uses the GM(1,1) model as a foundation for forecasting from existing data. It discovers the future dynamic conditions of multiple elements within a series [28].

2.2 DGM (2, 1) model

The DGM (2, 1) model [13] is a single sequence second-order linear dynamic model and is fitted by differential equations. Assume an original series to be $x^{(0)}$

$$X^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n) \right\}, \quad (13)$$

a new sequence $x^{(1)}$ is generated by the accumulated generating operation (AGO).

$$X^{(1)} = \left\{ x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n) \right\}, \quad (14)$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \; k = 1, 2, ..., n. \quad (15)$$

Setting up a second-order differential equation:

$$\frac{d^2 x^{(1)}(t)}{dt^2} + a \frac{dx^{(1)}(t)}{dt} = b. \quad (16)$$

Where

$$\left[a, b\right]^T = \left[B^T B\right]^{-1} B^T Y, \quad (17)$$

$$Y = \begin{bmatrix} (x^{(0)}(2) - x^{(0)}(1)) \\ (x^{(0)}(3) - x^{(0)}(2)) \\ \vdots \\ (x^{(0)}(n) - x^{(0)}(n-1)) \end{bmatrix}, \quad (18)$$

$$B = \begin{bmatrix} -x^{(0)}(2) & 1 \\ -x^{(0)}(3) & 1 \\ \vdots & \vdots \\ -x^{(0)}(n) & 1 \end{bmatrix} \quad (19)$$

According to Eq.(16), we have:

$$x_p^{(0)}(k+1) = \left[\frac{b}{a^2} - \frac{x^{(0)}(1)}{a} \right] e^{-ak} + \frac{b}{a}(k+1) + (x^{(0)}(1) - \frac{b}{a})(\frac{1+a}{a}) \quad (20)$$

The prediction values of original sequence can be obtained by applying inverse AGO to $x^{(1)}$. Namely,

$$x_p^{(0)}(k+1) = x_p^{(1)}(k+1) - x_p^{(1)}(k) = \left[\frac{b}{a^2} - \frac{x^{(0)}(1)}{a}\right]\left(1-e^{a}\right)e^{-ak} + \frac{b}{a}, k = 1,2,\dots,n-1. \quad (21)$$

3 Forecast accuracy measurements

The estimated difference between an actual value and the predicted value obtained using a prediction model is considered a prediction error. As a judgment method, determining the prediction error indicates the success of a forecasting model. In this study, some statistical measurements, including Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean of Average Percentage Error (MAPE), were applied to measure the performance of the proposed models [9,17]. The formula of these measurements is introduced as follows:
RMSE is part of a standard for evaluating forecasting accuracy that presents the sample standard deviation of the differences between actual values and predicted values. RMSE is calculated using the following equation:

$$RMSE = \sqrt{\frac{\sum_{k=1}^{n}(x^{(0)}(k) - x_p^{(0)}(k))^2}{n}} \quad (22)$$

MAE is a measure of difference between actual value and the predicted value the actual value. The formula of MAE is expressed as follows:

$$MAE = \frac{1}{n}\sum_{k=1}^{n}\left|x^{(0)}(k) - x_p^{(0)}(k)\right| \quad (23)$$

MAPE is an accuracy measurement which is popularly applied in forecasting. MAPE denotes the average relative size of the predicted error. MAPE is defined as follows:

$$MAPE = \left(\frac{1}{n}\sum_{k=1}^{n}\frac{x^{(0)}(k) - x_p^{(0)}(k)}{x^{(0)}(k)}\right)\times 100 \quad (24)$$

When MAPE is close to zero, the forecasting model is highly accurate and has provided good performance, and vice versa. Besides this, in accordance with the value of MAPE, the precision rate of forecasting model can be classified into four levels: excellent, good, qualified and unqualified.

Table 1 Forecasting power of methods

MAPE	Forecasting power
<10	excellent
10-20	good
20-50	qualified
>50	unqualified

4 Experimental Results and Discussion

The electricity consumption data from 2013 to 2017 in Mazandaran is used to set up the two grey prediction models GM(1,1) and DGM(2,1) to compare the forecasting trends and the accuracy of prediction of the two models. The actual and forecasted values are shown in Table 2.

The parameters for implementing the model GM (1,1) with 5 time series data are as follows:

$$a= - 0.678514344$$

$$b=4.0388056182$$

$$x_p^{(0)}(k+1) = 4.181926029 \times e^{-ak}$$

The parameters for implementing the model DGM (2,1) with 5 time series data are as follows:

$$a= - 0.079527399$$

$$b= - 0.1028861079$$

$$x_p^{(0)}(k+1) = 2.81672325338 \times e^{-ak} + 1.29371901$$

For calculations related to Eqs. (1)– (21) to obtain predicted values MATLAB software was used. Out of overall electricity consumption, was predicted to be 4.34 in 2014, 4.94 in 2015, 5.21 in 2016 and 5.39 in 2017.Table 2 shows additional details. The predicted values were 5.87 in 2018, 6.28 in 2019, and 6.72 in 2020.

Table 2 Electricity consumption (Terawatt hour)

year	Actual Values	GM(1, 1)	DGM(2, 1)
2013	4.223933	4.223933	4.223933
2014	4.340111	4.475524	4.343597
2015	4.939417	4.789734	4.596051
2016	5.209758	5.126003	4.869403
2017	5.394126	5.485881	5.165381
		Forecasting Results	
2018		5.871024	5.485857
2019		6.283207	5.832864
2020		6.724320	6.208593

In this study, the RMSE, MAE and MAPE error criteria were used. So that each of the predictive methods that have less error is more accurately predicted and can be used to predict future years.

Table 3 Comparative analysis of forecasting error

Model	GM(1,1)	DGM(2,1)
RMSE	0.0966	0.2674
MAE	0.0927	0.2290
MAPE	1.91%	4.46%

Excel was used for (Eqs. (22)–24) to calculate the forecast accuracy indicators of the GM(1,1) and DGM(2,1) models, yielding a RMSE of 0.097 and 0.267, an MAE of 0.093 and 0.229, an MAPE of 1.91 and 4.46, respectively. Table 3 shows additional details.

5 Conclusion

Electricity as one of the most important energy sectors, while having an effective role in production and consumption, is of particular importance in the economic decision making process. With proper prediction of power consumption, it is possible to plan well for power generation. In this study, by using power consumption statistics was projected for future. The results show that the GM(1,1) model is more accurate than DGM(2,1). It may be used for other real cases for energy consumption forecasting.

References

1. Chen, C.C., Kuo, Y.C., Huang, C.H. and Chen, A.P., (2014). Applying market profile theory to forecast Taiwan index futures market, Expert Systems with Application. 41, 4617-4624.
2. Cui, J., Liu, S.F., Zeng, B. and Xie, N., (2013). A novel grey forecasting model and its optimization, Applied Mathematical Modelling. 37, 4399–4406.
3. Deng, J.L., (1982). Control problems of grey systems, Systems and Control Letters. 1 (5), 288-294.
4. Deng, J.L., (1989). Introduction to grey system theory, The Journal of Grey System. 1 (1), 1-24.
5. Ediger, V.S. and Akar, S., (2007). ARIMA forecasting of primary energy demand by fuel in Turkey, Energy Policy. 35 (3), 17018-17031.
6. Ho, S. L., Xie, M. and Goh, T.N., (2002). A comparative study of neural network and Box–Jenkins ARIMA modeling in time series predictions, Computers and Industrial Engineering. 42, 371-375.
7. Hu, Y.C., (2017). Electricity consumption forecasting using a neural network based grey prediction approach, Journal of the Operational Research Society. 68, 1259–1264.
8. Hu, Y.C. and Jiang, P., (2017). Forecasting energy demand using neural network based grey residual modification models, Journal of the Operational Research Society. 68, 556–565.
9. Hsu, L.C., (2003). Applying the grey prediction model to the global integrated circuit industry, Technological Forecasting and Social Change. 70, 563–574.
10. Kaytez, F., Toplamacioglu, M.C., Cam, E. and Hardalac, F., (2015). Forecasting electricity consumption: A comparison of regression analysis, neural networks and least squares support vector machines, Electrical Power and Energy Systems. 67, 431-438.
11. Khashei, M. and Bijari, M., (2011). A novel hybridization of artificial neural networks and ARIMA models for time series forecasting, Applied Soft Computing. 11, 2664-2675.
12. Lee, Y. and Tong, L., (2011). Forecasting energy consumption using a grey model improved by incorporating genetic programming, Energy Convers Management. 52, 147–152.
13. Li, J., (2012). The optimization DGM(2,1) model and its application, Journal of Grey System. 24 (2), 181–186.
14. Liu, S.F., Yang, Y.G. and Wu, L.F., (2014). Grey System Theory and Application; Science Press: Beijing, China.
15. Lu, J., Xie, W., Zhou, H. and Zhang, A., (2016). An optimized nonlinear grey Bernoulli model and its applications, Neurocomputing. 177, 206–214.
16. Lu, I. J., Lewis C. and Lin, S. J., (2009). The forecast of motor vehicle, energy demand and CO2 emission from Taiwan's road transportation sector, Energy Policy: 37 (8), 2952–2961.
17. Ma, H. and Zhang, Z., (2009). Grey Prediction with Markov-Chain for crude oil production and consumption in china; Springer, Berlin.
18. Rathnayaka, R.M., (2014). Cross Cultural dimensions of business communication: Evidence from Sri Lanka, International Review of Management and Business Research. 3 (3), 1579-1587.
19. Shih, C.H., Hsu, Y.T., Yeh, J. and Lee, P.C., (2011). Grey number prediction using the grey modification model with progression technique, Applied Mathematical Modelling. 35, 1314–1321.

20. Sozen, A. and Arcaklıoglu, E., (2007). Prospects for future projections of the basic energy sources in Turkey, Energy Sources. 2 (2), 183-201.
21. Suganthi, L. and Samuel, A.A., (2012). Energy models for demand forecasting-a review, Renewable and Sustainable Energy Reviews. 16, 1223-1240.
22. Sun, X., Sun, W., Wang, J. and Gao, Y., (2016). Using a grey Markov model optimized by Cuckoo search algorithm to forecast the annual foreign tourist arrivals to China, Tourism Management. 52, 369–379.
23. Talebnejad, A. and Nadaf, A., (2011). The use of grey system theory in predicting the road traffic accident in fares province in Iran, Australian Journal of Business and Management Research. 1 (9), 18-23.
24. Tsai , S.B., Xue, Y. Zhang, J., Chen, Q., Liu ,Y., Zhou, J and Dong, W., (2017). Models for forecasting growth trends in renewable energy, Renewable and Sustainable Energy Reviews. 77, 1169-1178.
25. Yao A. W. L. and Chi S. C., (2004). Analysis and design of a taguchi grey based electricity demand predictor for energy management systems, Energy Conversion and Management. 45 (7–8), 1205–1217.
26. Yu, S., Zhu, K. and Zhang, X., (2012). Energy demand projection of China using a path coefficient analysis and PSO-GA approach, Energy Conversion Management. 53, 142-153.
27. Yumurtacı, Z. and Asmaz, E., (2004). Electric energy demand of turkey for the year 2050, Energy Sources. 26 (12), 1157–1164.
28. Zhou, P., Ang, W. B. and Poh, L. K., (2006). A trigonometric grey prediction approach to forecasting electricity demand, Energy. 31, 2839–2847.