Estimation the value at risk using an adaptive model

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Abstract The modeling of the volatility is one of the most problems in financial research areas such as pricing derivatives, risk estimation and financial decision-making. In recent years, many attempts have been made to identify the optimal model for better estimation and forecasting of volatility and risk in investment and policies economic. This study aimed to identify and investigate a suitable model for estimating the value at risk in the stock exchange data. Based on the four stock indices, the value at risk is estimated using an adaptive model. Using different criteria, it is observed that the adaptive model has a good performance.

Keyword: Adaptive Model; Risk Management; Conditional Autoregressive Value at Risk; Performance Indices; Stock Exchange.

1 Introduction

Volatility is one of the most important aspects of the development of financial markets and plays an essential role in portfolio management, arbitrage pricing, and market rules. Within the different forms of financial risk measurement tools, value at risk (*VaR*) is widely accepted as a fundamental tool for risk management and it has become a standard benchmark for measuring financial risk. There are several literatures discussing the *VaR* estimation. For instance, Shahmoradi and Zanganeh [1] obtained value at risk using the parametric method and the results show that these types of models are quite successful in estimating the *VAR*. Shahiki Tash et al. [2] calculated the *VaR* in the Tehran stock exchange. The results show that the risk level is high in Tehran stock exchange by comparing the estimated value of risk; the distribution of the generalized error has a better performance than the t-student distribution and normal distribution. Chen and Lu [3] estimated the *VaR* models and found that CAViaR and the NIG-based estimation is robust and deliver accurate *VaR* estimation, if the short forecasting interval is considered. Lima and Neri [4] compared the Value-at-Risk measure. The results indicate that the non-robust methodologies have higher probability to predict *VaRs* with too many violations. Taylor [5] forecasted the value at risk and expected shortfall using a semi-parametric approach based on the asymmetric Laplace distribution. Gaglianone et al. [6] evaluated value-at-risk models via quantile regression. The methodology allows to identify periods of an increased risk exposure based on a quantile regression model. Aloui and

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Mabrouk [7] evaluated the *VaR* and the expected shortfall for some major crude oil and gas commodities for both short and long trading positions. The *VaR* of *X* (with the cumulative distribution function $F_X(x)$) is given by

$$VaR_\alpha(X) = \inf\left\{x \in R \,\middle|\, F_X(x) \geq \alpha\right\} \tag{1}$$

In other words, the *VaR* is equal to $(1-\alpha)$ worst-case quantile of a loss distribution. Different aspects of the *VaR* have considered as a tool for managing risk. Conditional value at risk (*CVaR*) and the conditional autoregressive value at risk (*CAViaR*) are the two development of the *VaR*. The *CVaR* of a random variable *X*, as a conditional expectation of the loss within that quantile can be written as:

$$CVaR_\alpha(X) = E\left[X \,\middle|\, X \geq VaR_\alpha(X)\right] \tag{2}$$

Moreover, *CAViaR* specifies the evolution of the quantile over time using a special type of autoregressive process, and estimates the parameters with regression quantiles. This approach has strong appeal in that it focuses the tail of return distribution directly and does not rely on any distributional assumption. The *CVaR* and *CAViaR* have been studied by several authors; see for example, Chacha et al. [8] studied application of conditional autoregressive value at risk model to kenyan stocks. It was found that the asymmetric CAViaR slope specification works well for the Kenyan stock market and is best suited for estimating *VaR*. Jooyoung and James [9] show that the implied volatility has more explanatory power as the focus moves further out into the left tail of the conditional distribution of S&P500 daily returns. Huang at al. [10] studied the *CAViaR* and show that time-varying *CAViaR* models can do a better job for *VaR* prediction. Alexander and Baptista [11] compared the *VaR* and *CVaR* constraints on portfolio selection with the mean-variance model**.**

The main purpose of this paper is to consider the adaptive model for estimating the *VaR*. In view of above considerations, the rest of the article is organized as follows. In Section 2, the conditional autoregressive value at risk is briefly introduced. The adaptive model is presented in Section 3. Different performance indices are considered in Section 4. For illustration, four sets of real data are discussed in Section 5. Conclusions are made in Section 6.

2 Conditional Autoregressive Value at Risk

In this Section, we introduce the conditional autoregressive value at risk using the idea of the Engle and Manganelli [12]. Suppose that we observe a vector of portfolio returns $\left\{y_t\right\}_{t=1}^{T}$. Let θ be the probability associated with *VaR*, letting x_t be a vector of time *t* observable variables, and β be a vector of unknown parameters. The conditional autoregressive value at risk can be written as:

$$f_t(\beta) = \beta_0 + \sum_{i=1}^{q} \beta_i f_{t-i}(\beta) + \sum_{j=1}^{r} \beta_j l(x_{t-j}). \tag{3}$$

where $p = q + r + 1$ is the dimension of β and l is a function of a finite number of lagged values of observables. The autoregressive terms $\beta_i f_{t-i}(\beta)$; $i = 1,...,q$ creates a smooth path between time-oriented quantiles. The first term, β_0, is simply a constant and the role of $l(x_{t-j})$ is to link $f_t(\beta)$ to observable variables that belong to the information set [12].

3 Adaptive model

Using an autoregressive framework, conditional autoregressive value at riskmodel aims to derive the evolution of the desired quantile rather than extracting the quantile from an estimate of a complete distribution or from a volatility estimate. The adaptive model is presented in the following expression.

$$Q_t(p) = Q_{t-1}(p) + \beta\left(\left[1 + \exp(G[r_{t-1} - Q_{t-1}(p)])\right]^{-1} - p\right). \tag{4}$$

Here $Q_t(p)$ is the p-th quantile at time t, β is regression parameter, and r_t is the excess return at time t and G is a constant. Engle and Manganelli note that the structure of the adaptive conditional autoregressive value at risk model is such that the estimator learns nothing from the extent to which the quantile has or has not been exceeded since it considers only whether $Q_t(p)$ is larger than r_{t-1} or not [12].

4 Performance indices

In order to compare efficiency and accuracy of different methods, mean square error (MSE), mean absolute error (MAE), and mean error (ME) were used as performance indices. These performance indices are defined as,

- **Mean squared error**

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(\sigma_{t,f}^2 - \sigma_t^2)^2 \tag{5}$$

- **Mean absolute error**

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|\sigma_{t,f} - \sigma_t\right| \tag{6}$$

- **Me error**

$$ME = \frac{1}{n}\sum_{t=1}^{n}(\sigma_{t,f} - \sigma_t) \tag{7}$$

See, Chai and Draxler [13].

4 Data analysis

In order to demonstrate and compare the forecasting performance of the proposed model, we consider daily financial returns from 4 stock markets index in stock exchange: exchange price index, price and cash returns index, 50 active companies index, 30 largest companies index. Our sample is divided in two groups (in-sample and out-of-sample). We set, where G entered the definition of the adaptive model in Section 3. In principle, the parameter G itself could be

estimated; however, this would go against the spirit of this model, which is simplicity. We first obtained the return of portfolio with price process r_t over the time period [$t, t - 1$] as:

$$r_t = Ln(S_t / S_{t-1}) \tag{8}$$

where, S_t is the price of the underlying asset at time t. The Plots of the TEPIX, PCRI, 50CI and 30CI are presented in Figures 1-4 respectively. The parameter of the model is estimated and then determined the efficiency and superiority of this model in each of the stock exchange indices. All programming is written in R software.

Based on the in-sample data, we estimated the adaptive model parameter in 4 stock market indices. Based in-sample data, this estimate is performed in $Q_{(0.5)}$. Table 1 presents the results as obtained for the 5% *VaR*. The results indicate that the model works well.

Table 1 Estimation of In-Sample Parameters of Adaptive Model for 4 Stock Exchange Index.

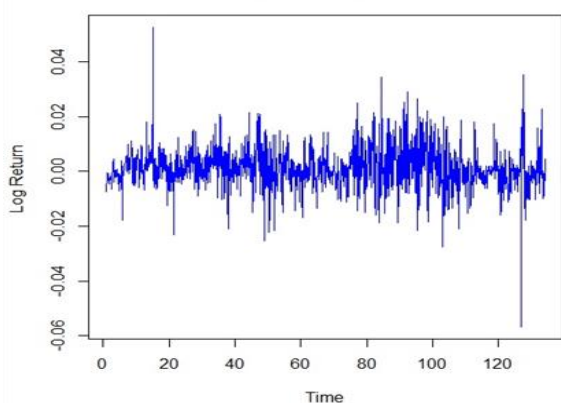| **Adaptive Model**				
parameter	**TEPIX**	**30CI**	**PCRI**	**50CI**

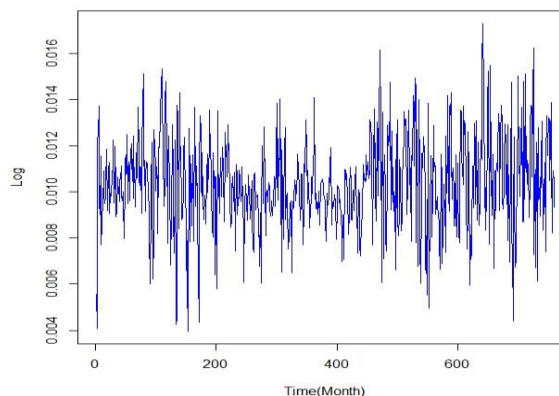**Fig. 1** Plot of stock returns (*TEPIX*).

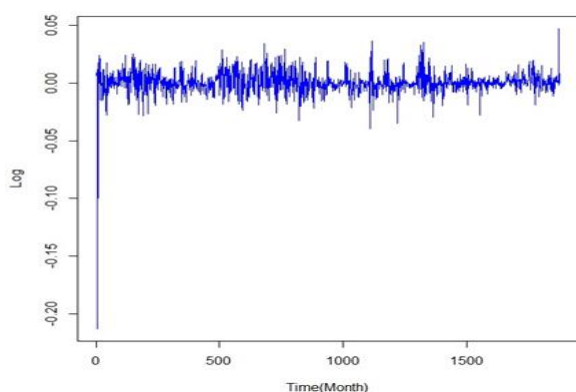**Fig. 2** Plot of stock returns (*PCRI*).

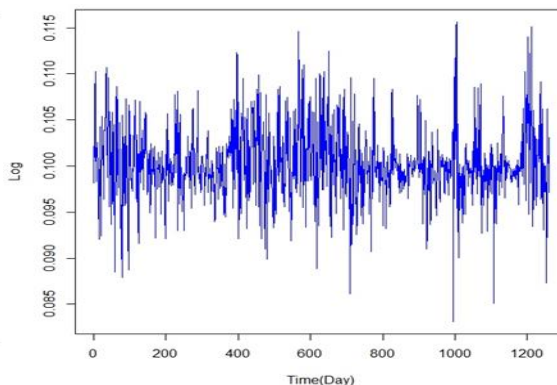**Fig. 3** Plot of stock returns (*30CI*).

Fig. 4 Plot of the stock returns (*50CI*).

Now for testing the performance of the adaptive model, we considered the model selection criteria, such as *MSE*, *MAE*, and *ME* based on the out-of-sample data. The results are reported in Table 2. The overall results show that this methodology provides very accurate measurement of the *VaR* for the proposed data. We also present the real and forecasting values for the *30CI* and *PCRI* in Figures 5 and 6 respectively. It is observed that the proposed model can be predicted the real values well. The results of the *TEPIX* and *50CI* are similar.

Table 2 The performance indices based on the Out-of-sample data.

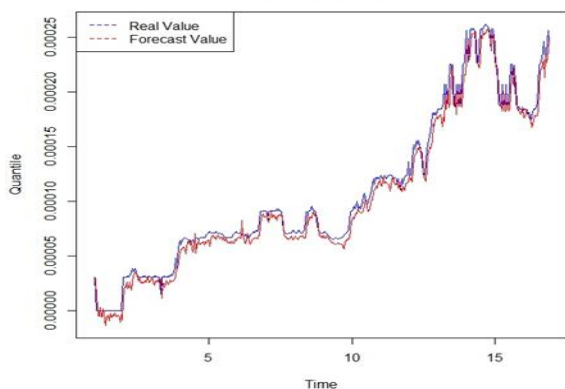|	Adaptive Model			
	TEPIX	***30CI***	***PCRI***	***50CI***
MSE	16439.72×10^{-11}	87156.35×10^{-11}	10090.51×10^{-10}	67695.27×10^{-11}
MAE	23.96999×10^{-6}	74.29467×10^{-6}	80.92356×10^{-6}	60.8817×10^{-6}
ME	6132.336×10^{-8}	57.62543×10^{-6}	51.05249×10^{-6}	22.03611×10^{-6}

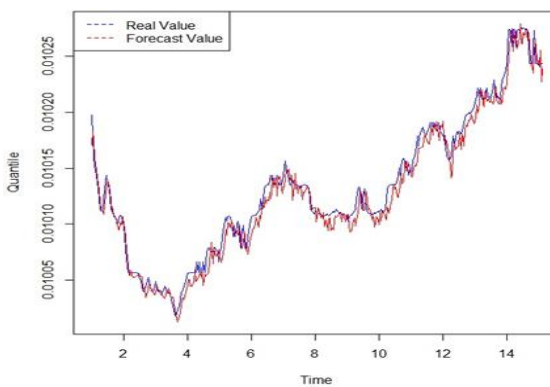**Fig. 5** Real data and forecasting value for the *30CI*.

Fig. 6 Real data and forecasting value for the *PCRI*.

5 Conclusion

Considering the importance of fluctuations in the stock market, this paper introduces an adaptive model for estimating the *VaR* and forecasting the volatility in stock exchange data. Using the *R* Software, the parameter of the model is estimated and predicted. The results obtained from the performance indices are indicative of the superiority of the consistent model for estimating stock predictions based on the terms and conditions of the period. Therefore, it can be concluded that the proposed model has good performance in predicting stock exchange indices.

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