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Multi-mode resource constrained time-cost trade off problem by including tardiness penalty cost of renewable resources and fuzzy duration

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Abstract Nowadays, the project scheduling problem is one of the most common issues in project control and planning, especially the time-cost trade-off problem. In this paper, a bi-objective problem is presented to minimize the cost and makespan simultaneously. Besides the common constraints in literature, it is assumed some renewable resources are hired. Each of them has a specific access time and deadline; they cannot be used before the access time but can be used after the deadline due to the cost of delay penalty. The project costs consist of direct costs, indirect costs, and tardiness penalty cost of renewable resources. Because of the uncertainty, the project times are considered as fuzzy numbers. Due to the NP-Hard nature of these problems, the Tabu search algorithm is used to solve them. The results are also compared with the genetic algorithm to check the quality of answers.

Keyword: Project Scheduling, Time-Cost Trade-Off, Tardiness Penalty Cost of Renewable Resources, Genetic Algorithm (GA), Tabu Search (TS) Algorithm.

1. Introduction

In today's competitive business conditions, the ability of the project manager for scheduling the activities has become increasingly important concerning the time and cost required for obtaining competitive priorities [1]. Critical Path Method is widely used for scheduling and planning in construction projects. This approach is about minimizing the project duration by considering the time and specifying critical activities. But it does not take into account the availability of resources. Each activity can start after the completion of prerequisite activities. But in practice, the availability of resources and their allocation can affect the scheduling of the project [2]. To overcome this limitation of the critical path approach, many techniques and optimizations presented in previous studies can be used. This research can be classified into four categories: scheduling with resource constraint, time-cost trade-off, resource leveling and resource allocation [3]; and three types of constrained resources, namely renewable resources, non-renewable resources, and constrained resource [4]. Project scheduling problems with resource constraints has been a common problem in recent decades. The general form of this problem is the Multi-Mode Resource-Constrained Project Scheduling Problem. In such

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problems, each activity can be performed by some modes. Each mode has its specific time and resources required [5].

The Multi-Mode Resource-Constrained Project Scheduling Problem is among NP-Hard problems, and none of the exact algorithms cannot achieve optimal scheduling large-scale problems within the logical duration [6]. That is why in practice innovative and meta-heuristic algorithms.

Cost and time are critical aspects of the project [7-8]. We can consider the time-cost trade-off problem to make the project schedule closer to the real-world conditions. These issues have been widely studied. It was introduced in 1979 by Harvey and Peterson, Hindelang and Muth [9, 10]. Many of these models can be divided into two types, certain and uncertain [11]. For the first one, we can refer to the articles by Kelly 1961, Siemens 1971, Philips and Desouki 1977 and Talbot in 1982 [12, 15]. Uncertainty in cost-time trade-off problems was raised in 2008 by Eshtehardian et al. [16]. Many probabilistic models with an uncertain duration of activities include the articles of Charnes and Cooper in 1962, and Glenco and Gonic in 1997, Vollmer in 1985, and Gyotjeher et al. in 2002 [17, 20]. Another approach to deal with uncertain data is applying fuzzy theory. The comparison of fuzzy approach and probabilistic approach can be studied in the paper by Shapley et al. in 1997 [21, 22].

Using the time-cost trade-off problem is common in project management. The presented approaches in previous researches classify into three categories:

The exact algorithms such as linear programming, integer programming, dynamic programming, branch and bound algorithm etc. [23, 26].

Innovative Algorithm [27, 30]

Meta-heuristic algorithm [31, 36]

Also, some researches there are about scheduling, resource-constrained, multi-mode, uncertainty and meta-heuristic which have been presented in comparing table as detail [37, 49].

Multi-Mode Resource-Constrained Project Scheduling Problem is a bi-objective problem aiming to minimize the cost and makespan of a project for determining the mode of activities on the precedence relationships and constrained resources.

In this paper, a Multi-Mode Resource-Constrained problem was developed by considering the tardiness penalty cost of renewable resources. Assume that renewable resources are not available during all periods of the project. Consider the access time and due date for these resources. But they may use it after the due date by including tardiness penalty cost. The project costs are direct, indirect and tardiness penalty costs. The direct costs are dependent on the mode of activities, and indirect costs are constant. The duration of implementing each activity can be a triangular fuzzy number. In practice, experts and decision-makers determine the duration of activities between the optimistic and pessimistic values. For example, if 15-month is pessimistic duration, 23-month is optimistic, and 19-month is the most likely value, then these numbers can be represented in the form of triangular fuzzy numbers:

$\widetilde{D}_1 = (D_1, D_2, D_3) = (15,19,23)$

A comparison table can be generated based on above mentioned references as follows:

Table 1 Comparing table of researches

Ref.	Scheduling	resource-constrained	renewable resources	time-cost trade-off	Multi-mode	Uncertainty	Meta-Heuristic
1	*						
2	*	availability of resources					
3		*		*			
4		*					
5	*	*					
6		*			*		*
7-10				*			
11						*	
12-15						Deterministic	
16						*	
17-20						Stochastic / Probabilistic	
21						Fuzzy	
22						Probabilistic and Fuzzy	
23-26							Exact
27-30							Heuristic
31-36							*
37						Fuzzy	
38	*						
39	*						*
40	*	*			*		
41-44	*	*					
45-47	*						
48	*	*					Exact
49	*	*					*
Current Research	*	*	*	*	*	Fuzzy	*

2 Problem Statement

The problem studied in this paper is the Multi-Mode Resource-Constrained time-cost trade-off. We display the project through a network of nodal activities G = (V, E) where the arcs (E) are precedence relationships, and the nodes (j = 1, 2, 3 ...., J) represent the activities. The activity J can be started after the completion of all prerequisite activities (Pj). Each activity jϵv can only be implemented in one of the several possible modes by the set Mj=1,…,M. Given the complexity of uncertainty factors, the uniqueness of the exact duration of every activity is difficult due to lack of information. In the real world, there are a lot of non-probabilistic factors affecting large-scale projects, and it is not possible to encounter these non-probabilistic factors by probabilistic approach. Hence, the duration of activities is considered as triangular fuzzy numbers. In this problem, we have both direct and indirect costs. The first type is dependent on the selection mode for the implementation of activities. But the second cost is a cost that is constant during the project execution. While renewable resources can be used after their due date by including tardiness penalty cost. The purpose of this paper is to schedule the activities and to determine the mode of execution of each activity so that the objective functions are optimized.

Table 2 The assumptions and parameters of the problem

Assumptions	Parameters
There is a prerequisite finish to start relationships between activities Renewable sources are not available during the entire project. No renewable resource is available before the access time, but it may be used after the due date by taking a tardiness penalty cost. The duration is considered as a triangular fuzzy number for each activity. An activity cannot be interrupted until the completion.	N: Number of project activities Mj: Set of executive modes in which the activity j can be performed Pj: Set of predecessors of activity j K: Number of resources djm: The duration of activity j executed in mode m rjmk: Units of resource k needed for implementation of activity j in mode m Cjm: Direct cost to executive activity j in mode m C: Indirect cost (constant) ESTj, LSTj: Earliest and Latest start time of the activity j si: Start time of activity i yk: The access time of renewable resource k dk: Due date of renewable resource k Pk: Tardiness penalty cost of renewable resource k per each period(USD/day) Rk: Availability of resource k RTk: The release time of renewable resource k by the project Nk: Set of activities using renewable resource k

For converting fuzzy numbers to the deterministic values, the expected value operator for the fuzzy criteria $\mathrm{E}^{\mathrm{Me}}[\xi]$ is introduced to deal with the uncertainty problem of the cost-time trade-off. The expected value for the triangular fuzzy number is defined as follows [37]:

$$\mathrm{E}^{\mathrm{Me}}[\xi]=\begin{cases}\frac{\lambda}{2}r_1+\frac{r_2}{2}+\frac{1-\lambda}{2}r_3\,, & \text{if } r_3\le 0\\ \frac{\lambda}{2}(r_1+r_2)+\frac{\lambda {r_3}^2-(1-\lambda){r_2}^2}{2(r_3-r_2)}\,, & \text{if } r_2\le 0\le r_3\\ \frac{\lambda}{2}(r_3+r_2)+\frac{(1-\lambda){r_2}^2-\lambda {r_1}^2}{2(r_2-r_1)}\,, & \text{if } r_1\le 0\le r_2\\ \frac{(1-\lambda)r_1+r_2+\lambda r_3}{2}\,, & \text{if } 0\le r_1\end{cases} \tag{1}$$

Where λ is a pessimistic-optimistic index to determine DM. Since all fuzzy variables in this study are non-negative triangular fuzzy numbers, the cost-time trade-off problem belongs to the r1≥0. For example, conversion of the duration for each activity which is a triangular fuzzy number will be as follows:

$$\widetilde{D_i}\rightarrow E\left[\widetilde{D_i}\right]=\frac{(1-\lambda)D_{i1}+D_{i2}+\lambda D_{i3}}{2}$$

2.1 Mathematical model

x_{jm} is a decision variable that is equal to one if and only if the activity j is implemented in the mode m, otherwise it will be zero.

The proposed mathematical model is as follows:

$$\min z_1 = \sum_{k=1}^{K}\sum_{j=1}^{n}\sum_{m=1}^{M_j}\left(c_{jm}.r_{jmk}\right)\sum_{t=EST_j}^{LST_j} x_{jmt} + C\sum_{t=EST_n}^{LST_n} t.x_{n1t} + \sum_{k=1}^{R} P_k.(RT_k - d_k) \qquad (2)$$

$$\min z_2 = \sum_{t=EST_n}^{LST_n} t.x_{n1t} \qquad (3)$$

St:

$$\sum_{m=1}^{M}\sum_{t=ESt_j}^{LSt_j} x_{jmt} = 1\ ,\quad \forall j = 1, \dots, n \qquad (4)$$

$$\sum_{m=1}^{M}\sum_{t=ESt_i}^{LSt_i} (t + d_{im})x_{imt} \le \sum_{m=1}^{M}\sum_{t=ESt_j}^{LSt_j} t.x_{jmt}\ ,\qquad \forall j = 0,1 \dots, n\ ,\quad i \in p_j \qquad (5)$$

$$\sum_{j=1}^{n}\sum_{m=1}^{M} r_{jmk} \sum_{z=t-d_{jm}+1}^{t} x_{jmz} \le R_k\ ,\qquad \forall\, k = 1,2 \dots R\ ,\quad t = 1,2 \dots, T \qquad (6)$$

$$s_j \ge \rho_k,\ \ \forall j \in N_k,\ k = 1,2 \dots R \qquad (7)$$

$$RT_k \ge s_j + d_j,\ \ \forall\, k = 1,2, \dots, R\ \ , j \in N_k \qquad (8)$$

$$s_j = \sum_{m=1}^{M}\sum_{t=EST_j}^{LST_j} t.x_{jmt}\ ,\quad \forall j = 0,1 \dots n, m \in M_j, t = ESt_j, \dots, LSt_j \qquad (9)$$

$$x_{jmt} \in (0,1),\quad j = 1,2, \dots, n, m \in M_j, t = ESt_j, \dots, LSt_j \qquad (10)$$

$$s_j, RT_k \ge 0,\qquad \forall j = 0,1 \dots n\ ,\ k = 1,2 \dots R \qquad (11)$$

Equation (2), explains the total project cost, which includes three parts: direct costs, indirect costs, and tardiness penalty cost of renewable resources. The first part is for the function of direct costs, the second part for indirect costs, and the third part calculates the tardiness penalty cost of renewable resources. Equation (3), expresses the second objective function to minimize the makespan. Equation (4) shows the obligation to perform every activity by one mode. Constraint (5), expresses the precedence relationships. Constraint (6), shows the amount of resource k needed to perform the activity j in mode m. Equation (7), ensures the starting time of activities that use the renewable resources k greater than or equal to the access time. Constraint (8), shows the release time of renewable resource k. Equation (9), shows the starting time of each activity. Equation (10) and (11) denotes the domain of the variables.

3 Tabu search algorithm

In this section, we propose our solution for minimizing both time and cost of a project. As explained already, each activity of the project is capable of being performed in multi modes and so the problem is how to assign each activity a mode to minimize the time and cost of the whole project simultaneously. So, if we consider N as the number of project activities, the number of all possible ways to assign each activity a mode will be equal to mN; while m is representing the number of modes for each activity. So, there will be a notable number of possibilities, and examining all of them will be exhausting from a time perspective and it will increase with an exponential function by increasing the number of activities. To make a solution possible for all kinds of problems with a different number of activities, we need to use an intelligent algorithm to reduce the number of examined possibilities and yet gain a good solution.

In this paper, we use Tabu search as this intelligent algorithm and in this section, we will explain the mechanism of its performing algorithm. Each activity with multiple modes will be a dimension in an N-dimensional space. So, for each point in the solution space, there will be 2N possible moves in N possible directions for this point to move toward one of its neighbors. Each point is a solution for the mode, assigning problem and so each point has a time and cost of a project. The problem as mentioned before is to find the best point with both minimum time and minimum cost. An intrinsic feature of this problem is that by minimizing time, the cost of the project will be elevated, and by minimizing cost, the time of the project will be increased; so the best point should have both minimum cost and time expense as it is possible.

Tabu search will start from an arbitrary point in the space, it then moves toward the best answer from its neighbors. To prevent from staying in a local best point, the new best answer and new best direction of movement will transport to a list, named Tabu list. For the next X moves, Tabu search cannot choose direction or point from the Tabu list and so the odds of being trapped in a local answer will be considerably lessened. After a certain number of iterations, or achieving the termination condition, the algorithm will stop searching and the best answer or point experienced until then will be extracted as the best solution. In this particular problem, we consider X (number of iterations for a point and direction to stay in the Tabu list) equal to N/2 and the maximum number of iterations sets as 200.

The lone dim part of this method is the way Tabu search chooses the best answer among the neighbors of each point. Each point has a cost and time feature, so we need a criterion to integrate these two features into a single feature so it will be comparable to another point. The purpose of implementing this method is to minimize both time and cost of the project, so we can minimize the bigger feature and so the criteria will be max (T, C), while T stands for time and C is an abbreviation for cost. So the best point from neighbors of a point will be the one with a minimum value of this criteria and the function will be min (max (T, C)).

The big question now is that time and cost are different measures and logically, the cost of a project will always be more than the time of the project. To solve this problem, we use a normalization function to smooth the values of time and cost and make them comparable. Normalization function used in this paper formulized as below:

$$normalized\ time = \frac{time - \min(time)}{\max(time) - \min(time)}$$
$$normalized\ cost = \frac{cost - \min(cost)}{\max(cost) - \min(cost)}$$

Min (time) is the minimum possible time of performing the project and logically, the point in possession of this time, cannot own the best cost too. This minimum time achieves from performing a one objective optimization problem that the purpose will be minimizing time. To find this point, we perform a Tabu search algorithm in solution space and the best answer will be the min (time) required for the normalization formula. The same procedure is implemented to find the values of max (time), min (cost), and max (cost).

4. The computational results

As mentioned at the beginning, these issues are included in NP-Hard problems. We cannot obtain the optimum solution within reasonable for large problems by an exact algorithm. Hence, in recent years, heuristic and meta-heuristic algorithms are mostly used. In this paper, the Tabu search algorithm is used to solve, and then the results are compared with the genetic algorithm to check the quality. There is an example of a project scheduling problems library (PSPLIB) selected here [38]. In this example, each activity is performed in three modes. Nonrenewable source costs are N1=20 and N2=17. In Table 2, the number of modes, a successor of each activity, and precedence relationships between the activities are listed. Both the duration of the activity and resources required for them have been separately listed in Table 3. Each activity uses two renewable and two nonrenewable resources. Activity 1 and 12 are virtual activities indicating the beginning and the end of the project. Finally, in Table 4, the available amount of each renewable source, the access time, due date, and tardiness penalty cost are included.

Table 3 Precedence relationships between activities

Activity No.	Number of modes	Number of precedence	successor of activities
1	3	3	2,3,4
2	3	2	5,6
3	3	2	10,11
4	3	1	9
5	3	2	7,8
6	3	2	10,11
7	3	2	9,10
8	3	1	9
9	3	1	12
10	3	1	12
11	3	1	12
12	3	0	0

Table 4 The duration of activity, resources required for each activity

Activity No.	Modes	Duration	Renewable resources required(R_1,R_2)	Nonrenewable resources required(NR_1,NR_2)
1	1	(0,0,0)	0,0	0,0
2	1 2 3	(0,3,6) (6,9,12) (7,10,13)	6,0 5,0 0,6	9,0 0,8 0,6
3	1 2 3	(0,1,2) (0,1,2) (2,5,8)	0,4 7,0 0,4	0,8 0,8 0,5
4	1 2 3	(0,3,6) (2,5,8) (6,8,10)	10,0 7,0 6,0	0,7 2,0 0,7
5	1 2 3	(1,4,7) (3,6,9) (8,10,12)	0,9 2,0 0,5	8,0 0,7 0,5
6	1 2 3	(0,2,4) (1,4,7) (3,6,9)	2,0 0,8 2,0	8,0 5,0 0,1

7	1	(0,3,6)	5,0	10,0
	2	(3,6,9)	0,7	10,0
	3	(6,8,10)	5,0	0,10
8	1	(1,4,7)	6,0	0,1
	2	(8,10,12)	3,0	10,0
	3	(8,10,12)	4,0	0,1
9	1	(0,2,4)	2,0	6,0
	2	(5,7,9)	1,0	0,8
	3	(8,10,12)	1,0	0,7
10	1	(0,1,2)	4,0	4,0
	2	(0,1,2)	0,2	0,8
	3	(6,9,12)	4,0	0,5
11	1	(3,6,9)	0.2	0,10
	2	(6,9,12)	0.1	0,9
	3	(8,10,12)	0.1	0,7
12	1 2 3	0	0,0	8

Table 5 The information of renewable resources

Renewable sources	Number of Available items	Access time	Due date	Tardiness penalty cost per time unit
1	20	2	20	8
2	10	0	17	6

5 Discussion

As mentioned above, the renewable resources are usable only in a determined time interval. These resources have an access time and due date. They have penalty costs after the due date. Notably, normalization was performed to compare the cost and time. Given the tardiness penalty cost of renewable resources, the longer is the duration of the project, the costs would be increased; because the renewable resources may not be delivered at a given time and the tardiness penalty would not apply to them. So, these data have been shown in table 4.
The efficient solution is inserted in Table 5 by Tabu search and genetic algorithm based on the value of λ= 0.2, λ= 0.5, and λ = 0.9(the value of λ is determined by an academic expert). The mode of activities will be different in any solution by these two algorithms. Normalization has been made for comparison concerning the bi-objective nature of the objective function.

Table 6 Efficient solution

Λ	Algorithm	Selected modes	makespan	Total project costs	Computational Time (s)
0.2	GA	(1,1,3,2,2,1,3,1,1,1,1,1)	16	3906	122.45
	TS	(1,1,2,2,2,1,1,1,1,1,1,1)	12	2912	53.12
0.5	GA	(1,1,2,1,1,1,1,1,2,1,1)	15	4057	128.26
	TS	(1,1,2,2,1,1,1,1,1,1,1,1)	15	3844	59.45
0.9	GA	(1,1,3,2,1,2,1,3,1,3,1,1)	24	6205	131.31
	TS	(1,1,2,2,1,3,1,1,1,1,1,1)	19	4536	60.11

According to the results obtained in each of three λ, the Tabu search algorithm has shown a better solution than the GA. In all three cases, the Tabu search algorithm has calculated the project schedule with less cost, makespan, and computational time. λ is the optimistic-pessimistic index. As observed, a change in this index has a significant impact on the optimum solution. The less is the value of λ, the expert's view would be more optimistic and it is expected that the project will be done in a shorter period. Based on computational results, the Tabu search takes less time from the genetic algorithm to solve the problem.

Figure (1) compares the efficient solutions of two algorithms based on the makespan, and figure (2) based on the total cost of the project.

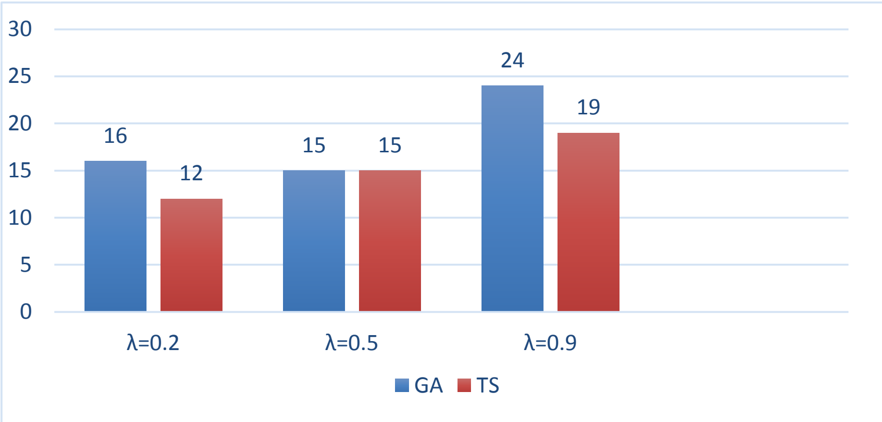

Fig. 1 The comparison between two algorithms based on the makespan

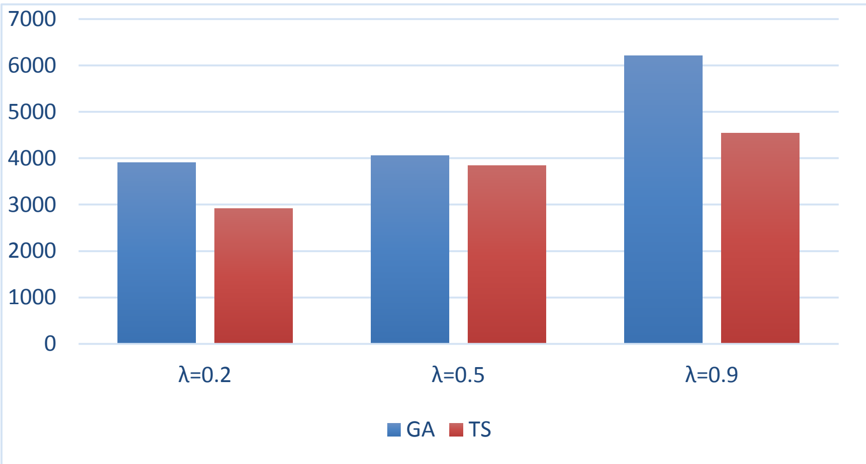

Fig. 2 The comparison between two algorithms based on the total costs

Both figures indicate better performance of Tabu search compared with genetic algorithms. Tabu search algorithm has presented lower time and cost for the efficient solution in all three λ.

Also, a small size statistical hypothesis can be presented for the above comparison table as below:

Table 7 Hypothesis1: Time

Hypothesis 1	
$\begin{vmatrix} H_0^1 : Time(GA) \geq Time(TS) \\ H_1^1 : Time(GA) < Time(TS) \end{vmatrix}$	
TS	GA
12	16
15	15
19	24

Table 8 Hypothesis2: Cost

Hypothesis 2	
$\begin{vmatrix} H_0^2 : Cost(GA) \geq Cost(TS) \\ H_1^2 : Cost(GA) < Cost(TS) \end{vmatrix}$	
TS	GA
2912	3906
3844	4057
4536	6205

In the above tests, both H_0^1, H_0^2 are accepted ($t_1 = 1.96 \not\leq -t_{2,0.01} = -6.96$, $t_2 = 2.28 \not\leq -t_{2,0.01} = -6.96$), so TS (in time and cost) is better than GA in this research. Of course, the innovation of the current paper is not that TS is better than GA and the above comparison could not be done.

6 Conclusions

In this paper, the time-cost trade-off problem was studied by considering the tardiness penalty cost of renewable resources. Here it was assumed that the resources are hired and should be delivered at a certain time. Given the amount of tardiness penalty cost, they can be used after the due date. Also, due to the unique and non-repeated nature of the projects, the duration of activities was considered to be triangular fuzzy numbers to include the terms of the uncertainty. These numbers will be decisive by the function expressed. The optimistic-pessimistic view of the expert has a great impact on efficient solutions. In continuation, the algorithms of Genetic and Tabu search were used for solving and their results were compared together. The obtained results show that the performance of the Tabu search algorithm is better than the Genetic algorithm in all three values of λ. Also by increasing λ, the duration of project time will increase too. The results show that due to including tardiness penalty costs of renewable resources by increasing the duration of project time, necessarily the project cost will not be increased, because the tardiness penalty costs will be added to the previous costs.

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