

Sequential Sampling Plan with Fuzzy Parameters

E. Baloui Jamkhaneh*, B. Sadeghpour Gildeh

Received: October 9, 2011 ; **Accepted:** February 16, 2012

Abstract In this paper a new sequential sampling plan is introduced in which the acceptable quality level (AQL) and the lot tolerance percent defective (LTPD) are a fuzzy number. This plan is well defined, since, if the parameters are crisp, it changes to a classical plan. For such a plan, a particular table of rejection and acceptance is calculated and compared with the classical one.

Keywords Statistical Quality Control, Sequential Sampling Plan, Fuzzy Number, Acceptable Quality Level, Lot Tolerance Percent Defective.

1 Introduction

Acceptance sampling plan have become an important field of statistical quality control (SQC). In this field, several sampling procedures are available for the application of attribute quality characteristics, namely, single, double, multiple and sequential sampling plan (SSP). SSP is an extension of multiple sampling plans, where there is no predetermined finite number of samples to be made. In sequential sampling, we take a sequence of samples from the lot and allow the number of samples to be determined by the results of the sampling process. If the inspected sample size at each sequence of them is greater than one, the plan is called group sequential sampling, but if at each stage it is one, the plan is called item by item sequential sampling [1]. Sequential sampling was developed in 1943 for use in rapid quality inspection in war research and production [2]. SSP has been studied by many researchers, and thoroughly elaborated on by Hardeo Sahai et al. [3]. Item by item sequential sampling is based on the sequential probability ratio test (SPRT) developed by Wald [4]. Decision making in Wald SPRT is based on exact hypotheses, but this precision is not true in the real world, and there also exists some uncertainty about the value of parameters obtained by experiments or estimation. In the design sequential plan the two-level quality AQL and LTPD, are crisp, as there are many different situations in which the AQL and LTPD are imprecise. In fact, with this problem we are defining the imprecise parameters as a fuzzy number, and we will define item by item SSP with fuzzy parameter, which is based on the SPRT for the fuzzy hypotheses.

Testing fuzzy hypotheses was discussed by Arnold [5] and [6], Delgado et al. [7], Watanabe and Imaizumi [8], Taheri and Behboodian [9], Torabi and Behboodian [10], Torabi et al. [11]. SPRT for the fuzzy hypotheses was discussed by Torabi and Mirhosseini [12].

*Corresponding author. (✉)

E-mail: e.baloui@qaemshahriau.ac.ir (E. B. Jamkhaneh)

E. Baloui Jamkhaneh

Department of Statistics, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran.

B. Sadeghpour Gildeh

Department of Statistics, Faculty of Basic Science, University of Mazandaran, Babolsar, Iran.

Baloui et al. [13-16] considered acceptance sampling plan under the conditions of the fuzzy parameter.

This paper is organized as follows. The next section introduces SPRT for fuzzy hypotheses testing. In the third section we provide the sequential sampling plan with fuzzy parameter, considered broadly, and its limiting lines in a special case are computed. Finally, operation characteristics, average sampling number curves, and rectifying inspection are introduced in sections 4 and 5. The results are summarized in the conclusion.

2 SPRT for fuzzy hypotheses testing

In this section, we discuss the sequential probability ratio test for fuzzy hypotheses testing (FHT) (See [12]). In FHT with crisp data, fuzzy hypotheses are

$$\begin{cases} H_0: \theta \text{ is } H_0(\theta), \\ H_1: \theta \text{ is } H_1(\theta), \end{cases} \quad (1)$$

where " $H_j : \theta \text{ is } H_j(\theta), j = 0, 1$ " implies that θ is in a fuzzy set of Θ (the parameter space) with membership function $H_j(\theta)$ i.e. a function from Θ to $[0,1]$. Note that the crisp hypothesis $H_j(\theta) : \Theta \rightarrow [0,1], j = 0, 1$ is a fuzzy hypothesis with membership function $H_j(\theta) = 1, \theta \in \Theta_j$, and zero otherwise.

Let random variable X have the probability density function (PDF) $f(x; \theta)$. Under the hypotheses $H_j, j = 0, 1$, the weighted probability density function (WPDF) of X is as follows

$$f_j(x) = \int_{\Theta} H_j^*(\theta) f(x; \theta) d\theta, \quad j = 0, 1, \quad (2)$$

where $H_j^*(\theta)$ is called the pseudo-membership function $H_j(\theta)$ and defined by

$$H_j^*(\theta) = \frac{H_j(\theta)}{\int_{\Theta} H_j(\theta) d\theta}, \quad j = 0, 1. \quad (3)$$

Substitute \int by \sum in the case that has just countable values. Note that $f_j(x)$ is a PDF. If $X = (X_1, X_2, \dots, X_n)$ is a random sample from a parametric population with the PDF $f(x; \theta)$; then the joint WPDF of X is

$$f_j(X) = \prod_{i=1}^n f_j(x_i), \quad j = 0, 1. \quad (4)$$

And $\Phi(X)$ is a test function, it is the probability of rejecting H_0 provided that $X = x$ is observed. The probability of type I and II error for the fuzzy testing problem (1) is $\alpha_\Phi = E_0(\Phi(X))$ and $\beta_\Phi = 1 - E_1(\Phi(X))$ respectively, in which $E_j(\Phi(X))$ is the mean of the expected value of $\Phi(X)$ over the joint WPDF $f_j(x)$, $j = 0, 1$. Therefore the fuzzy hypotheses testing problem (1) is equivalent to the following crisp hypotheses testing

$$\begin{cases} H_0: x \rightarrow f_0(x), \\ H_1: x \rightarrow f_1(x). \end{cases} \quad (5)$$

Let X_1, X_2, \dots denote a sequence of the iid random variables, from a population with PDF $f(x; \theta)$. First compute sequentially R_1, R_2, \dots . Where

$$\begin{aligned} R_n(x_1, x_2, \dots, x_n) &= \frac{L_0(x_1, x_2, \dots, x_n)}{L_1(x_1, x_2, \dots, x_n)}, \\ &= \prod_{i=1}^n \frac{f_0(x_i)}{f_1(x_i)}, \\ &= \prod_{i=1}^n \frac{\int H_0^*(\theta) f(x_i; \theta) d\theta}{\int H_1^*(\theta) f(x_i; \theta) d\theta}. \end{aligned} \quad (6)$$

For fixed k_0 and k_1 satisfy $0 < k_0 < k_1$, adopt the following procedure: Take observation x_1 and compute R_1 ; if $R_1 \leq k_0$, reject H_0 ; if $R_1 \geq k_1$, accept H_0 ; and if $k_0 < R_1 < k_1$, take observation x_2 , and compute R_2 ; if $R_2 \leq k_0$, reject H_0 ; if $R_2 \geq k_1$, accept H_0 ; and if $k_0 < R_2 < k_1$, take observation x_3 , etc. The idea is to continue sampling as long as $k_0 < R_n < k_1$ and stop as soon as $R_n \leq k_0$ or $R_n \geq k_1$, rejecting H_0 if $R_n \leq k_0$ and accepting H_0 if $R_n \geq k_1$. Consequently, the critical region of the described SPRT for fuzzy hypotheses testing (2) is $C = \bigcup_{n=1}^{\infty} C_n$, where

$$C_n = \{(x_1, \dots, x_n) \mid k_0 < R_j(x_1, \dots, x_j) < k_1, \quad j = 1, \dots, n-1, \quad R_n(x_1, \dots, x_n) \leq k_0\} \quad (7)$$

Similarly, the acceptance region can be as $A = \bigcup_{n=1}^{\infty} A_n$, where

$$A_n = \{(x_1, \dots, x_n) \mid k_0 < R_j(x_1, \dots, x_j) < k_1, \quad j = 1, \dots, n-1, \quad R_n(x_1, \dots, x_n) \geq k_1\} \quad (8)$$

Therefore in the SPRT for fuzzy hypotheses, the probability of type I and II errors is calculated by

$$\alpha = \sum_{n=1}^{\infty} \int_{C_n} L_0(X) dX, \quad \beta = \sum_{n=1}^{\infty} \int_{A_n} L_1(X) dX. \quad (9)$$

Let k_0 and k_1 be defined so that the SPRT for fuzzy hypotheses has the fixed probability of type I and II errors α and β . Then k_0 and k_1 can be approximated by $k_0' = \frac{\alpha}{1-\beta}$ and $k_1' = \frac{1-\alpha}{\beta}$. If α' and β' are the error size of the SPRT defined by k_0' and k_1' , then $\alpha' + \beta' \leq \alpha + \beta$. If

$$Z_i = \ln \left[\frac{\int_{\Theta} H_0^*(\theta) f(x_i; \theta) d\theta}{\int_{\Theta} H_1^*(\theta) f(x_i; \theta) d\theta} \right]. \quad (10)$$

With observed one sample and using (10), an equivalent test to the SPRT for fuzzy hypotheses is given by the following: Continue sampling as long as $\ln(k_0') < \sum_{i=1}^m Z_i < \ln(k_1')$, otherwise stop sampling, if $\sum_{i=1}^m Z_i \leq \ln(k_0')$ then we will reject H_0 and if $\sum_{i=1}^m Z_i \geq \ln(k_1')$ then we will accept H_0 . Let N be the random variable denoting the sample size of SPRT for fuzzy hypotheses, then

$$E(N | H_0 \text{ is true}) \approx \frac{\alpha L_n \frac{\alpha}{1-\beta} + (1-\alpha) L_n \frac{1-\alpha}{\beta}}{E(Z_i | H_0 \text{ is true})}, \quad (11)$$

$$E(N | H_1 \text{ is true}) \approx \frac{(1-\beta) L_n \frac{\alpha}{1-\beta} + \beta L_n \frac{1-\alpha}{\beta}}{E(Z_i | H_1 \text{ is true})}, \quad (12)$$

where

$$E(Z_i | H_j \text{ is true}) \approx \int_{\Theta} L_n \left[\int_{\Theta} H_0^*(\theta) f(x_i; \theta) d\theta / \int_{\Theta} H_1^*(\theta) f(x_i; \theta) d\theta \right] f_j(x_i) dx_i, \quad j = 0, 1. \quad (13)$$

3 Sequential sampling plans with fuzzy parameters

A decision criterion in an item by item sequential sampling plan is the acceptance and rejection of lines. These two limit lines are plotted in terms of the total number of items

selected up to that time, and show cumulative observed number of defectives items. The operation of such a plan is illustrated in Fig 1.

For each point in this figure, the x axis is the total number of items selected up to that time, and the y axis is the total number of observed defective items. Then the operation procedure is given in the following,

(i) If plotted points stay within the boundaries of the limit lines, another sample must be drawn,

(ii) When plotted point falls on or above the upper line, at that stage the lot is rejected,

(iii) When plotted point falls on or below the lower line, at that stage the lot is accepted [1].

Accepting or rejecting a lot in the SSP is analogous to not rejecting or rejecting the null hypotheses in a hypotheses test. The hypotheses for SSP as a kind of statistical test are [15]:

H_0 : The lot is of acceptable quality level (AQL)

H_1 : The lot is of rejectable quality level (RQL)

The AQL presents the poorest level of quality for the vendor's process that the consumer would consider to be acceptable as a process average [1]. An alternative name for the RQL is lot tolerance percent defective (LTPD). The LTPD is the poorest level of quality that the consumer is willing to accept in an individual lot. Probability of type I and II errors α and β for this hypotheses test is as follows

$$\alpha = P(\text{rejected } H_0 \mid H_0 \text{ is true}) , \quad \beta = P(\text{not rejected } H_0 \mid H_0 \text{ is false}).$$

A lot may be rejected that should be accepted and the risk of doing this is the producer's risk (α). The second error is that a lot may be accepted that should be rejected and the risk of doing so is called the consumer's risk (β).

If $AQL = p_1$ and $LTPD = p_2$, ($p_1 < p_2$) then the equivalent hypothesis is given by the following:

$$\begin{cases} H_0: p = p_1, \\ H_1: p = p_2. \end{cases} \quad (14)$$

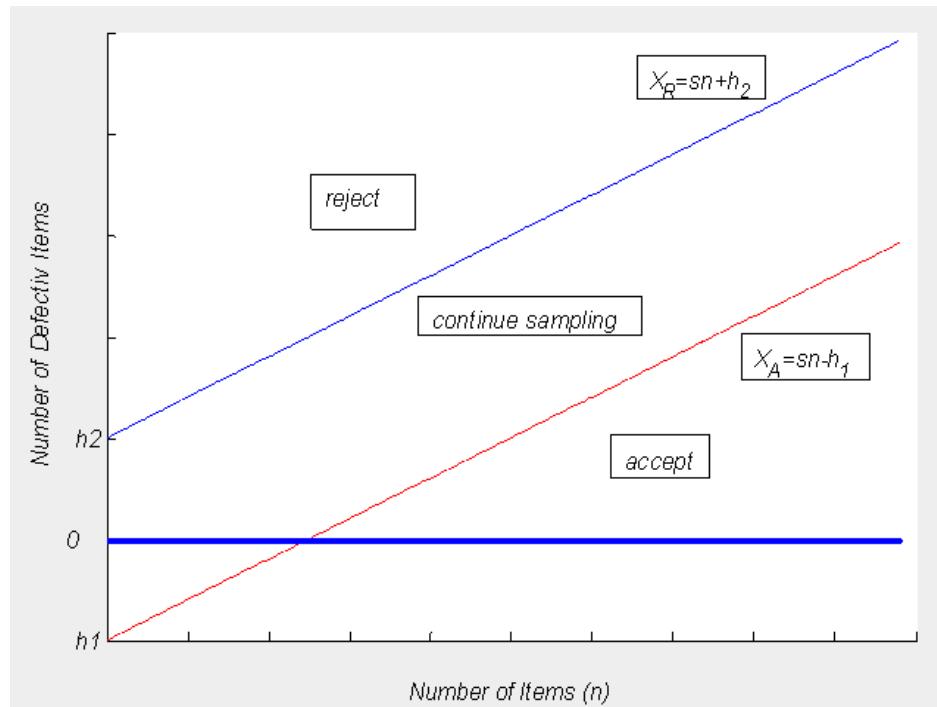


Fig. 1 Graphical performance of sequential sampling plan

When designing an item by item sequential sampling plan, four parameters of the AQL, the producer's risk α (the probability of rejecting a lot of AQL quality), LTPD and the consumer's risk β (the probability of accepting a lot of LTPD quality) must be determined prior to determining the acceptance and rejection lines. In the design SSP two levels of quality are crisp, but sometimes these are not exact and certain. So, we face a fuzzy hypothesis as follows

$$\begin{cases} H_0: p \approx p_1, \\ H_1: p \approx p_2. \end{cases} \quad (15)$$

We propose to design SSP with fuzzy AQL and fuzzy LTPD. Such a plan is based on the SPRT for fuzzy hypotheses. The sampling distribution of the number of defective items in every stage (X_i) is the Bernoulli distribution with parameter p (incoming quality level), because each item inspected is either defective or not. That is, $X_i = 1$, if the i^{th} inspected item is defective, otherwise $X_i = 0$. In FHT with crisp data, fuzzy hypotheses are

$$\begin{cases} H_0: p \text{ is } H_0(p), \\ H_1: p \text{ is } H_1(p), \end{cases} \quad (16)$$

where membership function of $H_j(p)$ is considered as follows:

$$H_j(p) = c_j p^{\alpha_j-1} (1-p)^{\beta_j-1}, \quad \forall p \in (0,1), \quad p_j = \frac{\alpha_j-1}{\alpha_j+\beta_j-2}, \quad j=0,1, \quad (17)$$

where,

$$c_j = \left[\left[\frac{\alpha_j-1}{\alpha_j+\beta_j-2} \right]^{\alpha_j-1} \left[\frac{\beta_j-1}{\alpha_j+\beta_j-2} \right]^{\beta_j-1} \right]^{-1}, \quad j=0,1. \quad (18)$$

α_j and β_j parameters are determined in a way that $H_j(p)$ would be an appropriate membership function of p_j , ($\alpha_1 > \alpha_0$). And pseudo membership function of $H_j(p)$ is

$$H_j^*(p) = \frac{\Gamma(\alpha_j + \beta_j)}{\Gamma(\alpha_j)\Gamma(\beta_j)} p^{\alpha_j-1} (1-p)^{\beta_j-1}, \quad (19)$$

according to (2) one has

$$f_j(x) = \begin{cases} \frac{\alpha_j}{\alpha_j + \beta_j}, & x = 1, \\ \frac{\beta_j}{\alpha_j + \beta_j}, & x = 0. \end{cases} \quad (20)$$

Finally by using (5) and (20), we obtain

$$\begin{aligned} R_n(x_1, x_2, \dots, x_n) &= \frac{\prod_{i=1}^n \left(\frac{\alpha_0}{\alpha_0 + \beta_0} \right)^{x_i} \left(\frac{\beta_0}{\alpha_0 + \beta_0} \right)^{1-x_i}}{\prod_{i=1}^n \left(\frac{\alpha_1}{\alpha_1 + \beta_1} \right)^{x_i} \left(\frac{\beta_1}{\alpha_1 + \beta_1} \right)^{1-x_i}}, \\ &= \prod_{i=1}^n \left(\frac{\alpha_0 \beta_1}{\alpha_1 \beta_0} \right)^{x_i} \left(\frac{\beta_0 (\alpha_1 + \beta_1)}{\beta_1 (\alpha_0 + \beta_0)} \right). \end{aligned} \quad (21)$$

If

$$Z_i = X_i \ln \left(\frac{\alpha_0 \beta_1}{\alpha_1 \beta_0} \right) + \ln \left(\frac{\beta_0 (\alpha_1 + \beta_1)}{\beta_1 (\alpha_0 + \beta_0)} \right), \quad (22)$$

then at the n^{th} stage of sampling, accept the lot if $\sum_{i=1}^n Z_i \leq \ln \frac{\alpha}{1-\beta}$, reject the lot if

$\sum_{i=1}^n Z_i \geq \ln \frac{1-\alpha}{\beta}$ and continue sampling by taking an additional observation if

$Ln \frac{\alpha}{1-\beta} < \sum_{i=1}^n Z_i < Ln \frac{1-\alpha}{\beta}$. Using rejection and acceptance regions, and the parameters p_1 , p_2 , α and β the SSP with fuzzy parameters is determined by the acceptance and rejection lines given as follows

$$X_A = sn - h_1 \text{ (Acceptance line)} , \quad X_R = sn + h_2 \quad \text{(Rejection line),}$$

here, we have

$$s = \frac{Ln(\frac{\beta_0(\alpha_1 + \beta_1)}{\beta_1(\alpha_0 + \beta_0)})}{k} , \quad k = Ln(\frac{\alpha_1 \beta_0}{\alpha_0 \beta_1}), \quad (23)$$

$$h_1 = \frac{Ln \frac{1-\alpha}{\beta}}{k} , \quad h_2 = \frac{Ln \frac{1-\beta}{\alpha}}{k}. \quad (24)$$

However instead of acceptance number, calculate the next integer less than or equal to X_A , and instead of rejection number calculate the next integer greater than or equal to X_R . For example, suppose we will be to find a SSP for which p_1 is close to 0.01, $\alpha = 0.05$, p_2 is close to 0.06 and $\beta = 0.1$, i.e. $\alpha_0 = 2$, $\beta_0 = 100$; $\alpha_1 = 7$, and $\beta_1 = 95$ thus, $k = 1.3041$, $s = 0.0393$, $h_1 = 1.7263$, $h_2 = 2.2164$, therefore, the acceptance and rejection lines are

$$X_A = 0.0393n - 1.7263 , \quad X_R = 0.0393n + 2.2164 \quad (25)$$

Now for $n=70$, according to (25) we obtain

- (i) Accept lot, if the number of defective items is less than or equal to 1.
- (ii) Continue sampling, if the number of defective item is in (1, 5).
- (iii) Reject lot, if the number of defective items is greater than or equal to 5.

Table 1 shows that the lot cannot be accepted until at least 44 items have been tested. Comparing Table 1 and the table related to SSP in its classical state in terms of common parameters (See [1]), one can conclude that this plan rejected lots cautiously, that is, it rejects lots by further investigation. Acceptance number of this plan is either greater or equal to acceptance numbers of SSP in its classical state with common parameters, that is, acceptance of lots happens easier.

Table 1 Sequential sampling plan $p_1 \approx 0.01$, $\alpha = 0.05$, $p_2 \approx 0.06$, $\beta = 0.1$

| n | Ac. Number | Re. Number | n | Ac. Number | Re. Number |
|-----|------------|------------|-----|------------|------------|
| 1 | a | b | ... | ... | ... |
| 2 | a | b | 69 | 0 | 5 |
| 3 | a | 3 | 70 | 1 | 5 |
| 4 | a | 3 | 71 | 1 | 6 |
| ... | ... | ... | ... | ... | ... |
| 19 | a | 3 | 94 | 1 | 6 |
| 20 | a | 4 | 95 | 2 | 6 |
| ... | ... | ... | 96 | 2 | 6 |
| 43 | a | 4 | 97 | 2 | 7 |
| 44 | 0 | 4 | ... | ... | ... |
| 45 | 0 | 4 | 120 | 2 | 7 |
| 46 | 0 | 5 | 121 | 3 | 7 |

^a mean acceptance not possible, ^b mean rejection not possible

However, changing α_j and β_j and correspondingly changing the skewed of $H_j(p)$; the difference between the acceptance and rejection numbers of these two plans would change. With the decreasing ambiguity in amount of p_1 (increasing the amount of α_0 and β_0), the continuation of sampling band will be narrower, and with decreasing ambiguity in amount of p_2 (increasing the amount of α_1 and β_1), the continuation of sampling band will be wider.

4 OC and ASN curves

The operation characteristic (OC) curve of the sequential sampling attribute plan is a plot of the probability of acceptance for a given lot versus the fraction of defective items (p). An approximation of OC curve for sequential sampling plan with fuzzy parameter can be obtained from five values of the fraction defective items including the two points defining the plan. These values and the corresponding probability of acceptance (p_a) are given as follows [3]:

| p | 0 | $\alpha_0 / (\alpha_0 + \beta_0)$ | s | $\alpha_1 / (\alpha_1 + \beta_1)$ | 1 |
|-------|---|-----------------------------------|---------------------|-----------------------------------|---|
| p_a | 1 | $1 - \alpha$ | $h_2 / (h_1 + h_2)$ | β | 0 |

A sequential sampling plan reduces the amount of inspection to the minimum possible. The average sample number (ASN) is defined as the average number of sample units per lot used for deciding acceptance or rejection. Average sample number (ASN) is useful to evaluate a sequential sampling plan. ASN taken under SSP with fuzzy parameter is

$$ASN = p_a A + (1 - p_a) B, \quad (26)$$

where the quantity A is the expected sample size for an accepted lot, and obtained as

$$A = \frac{\ln \frac{1-\alpha}{\beta}}{p \ln \left(\frac{\alpha_0(\alpha_1 + \beta_1)}{\alpha_1(\alpha_0 + \beta_0)} \right) + (1-p) \ln \left(\frac{\beta_0(\alpha_1 + \beta_1)}{\beta_1(\alpha_0 + \beta_0)} \right)}, \quad (27)$$

and the quantity B is the expected sample size for rejected lot, and obtained as

$$B = \frac{\ln \frac{\alpha}{1-\beta}}{p \ln \left(\frac{\alpha_0(\alpha_1 + \beta_1)}{\alpha_1(\alpha_0 + \beta_0)} \right) + (1-p) \ln \left(\frac{\beta_0(\alpha_1 + \beta_1)}{\beta_1(\alpha_0 + \beta_0)} \right)}. \quad (28)$$

The ASN curve is a plot of ASN versus the fraction of defective items, (p). As in the case of operating characteristics, approximation to the ASN can be obtained from the five values of the fraction items including the two points defining the plan.

5 Rectifying inspection

The main aim of rectifying inspection programming is restoring the lot's quality. Assume the size of the lot is N^* and the fraction of defective items is p . In one way rectifying inspection programming, all items in a rejected lot are inspected and defective items replaced with safe items. Consequently, the number of defective items of outgoing lots is equal to zero, and similarly, if the lot is accepted, the defective items found in samples from accepted lots also replaced with safe items, accordingly, on the average A items were inspected and found without defective items, and the rest of the items would be accepted without inspection; therefore, on the average the outgoing lot has $p(N^* - A)$ number of defective items. Therefore, in the outgoing inspection process the number of defective items, with the probability p_a is equal to $p(N^* - A)$ and with the probability $1 - p_a$ equals zero. Thus, the average outgoing quality (AOQ) for SSP with fuzzy parameter is given as follows

$$AOQ = \frac{p_a p (N^* - A)}{N^*}, \quad (29)$$

it is given approximately by $p_a p$.

The average total inspection is an important criterion in the rectifying inspection for SSP with fuzzy parameters. The amount of sampling is A when a lot is accepted (the probability being p_a), and N^* when it is rejected (the probability being $(1 - p_a)$), therefore, the average total inspection is

$$ATI = p_a A + (1 - p_a) N^*. \quad (30)$$

Decreasing the AQL or increasing the RQL, the average outgoing quality and the average total inspection would improve and increase, respectively.

6 Conclusions

In this paper a method to design SSP with fuzzy AQL and fuzzy LTPD has been presented. This plan is well defined since, if the parameters are crisp, it changes to a classical plan. Comparing Table1 and the table related to SSP in its classical state in terms of common parameters, one can conclude that this plan rejects lots cautiously, and acceptance of lots happens easier. However, changing α_j and β_j correspondingly, changing the skewed of $H_j(p)$, the difference between the acceptance and rejection numbers of these two plans would change. In such a plan, decreasing producer's risk or consumer's risk, AOQ and ATI would improve and increase, respectively.

References

1. Montgomery, D. C., (1991). *Introduction to statistical quality control*. Wiley New York.
2. Kao, S. S., (1984). Sequential sampling plan for insect pests. *Phytopathologist & Entomologist*, NTU, 102-110.
3. Hardeo, S., Anwer K., Muhammad A., (2003). A visual basic program to determine Wald's sequential sampling plan, 24(3).
4. Wald, A., (1947). *Sequential analysis*. John Wiley, New York.
5. Arnold, B. F., (1995). Statistical tests optimally meeting certain fuzzy requirements on the power function and on the sample size. *Fuzzy Sets and System*, 75(2), 365-372.
6. Arnold, B. F., (1996). An approach to fuzzy hypotheses testing. *Metrika*, 44, 119-126.
7. Delgado, M., Verdegay, M. A., Vila, M. A., (1985). Testing fuzzy hypotheses: a Bayesian approach. in: M. M. Gupta et al.(Eds), *Approximate Reasoning in Expert Systems*, North-Holland Publishing Co, Amsterdam, 307-316.
8. Watanabe, N., Imaizumi, T., (1993). A fuzzy statistical test of fuzzy hypotheses. *Fuzzy Sets and Systems*, 53, 167-178.
9. Taheri, S. M., Behboodian, J., (2001). A Bayesian approach to fuzzy hypotheses testing. *Fuzzy Sets and Systems*, 123, 39-48.
10. Torabi, H., Behboodian, J., (2007). Likelihood ratio test for fuzzy hypotheses testing. *Statistical Papers*, 48(3), 509-522.
11. Torabi, H., Behboodian, J., Taheri, S.M., (2006). Neyman-Pearson lemma for fuzzy hypotheses testing with vague data. *Metrika*, 64(3), 289-304.
12. Torabi, H., Mirhosseini, S. M., (2009). Sequential probability ratio tests for fuzzy hypotheses testing. *Applied Mathematical Sciences*, 3(33), 1609-1618.
13. Baloui, J. E., Sadeghpour Gildeh, B., Yari, Gh., (2011). Acceptance single sampling plan with fuzzy parameter. *Iranian Journal of Fuzzy Systems*, 8(2), 47-55.
14. Baloui, J. E., Sadeghpour Gildeh B., Yari, Gh., (2011). Inspection error and its effects on single sampling plans with fuzzy parameters. *Structural and Multidisciplinary Optimization*, 43(4), 555-560.
15. Baloui, J. E., Sadeghpour Gildeh B., (2010). Sequential sampling plan by variable with fuzzy parameters. *Journal of Mathematics and Computer Science*, 1(4), 392-401.
16. Baloui, J. E., Sadeghpour Gildeh B., (2008). Acceptance double sampling plan with fuzzy parameter. 11th Joint Conference on Information Sciences, China, Atlantis Press.
17. Dunicic, K., Bahovec, V., Zivadinovic, N. K., (2006). Studying an OC curve of an acceptance sampling: a statistical quality control tool. proceeding of the 7th WSEAS international conference on mathematics & computers in business & economics, cavtat, croatia, JUNE 13-15, 1-6.