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An Approach for Solving Traveling Salesman Problem

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Abstract In this paper, we introduce a new approach for solving the traveling salesman problems (TSP) and provide a solution algorithm for a variant of this problem. The concept of the proposed method is based on the Hungarian algorithm, which has been used to solve an assignment problem for reaching an optimal solution. We introduced a new fittest criterion for crossing over such problems, and illustrated it with analytical examples and by computer programming. The proposed method builds on the initial solution of the traveling salesman problem (TSP) which is very simple, easy to understand and apply.

Keywords Traveling Salesman Problem, Hungarian Method, Optimal Solution, Computer Algorithm.

1 Introduction

The Traveling Salesman Problem (TSP) is one of the well-known problems for finding an optimal path. It is the problem for finding the shortest closed route among n cities, having as input the complete distance matrix among all the cities. A common application of the TSP is the movement of people, equipment and vehicles around tours in aiming to minimize the total traveling cost. For example, let us consider that a salesman is planning a business trip that takes him to certain cities in which he has customers and then brings him back to the city from where he started. Between some of the pairs of cities that he has to visit, there is a direct air service; while between others there is not such a thing. Can he plan the trip so that he (a) begins and ends the same city while visiting every other city only once, and (b) pays the lowest price in airfare possible? The key to this is not just finding a solution, but an optimal solution, the one with the lowest airfare. This type of problem can be easily solved by using the TSP algorithm. Using the TSP algorithm a bank can improve its regular services. Suppose that a bank has many ATM machines. Each day, a courier goes from machine to machine to make collections, gather computer information, and service the machines. A problem which may arise in practice at many banks is that in what order should the machines be visited so that the courier's route is the shortest possible? Another application of the TSP is the school-bus routing problem. It is required to schedule a school bus to pick up waiting students from

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the pre-specified locations. A well-known application of the TSP is post routing. The postman problem is modeled as traversing a given set of streets in a city, rather than visiting a set of specified locations. Moreover, the TSP plays an important role in general post problems, where the houses or streets are far away from each other. The applications of the TSP are not limited to the examples described above. However, a detailed review of the applications of the TSP can be found in Applegate et al. [1], Dantzig et al. [2] and Lawler et al. [3].

Mathematical problems related to the Traveling salesman problems were treated in the 1800's by the Irish Mathematician W. R. Hamilton and by the British Mathematician Thomas Kirkman. Hamilton's Icosian Game was a recreational puzzle based on finding a Hamiltonian cycle. The general form of the TSP appears to have been first studied by mathematicians during the 1930's in Vienna and at Harvard, notably by Karl Menger, who defines the problem, considers the obvious brute-force algorithm, and observes the non-optimality of the nearest neighbor heuristic. In the 1950's and 1960's, the problem became increasingly popular in scientific circles in Europe and the U.S. Notable contributions were made by Dantzig et al. [2], who expressed the problem as an integer linear program and developed the cutting plane method for its solution. With these new methods they solved an instance with 49 cities to optimality by constructing a tour and proving that no other tour could be shorter. In the following decades, the problem was studied by many researchers from mathematics, computer sciences, chemistry, physics, and other sciences. Some of the well-known tour construction procedures are the nearest neighbor procedure by Rosenkrantz et al. [4], genetic algorithm by Ray et al. [5] and Khan et al. [6], Scheduling algorithm by Clarke and Wright [7], the partitioning approach by Karp [8], a foveating pyramid model by Pizlo et al. [9] and the minimal spanning tree approach by Christofides [10], etc. The branch exchange is perhaps the best known iterative improvement algorithm for the TSP. The 2-opt and 3-optheuristics were described by Lin [11] and Lin and Kernighan [12]. They made a great improvement in the quality of tours that can be obtained by heuristic methods. Even today, their algorithm remains the key ingredient in the most successful approaches for finding high-quality tours and is widely used to generate initial solutions for other algorithms.

One of the earliest exact algorithms is due to Dantzig et al. [2], in which linear programming (LP) relaxation is used to solve the integer formulation by adding suitably chosen linear inequality to the list of constraints continuously. Branch and bound algorithms are widely used to solve the TSPs. Several authors have proposed (B&B) algorithms based on assignment problem (AP) relaxation of the original TSP formulation. These authors include Eastman [13], Held and Karp [14], Smith et al. [15], Graham et al. [16], Carpaneto and Toth [17], Balas and Crhistofides [18]. Some branch- and cut-based (B&C) exact algorithms were developed by Crowder and Padberg [19], Padberg and Hong [20], Grotschel and Holland [21]. A zero suffix method was developed by Sudhakar and Kumar [22]. Besides, the above mentioned exact and heuristic algorithms, metaheuristic algorithms, have been applied successfully to the TSP by a number of researchers.

In this paper, based on modified Hungarian algorithm we introduce a new method for solving the traveling salesman problem (TSP), and develop computer program for that purpose. In real life the TSP can be very large, which is very difficult to solve analytically. Here we assign some real-life problems of the TSP type and solve them both analytically and with the aid of computer programming developed in this study.

2 Formulation of traveling salesman problem (TSP)

Let $1,2,.......,n$ be the labels of the n cities and $C = C_{i,j}$ be an $n \times n$ cost matrix where $C_{i.j}$ denotes the cost of traveling from city i to city j. Then, the general formulation of the traveling salesman problem (TSP), as described by Hungarian algorithm, is shown in Table 1.

Table 1 Formulation of the traveling salesman problem (TSP)

	1	2	3	……	n
1	*	C_{12}	C_{13}	……	C_{1n}
2	C_{21}	*	C_{23}	……	C_{2n}
3	C_{31}	C_{32}	*	……	C_{3n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	*	$\vdots$
n	C_{n1}	C_{n2}	C_{n3}	……	*

If $C_{i,j} = C_{j,i}$, the problem is called *symmetric traveling salesman problem* (STSP).

3 Working procedure for solving TSP

Based on Hungarian method, here we introduce a new method for solving traveling salesman problem which is described as follows:

Step 1. First, we use the Hungarian method to obtain an initial basic feasible solution.

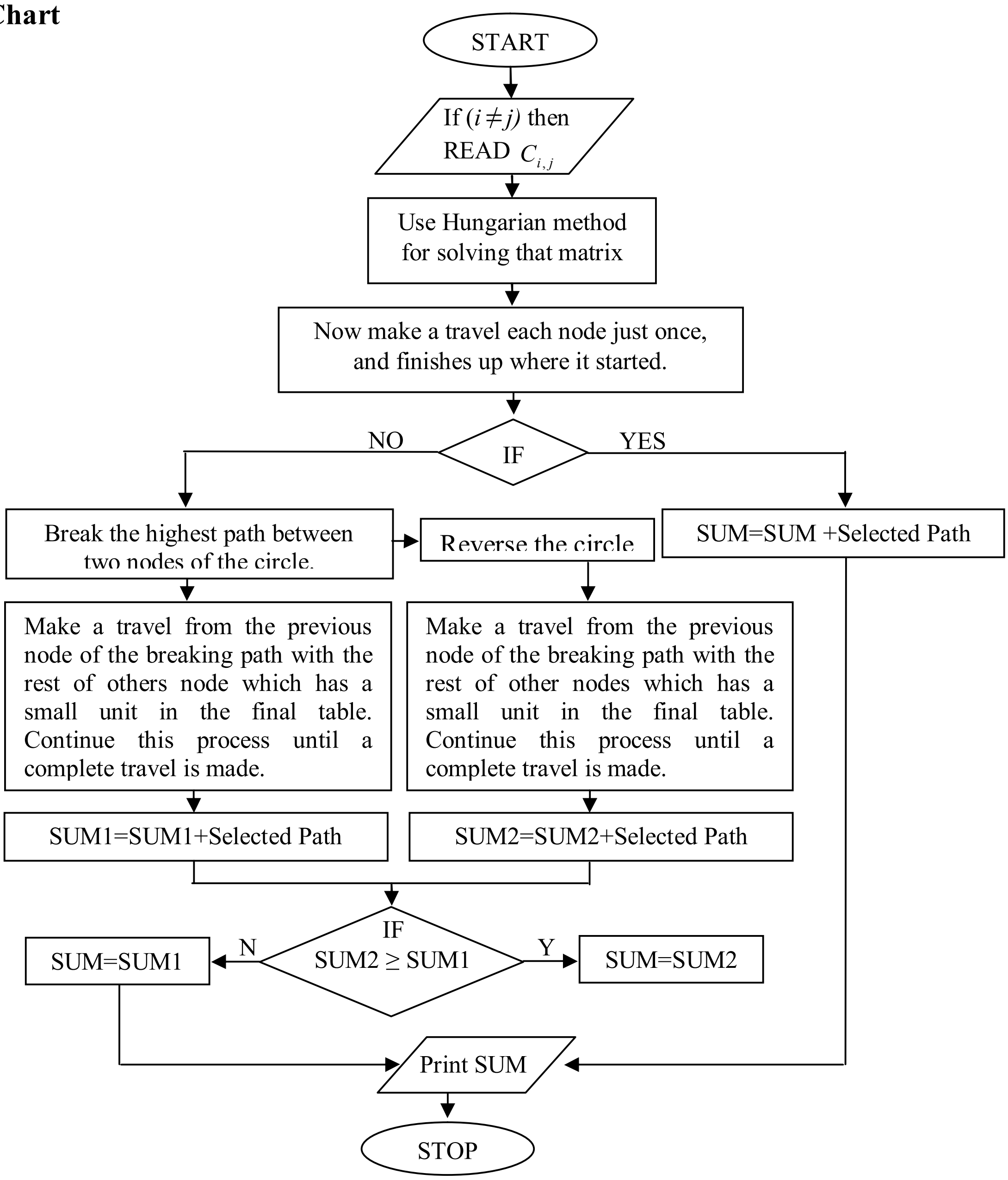
Step 2. Now, make a travel for each node just once, when it finishes up the place from where it started, and if not then it goes to step 3.

Step 3. (a) Break the highest path between the two nodes of the circle. Now make a travel from the previous node of the breaking path with the rest of the other nodes which has a small unit in the last table. Continue this process until a complete travel is made. Calculate the total travel cost.

(b) Again reverse the circle and break the highest path between the two nodes of the circle, and then do the same as discussed in step 3(a). Continue this process until a complete travel is made, and then calculate the total travel cost.

Step 4. Compare the travel costs between the steps in 3(a) and in 3(b) and the minimum travel cost, thus obtained, is the optimal solution.

4 Flow Chart



5 Computer technique (FORTRAN)

In this section, we develop a computer program for solving traveling salesman problem for the proposed problem. In real life, the TSP problem is so much large that it is too difficult to solve it by simple hand calculation, but using the computer technique, one can easily solve them.

```
!***********************************************************
!*                  TRAVELING SALESMAN PROBLEM
!*
!* LIST OF MAIN VARIABLES:
!*
!*  NJ:         NUMBER OF JOBS
```

```
!*  C(NJ,NJ):    APPOINTMENT COST MATRIX
!*  JM(NJ,NJ):    JOB/APPLICANT MATRIX
!*  IF1:         FLAG=1 IF OPTIMAL APPOINTMENT IS FOUND
!*  IZ:          FLAG TO MARK A ZERO
!*  XMIN:        MINIMUM VALUE
!*  XCASE:       APPOINTMENT OF A ZERO IN RELATION WITH
!*               THE ZEROES' MINIMUM
!*  NZ:          NUMBER OF ZEROES
!*  ZI,ZJ:       COORDINATES OF CURRENT CASE
!*  M,N:         COORDINATES OF A MARKED ZERO
!*  A,B:         AUXILIARY VARIABLES TO CHANGE
!*               APPOINTMENTS.
!**********************************************************
PARAMETER(MAX=20)
REAL C(0:MAX,0:MAX)
INTEGER NJ,JM(0:MAX,0:MAX),COUNT
  PRINT *,' '
  PRINT *,'      ############ OPERATIONS RESEARCH ############'
  PRINT *,'     ********TRAVELING SALESMAN PROBLEM*********'
  PRINT *,' '
  WRITE(*,10,ADVANCE='NO'); READ *,NJ
  PRINT *,' '
  PRINT *,' INPUT APPOINTMENT COSTS/REGRETS OF APPLICANTS:'
  DO I = 1,NJ
    PRINT *,' '
    WRITE(*,20) I
     DO J = 1,NJ
       WRITE(*,30,ADVANCE='NO') J
       READ *,C(I,J)
     END DO
  END DO
   PRINT *,' '
   PRINT *,' APPOINTMENTS:'
   PRINT *,' '
!*********************************************************
!CALL MODIFIED HUNGARIAN ALGORITHM
CALL MODHUNGARIAN(NJ,C,JM)
    DO i=1,nj
        WRITE(*,*)(c(i,j),j=1,nj)
    END DO
   I = 1
     DO J = 1,NJ
   IF (JM(I,J).NE.1) GOTO 50
   WRITE(*,40) I,J
   IF (J.NE.1) GO TO 60
50 END DO
60 I=J
     DO J = 1,NJ
   IF (JM(I,J).NE.1) GOTO 52
   WRITE(*,40) I,J
   IF (J.NE.1) GO TO 60
   PRINT *,' '
52 END DO
10 FORMAT('  INPUT THE NUMBER OF CITIES = ')
20 FORMAT('  FORM #',I1,':')
30 FORMAT('     TO #',I1,' ? ')
40 FORMAT('       FORM #',I2,' => TO #',I2)
STOP
END
```

6 Numerical Examples

Example 1 We solve the following problem by the proposed method and verify the results.

Table 2 Traveling salesman problem (example 1)

	1	**2**	**3**	**4**	**5**	**6**	**7**
1	-	75	99	9	35	63	8
2	51	-	86	46	88	29	20
3	100	5	-	16	28	35	28
4	20	45	11	-	59	53	49
5	86	63	33	65	-	76	72
6	36	53	89	31	21	-	52
7	58	31	43	67	52	60	-

Table 3 Analytical solution of Table 2 (example 1)

	1	**2**	**3**	**4**	**5**	**6**	**7**
1	-	77	91	Ø	27	46	[0]
2	22	-	66	25	68	[0]	Ø
3	86	Ø	-	[0]	13	11	13
4	[0]	44	Ø	-	48	33	38
5	44	40	[0]	31	-	34	39
6	6	42	68	9	[0]	-	31
7	8	[0]	2	25	15	10	-

Table 3 gives the optimal basic feasible solution obtained by the Hungarian method.
Therefore, the assignment is: $1 \to 7$, $2 \to 6$, $3 \to 4$, $4 \to 1$, $5 \to 3$, $6 \to 5$, $7 \to 2$
Now, make a travel, each node is traveled only once and finishes up the place from where it was started. For example, the journey was started from node 1 and finishes up the same node finally visiting the other nodes.

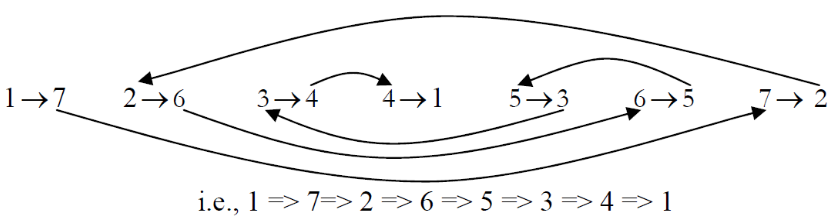


Fig. 1 Schematic diagram of the complete travel of Table 3

The above figure shows the feasible route and using the above procedure the minimum traveling cost for example 1 (obtained from Table 2) is 158 units.

Computer oriented solution

```
Output
     Form 1 =>  To 7
     Form 7 =>  To 2
     Form 2 =>  To 6
     Form 6 =>  To 5
     Form 5 =>  To 3
     Form 3 =>  To 4
     Form 4 =>  To 1
The minimum traveling cost is =158 units
```

Example 2 Let us consider that a tour company is planning to organize a tour trip that takes the tourists in several cities, and then brings them back to the city from where they started. It is recorded that between some of the pairs of cities there is direct train service available while between some of the cities there is not such a thing. The main goal of the company is to plan the trip in such a way that the tourists begin and end the same city visiting all other cities only once with the minimum traveling cost. The company sales three types of train tickets for different routes as given below.

Howrah to		2A	3A	SL
Delhi	12311 HWH DLI KLK MAI	₹1750 / $ 39	₹1113 / $ 25	₹406 / $ 10
Agra	12307 HWH JU EXPRESS	₹1650 / $ 37	₹1065 / $ 24	₹399 / $ 9
Mumbai	03061 HWH CSTM SP	₹2250 / $ 50	₹1394 / $ 31	₹506 / $ 12
Jaipur	12307 HWH JU EXP	₹1840 / $ 41	₹1175 / $ 27	₹438 / $ 10
Chandigarh	12311 HWH DLI KLK MAI	₹1960 / $ 44	₹1233 / $ 28	₹449 / $ 10
Goa	18047 AMARAVATHI EXP	₹2240 / $ 50	₹1388 / $ 31	₹504 / $ 12
Patna	12303 POORVA EXP	₹965 / $ 22	₹653 / $ 15	₹252 / $ 6
Guwahati	12303 POORVA EXP	₹1410 / $ 32	₹927 / $ 21	₹350 / $ 8
Puri	12887 HWH PURI EXPRES	₹930 / $ 21	₹631 / $ 15	₹244 / $ 6

Mumbai to		2A	3A	SL
Jaipur	12979 J.SUPER FAST	₹1555 / $ 35	₹1010 / $ 23	₹379 / $ 9
Chandigarh	12217 SAMPARK KRANTHI	₹1955 / $ 44	₹1242 / $ 28	₹461 / $ 11
Goa	10103 MANDOVI EXP	₹1190 / $ 27	₹782 / $ 18	₹288 / $ 7
Patna Jn	12141 RAJENDRA NGR EX	₹1990 / $ 45	₹1263 / $ 29	₹469 / $ 11
Guwahati	15635 GUWAHATI EXP	₹2445 / $ 55	₹1487 / $ 34	₹539 / $ 12
Puri	12745 LTT PURI SUP EX	₹2105 / $ 47	₹1330 / $ 30	₹493 / $ 11

Delhi to		2A	3A	SL
Agra	11078 JHELUM EXPRESS	₹ 585	₹ 301	₹ 120
		$ 13	$ 7	$ 3
Mumbai	12138 PUNJAB MAIL	₹ 1860	₹ 1187	₹ 442
		$ 42	$ 27	$ 10
Jaipur	04041 DLI AII SF SP	₹ 630	₹ 434	₹ 175
		$ 14	$ 10	$ 4
Chandigarh	12217 SAMPARK KRANTHI	₹ 615	₹ 401	₹ 163
		$ 14	$ 9	$ 4
Goa	12218 KERLA S KRANTI	₹ 2260	₹ 1412	₹ 522
		$ 51	$ 32	$ 12
Patna	12304 POORVA EXP	₹ 1430	₹ 938	₹ 354
		$ 32	$ 21	$ 8
Guwahati	15610 A.ASSAM EXP	₹ 2135	₹ 1333	₹ 484
		$ 48	$ 30	$ 11
Puri	12802 PURSHOTTA M EXP	₹ 2105	₹ 1330	₹ 493
		$ 47	$ 30	$ 11

Agra to		2A	3A	SL
Mumbai	12138 PUNJAB MAIL	₹ 1725	₹ 1109	₹ 414
		$ 39	$ 25	$ 10
Jaipur	12307 HWH JU EXP	₹ 615	₹ 379	₹ 155
		$ 14	$ 9	$ 4
Chandigarh	12687 DEHRADUN EXP SLIP	-	₹ 645	₹ 249
		-	$ 15	$ 6
Goa	12780 GOA EXP	₹ 2210	₹ 1390	₹ 514
		$ 50	$ 31	$ 12
Patna Jn	15635 GUWAHATI EXP	₹ 1295	₹ 845	₹ 311
		$ 29	$ 19	$ 7
Guwahati	15635 GUWAHATI EXP	₹ 2020	₹ 1266	₹ 461
		$ 45	$ 29	$ 11
Puri	18478 KALINGAUTK ALEXP	₹ 2135	₹ 1333	₹ 484
		$ 48	$ 30	$ 11

Jaipur to		2A	3A	SL
Chandigarh	05286 AII BJU SPL	-	₹ 469	-
		-	$ 11	-
Goa	12978 MARU SAGAR EXP	₹ 2125	₹ 1340	₹ 497
		$ 48	$ 30	$ 12
Patna Jn	12316 ANANYA EXPRESS	₹ 1575	₹ 1021	₹ 383
		$ 35	$ 23	$ 9
Guwahati	15631 BME GHY EXPRESS	₹ 2150	₹ 1344	₹ 488
		$ 48	$ 30	$ 11
Puri	18474 JU PURI EXPRESS	₹ 2230	₹ 1382	₹ 502
		$ 50	$ 31	$ 12

Chandigarh to		2A	3A	SL
Goa	12978 MARU SAGAR EXP	₹ 2370	₹ 1468	₹ 542
		$ 53	$ 33	$ 13
Patna Jn	-	-	-	-
		-	-	-
Guwahati	15904 CDG DBRG EXPRES	₹ 2230	₹ 1382	₹ 502
		$ 50	$ 31	$ 12
Puri	-	-	-	-
		-	-	-

Goa to		2A	3A	SL
Patna Jn	12741 VSG PATNA EXP	₹ 2395	₹ 1479	₹ 546
		$ 54	$ 33	$ 13
Guwahati	-	-	-	-
		-	-	-
Puri	-	-	-	-
		-	-	-

Patna Jn to		2A	3A	SL
Guwahati	15647 GUWAHATI EXPRES	₹ 1445	₹ 936	₹ 343
		$ 33	$ 21	$ 8
Puri	18450 B NATH DHAM EXP	₹ 1340	₹ 873	₹ 321
		$ 30	$ 20	$ 8

Guwahati to		2A	3A	SL
Puri	15640 GHY PURI EX	₹ 1830	₹ 1157	₹ 422
		$ 41	$ 26	$ 10

In the above table, there are three types of train-fair available for the passengers. However, for our mathematical problem, we have considered one type of train-fair (SL) to construct a traveling salesman problem as shown in Table 4. Similarly, for other type of train-fair similar results can be deduced based on this method.

Table 4 Formulation of Traveling salesman problem of example 2

	1:How	2:Del	3:Agr	4:Mum	5:Jai	6:Cha	7:Goa	8:Pat	9:Guw	10:Pur
1: How	*	406	399	506	438	449	504	252	350	244
2: Del	406	*	120	442	175	163	522	354	484	493
3: Agr	399	120	*	414	155	249	514	314	461	484
4:Mum	505	442	414	*	379	461	288	469	539	493
5: Jai	438	175	155	379	*	—	497	383	488	501
6:Chan	449	163	249	461	—	*	542	—	502	—
7: Goa	504	522	514	288	497	542	*	546	—	—
8: Pat	252	354	314	469	383	—	546	*	343	321
9:Guw	350	484	461	539	488	502	—	343	*	422
10:Pur	244	493	484	493	501	—	—	321	422	*

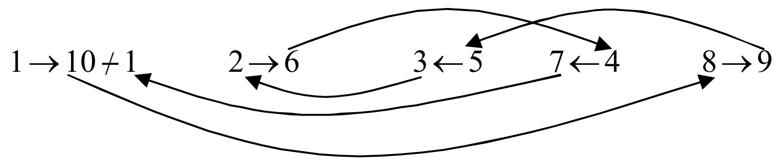
Table 5 Analytical solution of Table 4

	1:How	2:Del	3:Agr	4:Mum	5:Jai	6:Cha	7:Goa	8:Pat	9:Guw	10:Pur
1:How	*	162	155	262	159	162	260	8	15	[0]
2:Del	286	*	$\cancel{0}$	322	20	[0]	402	234	273	373
3:Agr	279	$\cancel{0}$	*	294	[0]	86	494	194	250	364
4:Mum	217	154	126	*	56	130	[0]	181	160	205
5:Jai	283	20	[0]	224	*	—	342	228	242	346
6:Chan	274	[0]	74	286	—	*	367	—	236	—
7:Goa	316	334	316	[0]	274	311	*	358	—	—
8:Pat	$\cancel{0}$	102	62	217	96	—	294	*	[0]	69
9:Guw	7	141	118	196	110	116	—	[0]	*	79
10:Pur	[0]	249	240	249	222	—	—	77	87	*

Table 5 gives the optimal basic feasible solution obtained by the Hungarian method, and the assignment is

$$1 \to 10,\ 2 \to 6,\ 3 \to 5,\ 4 \to 7,\ 5 \to 3,\ 6 \to 2,\ 7 \to 4,\ 8 \to 9,\ 9 \to 8,\ 10 \to 1.$$

Now, we will make a travel, each node just once and finishes up the node from where we got started. From the above assignment we find that one started from node 1, then goes to node 10 and then back to node 1 without traveling other nodes, i.e. $1 \to 10 \to 1$, but it does not complete the whole travel, and so we have to go through the following steps (Fig. 2) as described in our method.



i.e., 1 => 10=>8 =>9 => 5 => 3 => 2 => 6=> 4 => 7 => 1

Fig. 2 Schematic diagram of the complete travel of Table 5

Using our proposed method, the minimum traveling cost (obtained from Table 5) is $= 3087$ units.

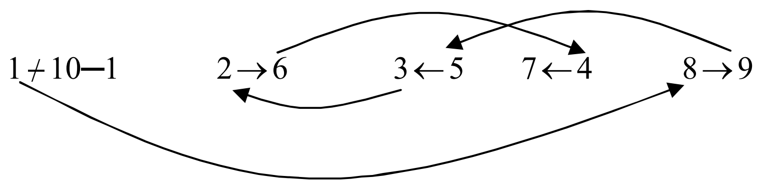


Fig. 3 Schematic diagram of the complete travel of Table 5 (reverse way)

From Fig. 3, it is seen that there are no available paths to return the starting point form Goa (shown in 1st column in Table 4). So, there are no solutions available for this condition. But on the other hand the minimum traveling cost is 3087 units, which are more flexible for all the passengers who are interested to visit all the cities at a time.

Computer oriented solution

```
Output
     Form 1 => To 10
     Form 10 => To 8
     Form 8 =>  To 9
     Form 9 =>  To 5
     Form 5 =>  To 3
     Form 3 =>  To 2
     Form 2 =>  To 6
     Form 6 =>  To 4
     Form 4 =>  To 7
     Form 7 =>  To 1
```

`The minimum traveling cost is = 3087 units`, which is the same as obtained by the analytical method.

7 Conclusions

In this paper, we have developed the modified Hungarian method for solving the traveling salesman problem (TSP) and developed computer program for solving such problems. A number of numerical examples have been selected and solved by using our proposed method. It is found that the proposed concept is helpful in solving present as well as future real-life problems for solving TSP in school bus routing problems, post routing, financial and corporate planning, health care and hospital planning, etc. So, our newly developed procedure saves time and energy and easy to apply for solving practical problems in traveling.

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