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Malmquist productivity index in several time periods

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Abstract The Malmquist productivity index evaluates the productivity change of a decision making unit (DMU) between two time periods. In this current study, a method is proposed to compute the Malmquist productivity index in several time periods (from the first to the last periods) in data envelopment analysis (DEA) and then, the obtained Malmquist productivity index is compared with Malmquist productivity index between two time periods (the first and the last time periods). The aim of this paper is to investigate progress and regress of decision making units (DMUs) in several time periods considering all time periods between the first and the last one. Consequently, when Malmquist productivity index is computed in several time periods, progress and regress of decision making units can be evaluated more carefully than before. At last, a numerical demonstration reveals the procedure of the proposed method then some conclusions are reached and directions for future research are suggested.

Keywords Data Envelopment Analysis (DEA), Malmquist Productivity Index (MPI), Returns To Scale (RTS), Efficiency.

1 Introduction

Productivity growth is one of the major sources of economic development and a thorough understanding of the factors affecting productivity is very important. Recently, research effort has focused on the investigation of the causes of productivity change and on its decomposition. Such decompositions promote the understanding of the determinants of better performance and provide valuable information for managers and planners in both the private and the public sectors. In early work in this field, productivity change was discussed in terms of technical change whereas recently it has become widely accepted that efficiency change can also contribute to it. In this framework, a DEA-based Malmquist productivity index was developed by Färe et al. [1] that it measures the productivity change over time. Malmquist first suggested the Malmquist index (MI) [2] as a quantity index for using in the analysis of consumption of inputs. These ideas were combined the measurement of efficiency from Farrell with the measurement of efficiency from Caves et al. [3] by Färe et al. for constructing the Malmquist productivity index.

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The Malmquist productivity index has proved that it can be a good tool for measuring the productivity change of DMUs. So far the Malmquist productivity index has been computed between two time periods for evaluating the productivity change of DMUs. As some researchers have already paid attention to the measurement of the productivity change of DMUs [4-21]. For instance, Chang et al. [22] investigated productivity measurement of the manufacturing process for outsourcing decisions. Furthermore, deriving the DEA frontier for two-stage processes was inspected by Chen et al. [23]. In addition, Kao [24] presented Malmquist productivity index based on common-weights DEA. Moreover, some researches have been done for performance evaluation of DMUs [25-41]. For example, Charnes et al. [42] introduced measuring the efficiency of decision making units. Also, some methods for estimating technical and scale inefficiencies in data envelopment analysis were presented by Banker et al. [43].

In this paper, we will propose a method to compute the Malmquist productivity index in several time periods to assess the productivity change of DMUs.

The rest of the paper is arranged as follows. The Malmquist productivity index is briefly described in Section 2. Section 3 documents the proposed method. An empirical example is presented in Section 4. Finally, in Section 5 the conclusion and some remarks are put forward.

2 Technical background

The Malmquist productivity index is computed in order to evaluate the productivity change of a DMU between two time periods. It is defined as the product of *Catch-up* and *Frontier-shift* terms. The catch-up term relates the efficiency change of the DMU, while the frontier-shift term reflects the change in the efficient frontiers between two time periods.

Suppose we have a set of DMUs $(\mathbf{x}_j, \mathbf{y}_j)$ $(j = 1,2,\ldots,n)$ each having m inputs denoted by a vector $\mathbf{x}_j \in \mathbb{R}^m$ and s outputs denoted by a vector $\mathbf{y}_j \in \mathbb{R}^s$ over the periods t and $t+1$. Moreover, we assume $\mathbf{x}_j > \mathbf{0}$ $(\forall j)$ and $\mathbf{y}_j > \mathbf{0}$ $(\forall j)$. The notations $(\mathbf{x}_o^t, \mathbf{y}_o^t) = (\mathbf{x}_o, \mathbf{y}_o)^t$ and $(\mathbf{x}_o^{t+1}, \mathbf{y}_o^{t+1}) = (\mathbf{x}_o, \mathbf{y}_o)^{t+1}$ are used to represent DMU_o $\left(o \in \{1,2,\ldots,n\}\right)$ in periods t and $t+1$, respectively. The production possibility set (PPS) T^l $(l = t$ and $t+1)$ is defined by $(\mathbf{x}_j, \mathbf{y}_j)^l$ $(j = 1,2,\ldots,n)$ as follows:

$$T^l = \left\{ (\mathbf{x},\mathbf{y}) \middle| \sum_{j=1}^{n} \lambda_j \mathbf{x}_j^l \le \mathbf{x},\ \mathbf{0} \le \mathbf{y} \le \sum_{j=1}^{n} \lambda_j \mathbf{y}_j^l,\ L \le \sum_{j=1}^{n} \lambda_j \le U,\ \lambda_j \ge 0\ (j = 1,2,\ldots,n) \right\}, \tag{1}$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n) \in \mathbb{R}^n$ is the intensity vector. $(L, U) = \{(0,\infty), (1,1), (1,\infty),$ and $(0,1)\}$ correspond to the CCR, BCC, IRS and DRS models, respectively.

The catch-up effect between two time periods t and $t+1$ is computed by the following formula.

$$Catch-up = \frac{{}^{o}\theta_{t+1}^{t+1}}{{}^{o}\theta_{t}^{t}}, \tag{2}$$

where ${}^{o}\theta_{t+1}^{t+1}$ is the efficiency of $(\mathbf{x}_o, \mathbf{y}_o)^{t+1}$ with respect to frontier of period $t+1$ and ${}^{o}\theta_{t}^{t}$ is the efficiency of $(\mathbf{x}_o, \mathbf{y}_o)^{t}$ with respect to frontier of period t [42].
Fig.1 depicts in the case of a single input and output $(m = s = 1)$ [43].

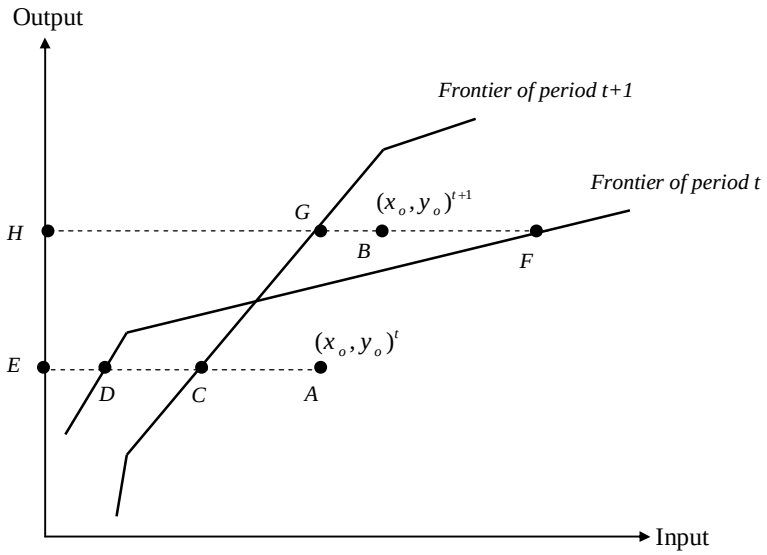

Fig. 1 Two time periods

The catch-up effect is computed in an input-orientation as:

$$Catch-up = \frac{HG}{HB} \Big/ \frac{ED}{EA}. \tag{3}$$

Furthermore, the frontier-shift effect is computed by the following geometric mean:

$$Frontier-shift = \sqrt{\varphi_1 \varphi_2}, \tag{4}$$

where $\varphi_1 = \dfrac{{}^{o}\theta_{t}^{t}}{{}^{o}\theta_{t}^{t+1}}$ and $\varphi_2 = \dfrac{{}^{o}\theta_{t+1}^{t}}{{}^{o}\theta_{t+1}^{t+1}}$ are the frontier-shift effect at $(\mathbf{x}_o, \mathbf{y}_o)^{t}$ and $(\mathbf{x}_o, \mathbf{y}_o)^{t+1}$, respectively. Note that, ${}^{o}\theta_{t}^{t+1}$ is the efficiency of $(\mathbf{x}_o, \mathbf{y}_o)^{t}$ with respect to frontier of period $t+1$ and also, ${}^{o}\theta_{t+1}^{t}$ is the efficiency of $(\mathbf{x}_o, \mathbf{y}_o)^{t+1}$ with respect to frontier of period t [43].
Associated with Fig. 1, the frontier-shift effect can be calculated as:

$$Frontier-shift = \sqrt{\frac{ED}{EC}\frac{HF}{HG}}, \tag{5}$$

where

$$\varphi_1 = \frac{ED}{EA} \Big/ \frac{EC}{EA} \text{ and } \varphi_2 = \frac{HF}{HB} \Big/ \frac{HG}{HB}.$$

Next, the Malmquist Index (MI) is computed as the product of *Catch-up* and *Frontier-shift*, i.e.,

$$MI = (Catch-up)\times(Frontier-shift). \tag{6}$$

Consequently, using (2) and (4), the Malmquist index for evaluating change of DMU_o is as follows:

$$\begin{aligned} MI = MI^o &= (Catch-up)^o \times (Frontier-shift)^o, \\ &= \frac{{}^o\theta_{t+1}^{t+1}}{{}^o\theta_t^t}\left(\frac{{}^o\theta_t^t}{{}^o\theta_t^{t+1}}\times\frac{{}^o\theta_{t+1}^t}{{}^o\theta_{t+1}^{t+1}}\right)^{\frac{1}{2}}, \end{aligned} \tag{7}$$

where the relative change in performance is represented by the first term and also the second term represents the relative change in the frontier used to evaluate these performances.
According to the Fig. 1, the Malmquist index is computed as:

$$MI^o = \frac{EA}{HB}\sqrt{\frac{HF}{ED}\frac{HG}{EC}}. \tag{8}$$

Note that, $MI^o > 1$ and $MI^o < 1$ indicate progress and regress in the total factor productivity of DMU_o between two time periods t and $t+1$, respectively. Moreover $MI^o = 1$ indicates no progress and no regress in the total factor productivity.

3 Proposed method

In this section, we propose a method to compute the Malmquist productivity index in p $(p \geq 3)$ time periods to evaluate the productivity change of a DMU. On the other hand, we will compute the Malmquist productivity index from period t to $t+p-1$. The notation $(\mathbf{x}_o, \mathbf{y}_o)^{t+i}$ $(i = 0,1,\ldots,p-1)$ is used to represent DMU_o in periods $t+i$.

In this method, using (7), the Malmquist index is first computed for evaluating productivity change of DMU_o between two time periods $t+i$ and $t+i+1$ $(i = 0,1,\ldots,p-2)$. Now we denote it MI_{i+1}^o.

Then, we compute the Malmquist index for evaluating productivity change of DMU_o in p time periods as the product of MI_{i+1}^o $(i = 0,1,\ldots,p-2)$, i.e.,

$$\begin{aligned} MI_{total} = MI_{total}^o &= (Catch-up)_{total}^o \times (Frontier-shift)_{total}^o, \\ &= \prod_{i=0}^{p-2} MI_{i+1}^o. \end{aligned} \tag{9}$$

It is worth stressing that the performance of DMU_o between each two consecutive time periods does not depend on its performance between each two another consecutive time periods.

Note that, $(Catch-up)^o_{total}$ and $(Frontier-shift)^o_{total}$ are the catch-up and frontier-shift effects of the DMU_o from period t to $t+p-1$, respectively. Then, they can be computed as follows:

$$(Catch-up)^o_{total} = \prod_{i=0}^{p-2} (Catch-up)^o_{i+1}, \tag{10}$$

$$(Frontier-shift)^o_{total} = \prod_{i=0}^{p-2} (Frontier-shift)^o_{i+1}. \tag{11}$$

$(Catch-up)^o_{total} > 1$ and $(Catch-up)^o_{total} < 1$ indicate progress and regress in relative efficiency of DMU_o from period t to $t+p-1$, respectively. Meanwhile, $(Catch-up)^o_{total} = 1$ indicates no change in efficiency.

Furthermore, $(Frontier-shift)^o_{total} > 1$ and $(Frontier-shift)^o_{total} < 1$ indicate progress and regress in the frontier technology around DMU_o from period t to $t+p-1$, respectively. In addition, $(Frontier-shift)^o_{total} = 1$ indicates the status quo in the frontier technology.

Fig. 2 highlights the illustration in the case of a single input and output [43].

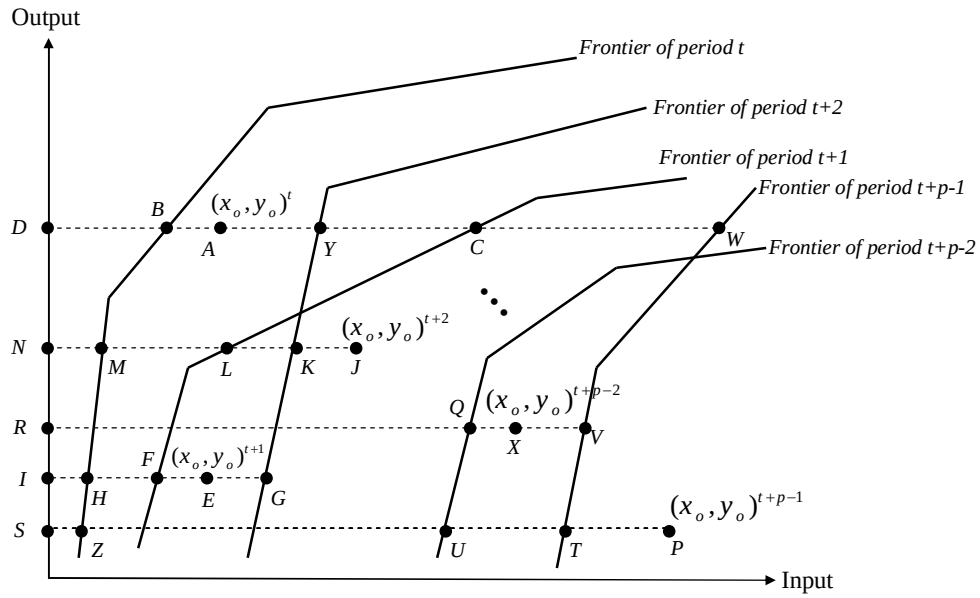

Fig. 2 Several time periods.

Then using (7) and (9), we get

$$MI^{o}_{total} = MI^{o}_{1} \times MI^{o}_{2} \times \ldots \times MI^{o}_{p-1},$$
$$= \left(\frac{DA}{IE} \sqrt{\frac{IH}{DB}\frac{IF}{DC}} \right) \times \left(\frac{IE}{NJ} \sqrt{\frac{NL}{IF}\frac{NK}{IG}} \right) \times \ldots \times \left(\frac{RX}{SP} \sqrt{\frac{SU}{RQ}\frac{ST}{RV}} \right). \quad (12)$$

Now, using (7) we compute the Malmquist index for evaluating productivity change of DMU_o between two time periods t and $t+p-1$ that is represented by notation $MI^{o}_{1,p}$, i.e.,

$$MI_{1,p} = MI^{o}_{1,p} = (Catch-up)^{o}_{1,p} \times (Frontier-shift)^{o}_{1,p}, \quad (13)$$

where $(Catch-up)^{o}_{1,p}$ and $(Frontier-shift)^{o}_{1,p}$ are the catch-up and frontier-shift effects of DMU_o between two time periods t and $t+p-1$, respectively.

According to Fig. 2, $MI^{o}_{1,p}$ can be computed as:

$$MI^{o}_{1,p} = \frac{DA}{SP} \sqrt{\frac{SZ}{DB}\frac{ST}{DW}}. \quad (14)$$

Theorem 1. *The catch-up effect of DMU_o between two time periods t and $t+p-1$ equals its catch-up effect from period t to $t+p-1$, i.e.,*

$$(Catch-up)^{o}_{1,p} = (Catch-up)^{o}_{total}. \quad (15)$$

Proof. The proof is straightforward from (2). □

Theorem 2. *The relation between $(Frontier-shift)^{o}_{total}$ and $(Frontier-shift)^{o}_{1,p}$ is as follows*

$$(Frontier-shift)^{o}_{total} = \left(\frac{{}^{o}\theta^{t+p-1}_{t}}{{}^{o}\theta^{t}_{t+p-1}} \times \frac{\prod_{i=0}^{p-2} {}^{o}\theta^{t+i}_{t+i+1}}{\prod_{i=0}^{p-2} {}^{o}\theta^{t+i+1}_{t+i}} \right)^{\frac{1}{2}} \times (Frontier-shift)^{o}_{1,p}. \quad (16)$$

Proof. According to relations (4) and (11), the frontier-shift effect of DMU_o from period t to $t+p-1$ is as follows:

$$(Frontier-shift)^{o}_{total} = \left(\frac{{}^{o}\theta^{t}_{t}}{{}^{o}\theta^{t+p-1}_{t+p-1}} \times \frac{\prod_{i=0}^{p-2} {}^{o}\theta^{t+i}_{t+i+1}}{\prod_{i=0}^{p-2} {}^{o}\theta^{t+i+1}_{t+i}} \right)^{\frac{1}{2}}. \quad (17)$$

Thus,

$$(Frontier-shift)^{o}_{total}=\left(\frac{{}^{o}\theta^{t}_{t}}{{}^{o}\theta^{t+p-1}_{t+p-1}}\times\frac{{}^{o}\theta^{t}_{t+p-1}}{{}^{o}\theta^{t+p-1}_{t}}\times\frac{{}^{o}\theta^{t+p-1}_{t}}{{}^{o}\theta^{t}_{t+p-1}}\times\frac{\prod_{i=0}^{p-2}{}^{o}\theta^{t+i}_{t+i+1}}{\prod_{i=0}^{p-2}{}^{o}\theta^{t+i+1}_{t+i}}\right)^{\frac{1}{2}},$$
$$=\left(\frac{{}^{o}\theta^{t+p-1}_{t}}{{}^{o}\theta^{t}_{t+p-1}}\times\frac{\prod_{i=0}^{p-2}{}^{o}\theta^{t+i}_{t+i+1}}{\prod_{i=0}^{p-2}{}^{o}\theta^{t+i+1}_{t+i}}\right)^{\frac{1}{2}}\times(Frontier-shift)^{o}_{1,p}. \quad (18)$$

Hence, the proof is complete. □

Now, after computing MI^{o}_{total} and $MI^{o}_{1,p}$, we compare them to evaluate progress and regress of DMU_o from period t to $t+p-1$ as follows.

(a) If $MI^{o}_{total}>MI^{o}_{1,p}>1$, then both MI^{o}_{total} and $MI^{o}_{1,p}$ indicate progress for DMU_o and also, MI^{o}_{total} indicates more progress.

(b) If $MI^{o}_{total}>MI^{o}_{1,p}$ and $MI^{o}_{1,p}=1$, then MI^{o}_{total} indicates progress for DMU_o, while $MI^{o}_{1,p}$ indicates no progress and no regress.

(c) If $MI^{o}_{total}>1$ and $MI^{o}_{1,p}<1$, then MI^{o}_{total} and $MI^{o}_{1,p}$ indicate progress and regress for DMU_o, respectively.

(d) If $MI^{o}_{total}>MI^{o}_{1,p}$ and $MI^{o}_{total}=1$, then MI^{o}_{total} indicates no progress and no regress for DMU_o, while $MI^{o}_{1,p}$ indicates regress.

(e) If $MI^{o}_{1,p}<MI^{o}_{total}<1$, then both of MI^{o}_{total} and $MI^{o}_{1,p}$ indicate regress for DMU_o and also, MI^{o}_{total} indicates less regress.

(f) If $MI^{o}_{1,p}>MI^{o}_{total}>1$, then both of MI^{o}_{total} and $MI^{o}_{1,p}$ indicate progress for DMU_o and also, MI^{o}_{total} indicates less progress.

(g) If $MI^{o}_{1,p}>MI^{o}_{total}$ and $MI^{o}_{total}=1$, then MI^{o}_{total} indicates no progress and no regress for DMU_o, while $MI^{o}_{1,p}$ indicates progress.

(h) If $MI^{o}_{total}<1$ and $MI^{o}_{1,p}>1$, then MI^{o}_{total} and $MI^{o}_{1,p}$ indicate regress and progress for DMU_o, respectively.

(i) If $MI^{o}_{total}<MI^{o}_{1,p}$ and $MI^{o}_{1,p}=1$, then MI^{o}_{total} indicates regress for DMU_o, while $MI^{o}_{1,p}$ indicates no progress and no regress.

(j) If $MI^{o}_{total}<MI^{o}_{1,p}<1$, then both of MI^{o}_{total} and $MI^{o}_{1,p}$ indicate regress for DMU_o and also, MI^{o}_{total} indicates more regress.

Hence, based on the above discussion, we conclude that the obtained results from MI^{o}_{total} about progress and regress of DMU_o are more careful than the obtained results from $MI^{o}_{1,p}$, since we assume all of the time periods between two time periods t and $t+p-1$ computing MI^{o}_{total}, while they are not considered computing $MI^{o}_{1,p}$.

4 Empirical example

To illustrate how the proposed method is applied, let us consider a realistic application to Iranian commercial banks. We want to survey progress and regress of these banks during 38 months. Using Expert advice from a banking specialist, inputs and outputs are used in this study shown in Table 1.

Table 1. The set of inputs and outputs

Inputs		Outputs	
(I_1)	Number of year of establishment	(O_1)	Savings
(I_2)	Area	(O_2)	Deposits
(I_3)	Privilege of staff	(O_3)	Current account
(I_4)	Equipment	(O_4)	Invest for long time
		(O_5)	Invest for short time

Note that, the data of inputs and outputs have not been shown for the sake of their voluminous. Moreover, the evaluation results are shown in Table 2.

Table 2 The results of evaluating.

Branch	MI_{total}	The obtained results from MI_{total}	$MI_{1,38}$	The obtained results from $MI_{1,38}$
1	2.2892	more progress	2.1067	progress
2	1.5407	more progress	1.1458	progress
3	0.5011	more regress	0.7757	regress
4	2.1075	more progress	1.3590	progress
5	2.1052	more progress	2.0077	progress
6	0.9742	regress	1.5728	progress
7	2.3038	less progress	2.4380	progress
8	2.6247	more progress	2.6230	progress
9	1.1505	less progress	1.4254	progress
10	6.3326	more progress	6.1107	progress
11	2.2997	more progress	2.2620	progress
12	1.9871	less progress	2.0711	progress
13	2.2333	more progress	2.0531	progress
14	2.6682	more progress	2.2334	progress
15	1.8121	more progress	1.5403	progress
16	1.8693	more progress	1.7450	progress
17	1.9310	more progress	1.8258	progress
18	2.3139	more progress	1.8040	progress
19	2.8677	more progress	2.0916	progress
20	3.0757	more progress	2.5855	progress

As presented in Table 2, MI_{total} indicates more progress for 15 branches and also, it indicates less progress for 3 branches. Moreover, MI^{3}_{total} indicates more regress for the third branch.

Note that, $MI^{6}_{1,38}$ indicates progress for the sixth branch, while our proposed method indicates regress for the DMU under evaluation.

In this case study, we have used the CCR DEA model (in an input-orientation) [43] to compute the efficiency of branch banks in different months. It is necessary to mention that other models can be appropriately extended to all other DEA variants.

5 Conclusions

The main objective of this note has been to present a method computing the Malmquist productivity index in order to calculate productivity change of a DMU in several time periods time (from the first to the last periods). Then, we compared it with the Malmquist productivity index between two time periods (the first and the last time periods) in order to evaluate progress and regress of the DMU under evaluation.

By considering all time periods between the first and the last time, the aim of this research is to investigate progress and regress of DMUs in several time periods. Hence, it is striking to observe that the obtained results from the Malmquist index in several time periods in order to evaluate progress and regress of the target DMU are more careful than the obtained results from the Malmquist index between two time periods.

At last, to illustrate the proposed approach, we apply it to compute the Malmquist productivity index of bank branches to evaluate progress and regress of the target DMU. We suggest considering special data such as stochastic, interval, integer, fuzzy, etc. for future researches.

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