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Portfolio performance evaluation in modified mean-variance models

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Abstract The present study is an attempt toward evaluating the performance of portfolios and assets selecting using modified mean-variance models by utilizing a non-parametric efficiency analysis tool, namely Data Envelopment Analysis (DEA). Huge amounts of money are being invested in financial market. As a result, portfolio performance evaluation has created a great deal of interest among people. We know that, for calculating portfolio variance measure based on mean-variance model, the covariance between each pair of the assets is not equal to zero. Consequently sharp's single factor model is used with linear regression for efficiency evaluation in modified mean-variance models. Since the covariance between two stocks are not merely bound to the characteristics of the two stocks but these stocks are connected together through their relations to the market return, the total number of parameters that needs to be estimated is reduced.

Keywords: Data Envelopment Analysis, Portfolio, Linear regression.

1 Introduction

In financial literature, a portfolio is an appropriate mix investments held by an institution or private individuals. Evaluation of portfolio performance has created a large interest among employees also academic researchers because of huge amount of money are being invested in financial markets. The mean – variance theory by Markowitz [1] is considered the basis of many current models and this theory is widely used to select portfolios. This model is due to the nature of the variance in quadratic form. Due to quadratic form, a study by Arditti [2], Kane [3] and Ho and Cheung [4] shown that investors prefer skewness which means that utility functions of investors are not quadratic. Other problem in Markowitz model is that increasing the number of assets will be developed the covariance matrix of asset returns and will be added to the content of data. Due to these problems sharp one- factor model is proposed by Sharp [5]. This method reduces the number of data required information for the decision. Data envelopment analysis (DEA) has proved the efficiency for assessing the relative efficiency of Decision Making Units (DMUs) that employing multiple inputs to

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produce multiple outputs [6]. Morey and Morey [7] proposed mean – variance framework based on Data Envelopment Analysis, which the variance of the portfolios is used as an input to the DEA and expected return is the output. Joro and Na [8] introduced mean - variance – skewness framework and skewness of returns are also considered as an output. Briec et al. [9] introduced shortage function. This shortage function obtains an efficiency measure that looks to improve in both mean and skewness and decreases in variance. Kerstence et al. [10] introduced a geometric representation of the MVS frontier related to a new tool introduced in the literature by Briec. Mihiri and Prigent [11] analyze the portfolio optimization problem by introducing the higher moments of the main financial index returns. In mentioned models instead of estimating the whole efficient frontier, only the projection points of the assets are calculated. In these models are used a non-linear DEA-like framework where the correlation structure among the units is taken into account.

The philosophy behind our approach is inspired by Sharp one-factor model, which reduces the number of information and the amount of required data for decision making. The main assumption here is that the return existing between the two financial assets that are not merely bound to the characteristics of the two stocks, but these two are connected together through their relations to the market return. The aim of paper is to evaluate portfolio performance measurement which is based on mean-variance and can overcome the difficulties of the existing for pervious methods. In this approach, instead of estimating the whole efficient frontier, only the projection points of the assets are calculated, too .The new models of mean- variance are proposed to employ this new analysis here. The rest of the paper is organized as follows. The next section represents DEA models; mean - variance models of Markowitz and Morey and mean - variance - skewness briefly. The second and third Sections develop nonlinear modified mean – variance models contain of input oriented model, output oriented model and combination oriented model. The fourth Section is a real global application and the proposed models are applied to evaluate the portfolios performance. Finally conclusions are given.

2 Background

Data Envelopment Analysis is a nonparametric method for evaluating the efficiency of systems with multiple inputs and multiple outputs. In this section we present some basic definitions, models and concepts that will be used in other sections in DEA. They will not be discussed in details. Consider DMU_j , $(j = 1,...,n)$ where each DMU consumes m inputs to produce s outputs. Suppose that the observed input and output vectors of DMU_j are $X_j = (x_{1j},...,x_{mj})$ and $Y_j = (y_{1j},...,y_{sj})$ respectively, and let $X_j \geq 0$ and $X_j \neq 0$, $Y_j \geq 0$ and $Y_j \neq 0$. A basic DEA formulation in input orientation is as follows:

$$
\begin{aligned}
& Min\ \theta-\varepsilon\left(\sum_{r=1}^{s} s_r^{+}+\sum_{i=1}^{m} s_i^{-}\right) \\
& s.t. \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij}+s_i^{-}=\theta x_{io} \quad i=1,\ldots,m, \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj}-s_r^{+}=y_{ro} \quad r=1,\ldots,s, \\
& \quad \lambda \in \Lambda, \\
& \quad s^{+}, s^{-} \geq 0, \\
& \quad \varepsilon \geq 0
\end{aligned} \tag{1}
$$

where λ is a n-vector of λ variables, s^{+} as-vector of output slacks, s^{-} an m-vector of input slacks and set Λ is defined as follows:

$$
\Lambda=\begin{cases}\{\lambda \in R_{+}^{n}\} & \text{with constant returns to scale,} \\ \{\lambda \in R_{+}^{n}, 1\lambda \leq 1\} & \text{with non-increasing returns to scale,} \\ \{\lambda \in R_{+}^{n}, 1\lambda=1\} & \text{with variable returns to scale}\end{cases}
$$

Note that subscript 'o' refers to the unit under the evaluation. A DMU is efficient if $\theta=1$ and all slack variables s^{-}, s^{+} equal zero; otherwise it is inefficient [12]. In the DEA formulation above, the left –hand sides in the constraints define an efficient portfolio. θ is a multiplier defines the distance from the efficient frontier. The slack variables are used to ensure that the efficient point is fully efficient. Another model is in output oriented as follows:

$$
\begin{aligned}
& Max\ \varphi-\varepsilon\left(\sum_{r=1}^{s} s_r^{+}+\sum_{i=1}^{m} s_i^{-}\right) \\
& s.t. \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij}+s_i^{-}=x_{io} \quad i=1,\ldots,m, \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj}-s_r^{+}=\varphi y_{ro} \quad r=1,\ldots,s, \\
& \quad \lambda \in \Lambda, \\
& \quad s^{+}, s^{-} \geq 0, \\
& \quad \varepsilon \geq 0
\end{aligned} \tag{2}
$$

Also combination oriented model is proposed for evaluation efficiency goes as follows:

$$
\begin{aligned}
&Max\ \beta \\
&s.t. \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} - \beta x_{io} \quad i = 1,\dots,m, \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \beta y_{ro} \quad r = 1,\dots,s, \\
&\quad \lambda \in \Lambda,
\end{aligned}
\tag{3}
$$

Portfolio theory to investing is published by Markowitz [1]. This approach starts by assuming that an investor has a given sum of money to invest at the present time. This money will be invested for a time as the investor's holding period. The end of the holding period, the investor will sell all of the assets that were bought at the beginning of the period and then either consume or reinvest. Since portfolio is a collection of assets, it is better that to select an optimal portfolio from a set of possible portfolios. Hence the investor should recognize the returns (and portfolio returns), expected (mean) return and standard deviation of return. This means that the investor wants to both maximize expected return and minimize uncertainty (risk). Rate of return (or simply the return) of the investor's wealth from the beginning to the end of the period is calculated as follows:

$$Return = \frac{(end-of-period\ wealth) - (beginning-of-period\ wealth)}{beginning-of-period\ wealth}$$

Or

$$Return = \log(end\ of\ period\ wealth) - \log(beginning\ of\ period\ wealth)$$

Since Portfolio is a collection of assets, its return r_p can be calculated in a similar manner. Thus according to Markowitz, the investor should view the rate of return associated to any one of these portfolios as what is called in statistics a random variable. These variables can be described expected the return (mean or $\overline{r}_p$) and standard deviation of return. Expected return and deviation standard of return are calculated as follows:

$$\overline{r}_p = \sum_{i=1}^{n} \lambda_i \overline{r}_i \ , \ \sigma_p = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \sigma_{ij} \right]^{1/2}$$

Where:

n=the number of assets in the portfolio

$\overline{r}_p$ =The expected return of the portfolio

λ_i =The proportion of the portfolio's initial value invested in asset i

$\overline{r}_i$ =The expected return of asset i

σ_p = The deviation standard of the portfolio

σ_{ij} = The covariance of the returns between asset i and asset j

In the above, optimal portfolio from the set of portfolios will be chosen that maximum expected return for varying levels of risk and minimum risk for varying levels of expected return. One of the Markowitz model's problems is that increasing the number of assets will be developed the covariance matrix of asset returns and will be added to the content of data. Due to these problems, sharp one-factor model is proposed by Sharp [5]. This model reduces the number of information and the amount of data required for decision making. The main assumption in this model is that the expected return existing between the two financial assets is not merely bound to the characteristics of the two stocks but these two are connected together through their relations to the market expected return. Suppose that the return on a common stock over a given time period (say, a month) is related to the return over the same period that is earned on a market index such as the widely cited S&P 500. That is if the market has gone up then it is likely that the stock has gone up and if the market has gone down then it is likely that the stock has gone down. One method to capture this relation is with the sharp model [13]:

$$r_i = \alpha_i + \beta_i I + C_i$$

Where:

r_i= return on asset i

α_i = intercept of a straight line or alpha coefficient

β_i = slope of straight line or beta coefficient

I= return on index (market)

C_i = random error term with a mean of zero and Q_i variance.

The amount of I in the future of a random variable is defined as follows:

$$I = \alpha_{n+1} + C_{n+1}$$

Where α_{n+1} is affixed number and C_{n+1} represents a random variable, with **zero** mean and Q_{n+1} variance.

Given the above relation the mean of the random variable r_i is equal to:

$$E(r_i) = E(\alpha_i + \beta_i I + C_i) = \alpha_i + \beta_i E(I) + E(C_i)$$

$$\alpha_i + \beta_i E(\alpha_{n+1} + C_{n+1}) + E(C_i) = \alpha_i + \beta_i \alpha_{n+1}$$

and the variance of the random variable equals to:

$$\sigma^2_{r_i} = \sigma^2_{\alpha_i + \beta_i I + C_i} = \beta^2_I \sigma^2_I + \sigma^2_{C_i} = \beta^2_I Q_{n+1} + Q_i$$

Sharp stated that the variance explained by the index could be referred to as the systematic risk and the variance is related to the characteristics of the assets that are called unsystematic risk. Return on a portfolio of assets consisting of n is a random variable as following:

$$r_p = \sum_{i=1}^{n} \lambda_i r_i = \sum_{i=1}^{n} \lambda_i (\alpha_i + \beta_i I + C_i) = \sum_{i=1}^{n} \lambda_i (\alpha_i + C_i) + [\sum_{i=1}^{n} \lambda_i \beta_i] I$$

Where the proportion of funds invested in asset i for a given portfolio p is denoted λ_i .Then we have:

$$E(r_p) = E\left(\sum_{i=1}^{n} \lambda_i (\alpha_i + C_i)\right) + E\left(\left[\sum_{i=1}^{n} \lambda_i \beta_i\right] I\right) \ , \ \sum_{i=1}^{n} \lambda_i \beta_i = \lambda_{n+1}, E(I) = \alpha_{n+1}$$

Therefore

$$E(r_p) = \sum_{i=1}^{n} \lambda_i \alpha_i + \lambda_{n+1}\alpha_{n+1} = \sum_{i=1}^{n+1} \lambda_i \alpha_i$$

And portfolio variance

$$\sigma_{r_p}^2 = \sum_{i=1}^{n} \lambda_i^2 Q_i + \left[\sum_{i=1}^{n} \lambda_i \beta_i\right]^2 Q_{n+1} = \sum_{i=1}^{n+1} \lambda_i^2 Q_i$$

The portfolio performance evaluation literature is vast. In recent years models have been used to evaluate the portfolio efficiency. In these models, instead of estimating the whole efficient frontier, only the projection points of the assets are calculated. The distance between the asset and its projection which means the ratio between the variance of the projection point and the variance of the asset is considered as an efficiency measure (θ) .In this framework, there is n assets, λ_j is the weight of asset j in the projection point, r_j is a random variable representing the rate of return of asset j , μ_o and σ_o^2 are the expected return and variance of the asset under evaluation respectively. Efficiency measure θ can be solved via following model [7]:

$$\begin{aligned} & Min \quad \theta - \varepsilon(s_1 + s_2) \\ & s.t. \\ & \quad E\left[\sum_{j=1}^{n} \lambda_j r_j\right] - s_1 = \mu_o, \quad (1) \\ & \quad E\left[(\sum_{j=1}^{n} \lambda_j (r_j - \mu_j))^2\right] + s_2 = \theta\sigma_o^2 \quad (2) \\ & \quad \sum_{j=1}^{n} \lambda_j \le 1 \quad \forall \lambda \ge 0 \end{aligned} \tag{4}$$

Model (4) is revealed by the non-parametric efficiency analysis Data Envelopment Analysis. Joro and Na [8] extended the described approach in (4) into mean-variance-skewness framework where κ_o is the skewness of the asset under evaluation. The efficiency measure θ can be solved through using the following model:

$$\begin{aligned} & Min \quad \theta - \varepsilon(s_1 + s_2 + s_3) \\ & s.t. \\ & \quad E\left[\sum_{j=1}^{n} \lambda_j r_j\right] - s_1 = \mu_o, \\ & \quad E\left[(\sum_{j=1}^{n} \lambda_j (r_j - \mu_j))^2\right] + s_2 = \theta\sigma_o^2 \\ & \quad E\left[(\sum_{j=1}^{n} \lambda_j (r_j - \mu_j))^3\right] - s_3 = \kappa_o \\ & \quad \sum_{j=1}^{n} \lambda_j \le 1 \quad \forall \lambda \ge 0 \end{aligned} \tag{5}$$

Model (5) projects the asset with the efficient frontier by fixing the expected return and skewness levels and minimizing the variance. In proposed models the covariance between each pair of the assets is not equal to zero, then the content of data is high.

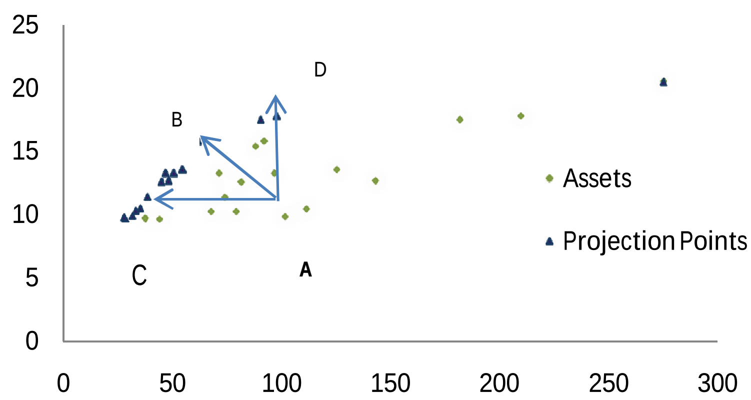

Fig. 1 Different projections (input oriented, output oriented and combination oriented

Hence it is better to use Sharp one-factor model [5] which reduces the number of information and the amount of data required for decision making. The main assumption here is that the expected return existing between the two financial assets is not merely bound to the characteristics of the two stocks but these stocks are connected together through their relations to the market expected return. Here the modified model of mean- variance (input oriented) is proposed applying this new method. In this model projects the asset in to the efficient frontier by fixing the expected return and minimizing the variance using sharp's single factor model. In DEA terminology, this corresponds to input orientation. Fig 1 illustrates different projection that consist of input oriented, output oriented and combination oriented. C is the projection point obtained via fixing expected return and minimizing variance (model 1), B via maximizing return and minimizing variance simultaneously (model 3), and D via fixing variance and maximizing return (model 2).

3 Modified mean-variance model

Let us assume that we are evaluating the efficiency score of the asset 'o' in modified mean-variance model. In model 4, consider relation 2, we can have:

$$E\left[\left(\sum_{j=1}^{n}\lambda_j(r_j-\mu_j)\right)^2\right]=\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\lambda_j\sigma_{ij}$$

Where σ_{ij} denotes the covariance of the returns between asset i and asset j. With considering the above relation, the units (stocks, here) are not independent while the units act independently in the Data Envelopment Analysis model. Also in model 4 for N assets, required information details are as follows; N expected return, N Standard deviation, $(N^2-N)/2$ covariance between the return of assets, and therefore totally N (N+3)/2 information details are needed. In the Sharp model, however, the number of needed covariance is reduced to N from $(N^2-N)/2$. It means that required information will be N expected return for N stocks, N Standard deviation, N covariance between the stocks and the index, which reduces the number of items to 3N+2. As it is seen, this amount of information is really less than N (N+3)/2 and shortens required data as well. There is no correlation between the return of the assets either. The main assumption here is that the return existing between the two financial assets that are not merely bound to the characteristics of the two assets, but these two are connected together through their relations to the market return. Thus in the modified mean-variance model, in addition to the independent units and these two stocks are connected together through their relations to the market return, the number of data is also drastically reduced. Let us assume that we are evaluating the efficiency of the asset A in modified mean- variance model. C is the projection point obtained via fixing expected return and minimizing variance in this model. The model goes as follows:

$$
\begin{aligned}
& Min\ \ \theta - \varepsilon(s_1 + s_2) \\
& s.t. \\
& \quad \sum_{j=1}^{n+1} \lambda_j \alpha_j - s_1 = E(r_o) = \mu_o \\
& \quad \sum_{j=1}^{n+1} \lambda_j^2 Q_j + s_2 = \theta \sigma_o^2 \\
& \quad \sum_{j=1}^{n} \lambda_j \beta_j = \lambda_{n+1} \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \quad \lambda_j \ge 0
\end{aligned}
\tag{6}
$$

In this model μ_o and σ_o^2 are the expected return and variance of the asset under evaluation, respectively, λ_j is the weight of asset j in the projection point, α_i and β_i are the fixed numbers that define the regression line between r_i and I.

Also α_i, β_i and Q_i pertained to the financial assets which are calculated based on the previous information and the amounts of α_{n+1} and Q_{n+1} pertained to the index which should be speculated. θ in (6) ranges from 0 to 1. Asset with θ equal to 1 and slack variables s_1 and s_2 equal to zero is said to be efficient. Otherwise is said to be inefficient. In model 6, we assume no lending or borrowing at a risk-free rate.

Define a set Λ:

$$
\Lambda = \begin{cases}
\lambda \mid \lambda \in \mathrm{R}_+^n & \text{with lending and borrowing} \\
\lambda \mid \lambda \in \mathrm{R}_+^n, \sum_{j=1}^{n} \lambda_j \le 1 & \text{with lending but no borrowing} \\
\lambda \mid \lambda \in \mathrm{R}_+^n, \sum_{j=1}^{n} \lambda_j = 1 & \text{with no lending or borrowing}
\end{cases}
$$

Where λ is an n-vector of λ variables.

Define a set F:

$$
\mathrm{F} = \left\{ (\mu, \sigma^2) \mid \mu = \sum_{j=1}^{n+1} \lambda_j \alpha_j - s_1,\ \sigma^2 = \sum_{j=1}^{n+1} \lambda_j^2 Q_j + s_2,\ \sum_{j=1}^{n} \lambda_j \beta_j = \lambda_{n+1}, \lambda \in \Lambda \right\}
$$

All efficient units lie on the efficient frontier, which is defined as a subset of points of set F satisfying the efficiency condition as defined in Definitions 1, 2.

Definition1:

A point $(\mu^*, \sigma^{2*}) \in \mathrm{F}$ is efficient if there dose not exist another $(\mu, \sigma^2) \in \mathrm{F}$ such that $\mu \geq \mu^*, \sigma^2 \leq \sigma^{2*}$ and $(\mu, \sigma^2) \neq (\mu^*, \sigma^{2*})$.

Definition 2:

A point $(\mu^*, \sigma^{2*}) \in \mathrm{F}$ is weakly efficient if there does not exist another $(\mu, \sigma^2) \in \mathrm{F}$ such that $\mu > \mu^*, \sigma^2 < \sigma^{2*}$.

This model is presented as follows after calculations:

$$
\begin{aligned}
& Min \;\; \theta - \varepsilon(s_1 + s_2) \\
& s.t. \\
& \quad \sum_{j=1}^{n} \lambda_j \mu_j - s_1 = \mu_o \\
& \quad \sum_{j=1}^{n} \lambda_j^2 \sigma_j^2 + \sum_{j=1}^{n} \sum_{\substack{i=1 \\ i \neq j}}^{n} \lambda_j \lambda_i \beta_i \beta_j Q_{n+1} + s_2 = \theta \sigma_o^2 \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \quad \lambda_j \geq 0 \quad j = 1, \dots, n
\end{aligned}
\tag{7}
$$

In above model the assets are projected in to the efficient frontier by fixing the expected return level and minimizing the variance. Now, we introduce other models which the assets are projected in to the efficient frontier by fixing the variance and maximizing the expected return and also maximum proportional reduction in variance, while return is increased in the same proportion to the initial assets respectively. In these models, the number of calculations is also drastically reduced, too.

4 Another modified Mean-Variance models

Let us continue from the situation presented in Fig 1. Let us assume that we are evaluation the efficiency of the asset A in modified mean-variance models in output oriented and combination oriented. The point D represents the projection of asset A into the efficient frontier via fixing variance and maximizing return. The point B represents the projection of asset A into the efficient frontier via maximizing return and minimizing variance simultaneously. Output oriented model and combination oriented model are proposed for evaluation efficiency goes as follows:

$$
\begin{aligned}
& Max \quad \varphi - \varepsilon(s_1 + s_2) \\
& s.t. \\
& \quad \sum_{j=1}^{n+1} \lambda_j \alpha_j - s_1 = \varphi E(r_o) = \varphi \mu_o \\
& \quad \sum_{j=1}^{n+1} \lambda_j^2 Q_j + s_2 = \sigma_o^2 \\
& \quad \sum_{j=1}^{n} \lambda_j \beta_j = \lambda_{n+1} \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \quad \lambda_j \geq 0
\end{aligned} \tag{8}
$$

and

$$
\begin{aligned}
& Max \quad \beta \\
& s.t. \\
& \quad \sum_{j=1}^{n} \lambda_j \mu_j \leq \mu_o - \beta \mu_o \\
& \quad \sum_{j=1}^{n} \lambda_j^2 \sigma_j^2 + \sum_{j=1}^{n} \sum_{\substack{i=1 \\ i \neq j}}^{n} \lambda_j \lambda_i \beta_i \beta_j Q_{n+1} \geq \sigma_o^2 + \beta \sigma_o^2 \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \quad \lambda_j \geq 0 \quad j = 1, \ldots, n
\end{aligned} \tag{9}
$$

Asset with φ equal to 1 and slack variables s_1 and s_2 equal to zero is said to be efficient. Also if β equals zero, then under evaluation asset is part of the efficient frontier and it is efficient. In proposed models are assumed no lending or borrowing at risk-free rate by requiring the sum of λs to be equal unity: $\sum_{j=1}^{n} \lambda_j = 1$ (in DEA this is known as variable returns to scale formulation).

It is better that starts with asset selection. The choice of the asset can be random or discrete. The random choice of assets is usually biased and do not promise an optimum portfolio; hence it is more rational to have an objective choice while selecting the assets to be included in the portfolio. We chose the variance-minimizing approach because we feel it is closest to the original mean-variance framework [1]. Based on these overall performance values, the n assets can be compared or fully ranked.

5 Application in stocks

We shall use an example to show how efficiency measure stocks might be constructed using above models. Seventeen common stocks are considered. Let us assume that these stocks have emerged from the security-analysis stage as candidates for portfolios. A uniform holding period was used in estimating return and risk for each stock. Specifically each stock was examined as a possible holding for a one-year period. Under model 6, we need information for each stock: (1) expected return for the holding period, (2) expected risk for the holding period, (3) covariance between the stocks and the stock relative to the market (index). In addition, for the model 6 is needed to estimate the return and variance on the index for the holding period. For each of the seventeen common stocks, the regression coefficients (α_i, β_i) and the residual variance (Q_i) were calculated from historical data. Monthly rates of return on each stock were regressed against the standard and Poor's 500 stock index monthly rates of return for a five-year period. The results are shown in Table1. The most crucial input before beginning to generate efficient portfolios was an estimate of the return and risk on the S&P Index for the holding period (one year ahead). The return on the S&P was estimated via the projection an estimated level of the index one year ahead plus expected dividends on the index. The return was estimated at 11 percent, with a risk (variance) of 26 percent. These two estimates, return and risk on the S&P, serve as the focal point for estimating return and risk for each stocks [13]. Assume that for these stocks, and considering their past information, the amounts of α_i, β_i, Q_i , $E(r_i)$ and $\sigma^2_{r_i}$, are calculated and presented in the Table 1.

Table 1 the regression coefficients (α_i, β_i) and the residual variance Q_i , $E(r_i)$ and $\sigma^2_{r_i}$

	α_i	β_i	$E(r_i)$	Q_i	${\sigma_{r_i}}^2$
A ten Life & Casualty	0.17	0.93	10.4	45.15	67.96
Citicorp	-0.59	1.26	13.3	29.48	71.41
High Voltage Engineering Co.	1.27	1.50	17.8	150.30	209.63
K Mart	-0.28	1.17	12.6	45.42	81.55
McDermott	1.02	10.05	12.5	114.66	143.07
McDonald's Corp	0.85	1.36	15.8	43.29	92.07
Nucor Corporation	2.48	1.37	17.5	132.25	181.83
Pargas	0.47	0.86	9.90	82.08	101.59
Pitney Bowes, Inc.	1.55	1.07	13.30	66.56	96.64
Quaker Oats	-0.16	0.97	10.5	86.40	111.28
Raytheon Co	2.52	1.17	15.4	51.98	88.11
Southwest Forest Products Co.	0.76	0.87	10.3	59.28	79.32
Texaco	-0.28	0.91	9.70	22.27	44.16
Trans world Corp.	1.47	1.73	20.5	166.28	275.13
United States Shoe	1.63	1.09	13.6	94.09	125.47
United States Steel	0.064	0.98	11.4	48.86	74.18
Wisconsin Gas co.	0.28	0.87	9.80	17.64	37.68

Table 2 represents the calculated and compared the results of efficiency of the modified Mean- Variance model (input oriented model) to standard linear DEA model (MV DEA) (model 1) with expected return as output and variance as input. As seen in Table 2, modified mean-variance model scores are as a conservative estimate of the MV scores. In this example all the linear DEA scores are greater than the non-linear modified mean-variance model. The results are obtained by General Algebraic Modeling System (GAMS) software.

Table 2 The results of modified mean-variance model and standard linear DEA model where return is treated as output and variance as input

	MV DEA Model 1	MV efficiency Measure Model7	Ranks
Aten Life & Casualty	0.63395	0.48956	11
Citicorp	0.96903	0.65416	5
High Voltage Engineering Co.	0.81080	0.46610	12
K Mart	0.77124	0.55239	7
McDermott	0.43332	0.33749	15
McDonald's Corp	1/00	0.67032	4
Nucor Corporation	0.87050	0.49794	10
Pargas	0.37977	0.31192	17
Pitney Bowes, Inc.	0.71605	0.52193	8
Quaker Oats	0.39525	0.31694	16
Raytheon Co.	1.00	0.68103	3
Southwest Forest Products Co.	0.53180	0.41972	14
Texaco	0.84455	0.64220	6
Trans world Corp.	1.00	1.00	1
United States Shoe	0.57305	0.43422	13
United States Steel	0.70219	0.51985	9
Wisconsin Gas Co.	1.00	0.73925	2

Table3 The results of output oriented models and combination oriented models in form linear DEA and nonlinear models (models8, 9) respectively

	MV DEA Model 2	MV DEA Model 3	Model 9	Model 8	Ranks
Aten Life & Casualty	0.790	0.84607	0.75672	0.70261	12
Citicorp	0.98187	0.98843	0.85877	0.83424	7
High Voltage Engineering Co.	0.94588	0.95607	0.90635	0.90676	2
K Mart	0.85880	0.90434	0.80015	0.77466	9
McDermott	0.73059	0.715	0.6564	0.67795	15
McDonald's Corp	1	1	0.90499	0.89802	3
Nucor Corporation	0.96660	0.97273	0.90431	0.90442	4
Pargas	0.61703	0.66966	0.60607	0.58114	17
Pitney Bowes, Inc.	0.83556	0.8732	0.7906	0.77914	10
Quaker Oats	0.64444	0.67306	0.61125	0.59972	16
Raytheon Co.	1	1	0.8975	0.88869	6
Southwest Forest Products Co.	0.71409	0.78418	0.70446	0.65737	14
Texaco	0.92209	0.94388	0.84972	0.80137	8
Trans world Corp.	1	1	1	1	1
United States Shoe	0.81644	0.81824	0.75076	0.75533	13
United States Steel	0.82291	0.87508	0.77725	0.73390	11
Wisconsin Gas Co.	1	1	0.8979	0.85979	5

Table 2 shows the results of modified mean-variance model and form standard linear DEA model where return is treated as output and variance as input. Also table 3 shows the results of output oriented model and combination oriented model in form linear DEA and nonlinear models (models 8,9) respectively. As it is mentioned, in these models all the linear DEA scores are greater than the non-linear models too. As it seen in Tables 2, ranks are not the same. We calculated these ranks for input oriented model (modified mean-variance model) and combination oriented model. Some of the best ranks are designated according to investor. We consider 6 of the best ranks. Four of the best ranks become the same, in this example incidentally. Selecting of stocks to be included in portfolio is followed by six of the best ranks in Tables 2, 3. We chose the variance-minimizing approach because we feel it is closest to the original mean-variance framework [1].

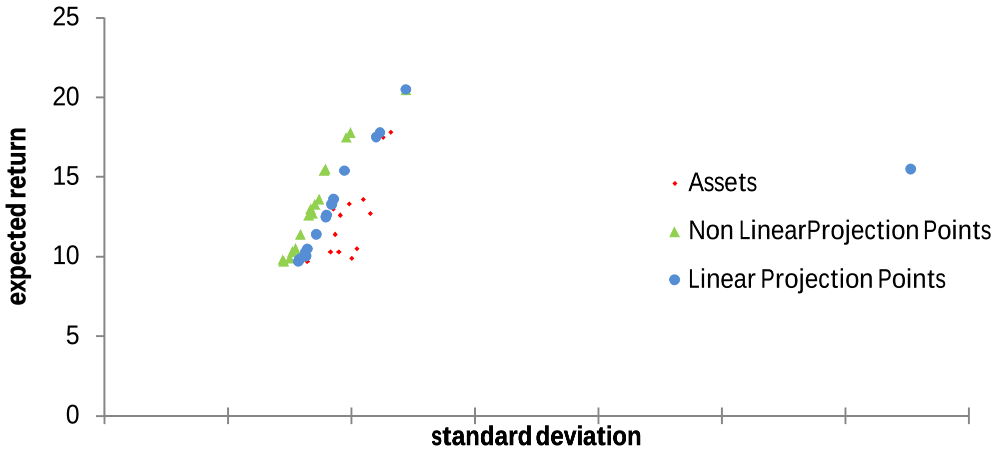

Fig. 2 Proposed method in modified mean-variance model (input oriented model)

Also Fig 2 demonstrates our method in a modified mean-variance model (input oriented model, model 7). In this Figure the horizontal axis represents the standard deviation and the vertical axis the expected return. Here, similar to model 4, instead of estimating the whole efficient frontier, only the projection points of assets are calculated.

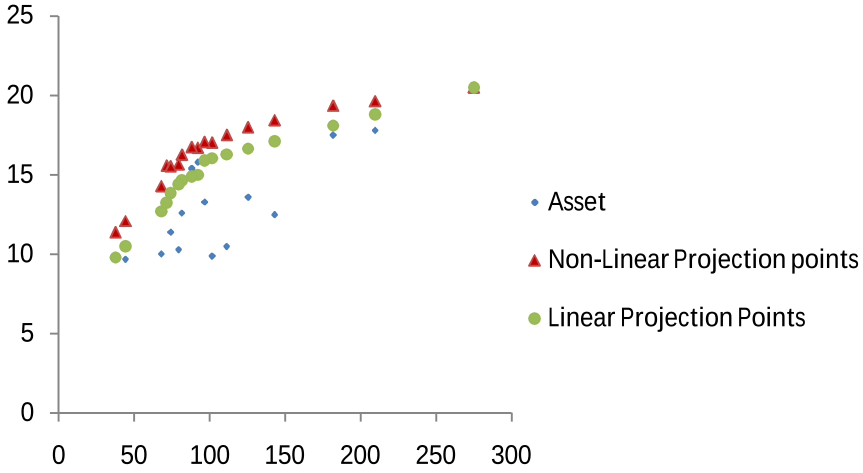

Fig 3 Proposed method in mean-variance model (output oriented model) in form linear and nonlinear models

Fig 3 demonstrates our method in a modified mean-variance model (output oriented nonlinear model, model 8) and linear model (output oriented linear model, model 2). Here, similar to model 6, instead of estimating the whole efficient frontier, only the linear and nonlinear projection points of assets are calculated too.

6 Conclusion

This paper introduced a measure for portfolio performance using modified mean-variance model. Joro and Na [8] had proposed models for evaluated portfolio efficiency in which Data Envelopment Analysis model was employed. In these models was used a non-linear DEA-like framework where the correlation structure among the units was taken into account. In the modified Mean-Variance model, return existing between the two assets is not merely bound to the characteristics of the two stocks but these two are connected together through their relations to the market return. In addition, in this model the total number of parameters that need to be estimated is also eyecatchingly reduced. We have applied model 6, and the linear model DEA with return as output and the variance as the input to 17 stocks. The detailed results are presented in Table 2. In the numerical example is also observed that compared with MV DEA, this model is highly exact in all the units, that is, all the linear DEA scores are greater than the non-linear modified mean-variance model. This means that the DEA frontier is always dominated via the non-linear modified mean-variance frontier. As it can be seen in Fig 2, in modified mean-variance model the projection is defined to be an efficient portfolio having the same return as the asset under evaluation, and deviation standard is reduced. The distance between the asset and its projection distinguishes an efficiency score. In addition, these points can be obtained through multiplying asset's variance with the efficiency measure. In Fig2, we see that Tran world Corp. is a part of efficient frontier. It is better to use a portfolio of stocks Tran world Corp., Wisconsin Gas Co., Raytheon Co., McDonald's, Citicorp and Texaco which are formed with higher efficiency scores.

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Reference

1. Markowitz, H. M., (1952). Portfolio selection. Journal of Finance, 7, 77–91.
2. Arditti, (1975). Skewness and investors_ decisions: A reply. Journal of Financial and Quantitative Analysis, 10, 173–176.
3. Kane, (1982). Skewness preference and portfolio choice. Journal of Financial and Quantitative Analysis, 17, 15–25.
4. Ho, Y. K., Cheung, Y. L., (1991). Behavior of intra-daily stock return on an Asian emerging market—Hong Kong. Applied Economics, 23, 957–966.
5. Sharpe, W. F., (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance, 19, 425–442,
6. Charnes, W., Cooper, W., Rhodes, E., (1978). Measuring Efficiency of Decision Making Units. European Journal of Operational Research, 2, 429–444.
7. Morey, M. R., Morey, R. C., (1999). Mutual fund performance appraisals: A multi-horizon perspective with endogenous benchmarking. Omega, 27, 241–258.
8. Joro, T., Na, P., (2005). Portfolio performance evaluation in a mean-variance-skewness framework. European Journal of Operational Research, 175, 446–461.
9. Briec, W., Kerstens, K., Jokung, O., (2007). Mean-Variance-Skewness Portfolio Performance Gauging: A General Shortage Function and Dual Approach. Management Science, 53, 135–149.
10. Kerstens, K., Mounir, A., Woestyne, I., (2013). Geometric representation of the mean-variance-skewness portfolio frontier based upon the shortage function. European Journal of Operational Research, 1-33.

11. Mhiri, M., Prigent, J., (2010). International portfolio optimization with higher moments. International Journal of Economics and Finance, 2(5), 157-169.
12. Charnes, W., Cooper, W., Lewin, A. Y. Seiford, (1994). Data Envelopment Analysis: Theory, Methodology and Applications. Kluwer Academic Publishers, Boston.
13. Sharpe, W. F., (1985). Investment. Third Edition, Prentice-Hall.