International Journal of Applied Operational Research
Vol. 7, No. 2, pp. 1-12, Spring 2017

Journal homepage: ijorlu.liau.ac.ir

An Interval Assignment Problem with Multiple Attributes: A DEA-Based Approach

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Received: 6 September 2016 ; **Accepted:** 8 February 2017

Abstract One of the basic combinatorial optimization problems is the assignment problem that deals with the assigning of jobs to individuals. In traditional assignment problems, n jobs usually are assigned to n individuals such that the total cost is minimized or the total profit is maximized. However, in numerous real-life applications, various attributes could be considered in assignment problems while data (objective function coefficients) may be uncertain. Therefore, in the current paper, interval factors are taken in assignment problems where multiple attributes, inputs and outputs, are present. Indeed, an approach based on Data Envelopment Analysis (DEA) is proposed to solve the interval assignment problem with multiple attributes. To illustrate, the non-parametric technique (DEA) is utilized to specify the lower and upper bounds of the best final efficiency scores of an assignment plan. Also, the method suggested herein is illustrated and clarified by an application.

Keyword: Assignment Problem, Data Envelopment Analysis, Interval Data.

1 Introduction

The assignment problem is a special case of transportation problems and is one of the basic and fundamental models in operations research, management science, economics, etc. Its standard version concerns minimizing cost or maximizing profit of assigning n jobs to n individuals wherein a deterministic cost or profit from each possible assignment is considered. Nevertheless, in many real-world applications, occasions exist in which for each assignment, various inputs and outputs are involved in an assignment problem while attributes (costs and profits) are not deterministic numbers. For instance, in a problem of assigning n projects to n teams, factors like cost(s), time(s) and profit(s) can be regarded as interval data. Actually, solving an assignment problem while several attributes and interval data are present is an important topic.

In the existing literature, there have been studies of assignment problems. Zarafat Angiz, *et al.* [1] proposed an alternative approach to the assignment problem with non-homogeneous costs using a common set of weights in data envelopment analysis (DEA). DEA is a popular non-parametric technique to evaluate the relative efficiency of decision making units (DMUs) where multiple inputs and outputs are present. Chen and Lu [2] extended the assignment

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problem by considering multiple inputs and outputs where data were precise. They reconstructed the assignment problem as a classical integer linear programming problem. Emrouznejad, *et al.* [3] introduced an alternative formulation for the fuzzy assignment problem. They developed a procedure based on the DEA method to solve the assignment problems with fuzzy costs or fuzzy profits for each possible assignment. Exact and heuristic algorithms for the interval data robust assignment problem have been suggested by Pereira and Averkakh [4]. They presented and compared computationally several exact and heuristic methods, including Benders decomposition, using CPLEX, variable depth neighborhood local search and two hybrid population-based heuristics. Tapkan, *et al.* [5] have proposed an approach for solving fuzzy multiple objective generalized assignment problems via bees algorithm and fuzzy ranking. Recently, Shirdel and Mortezaee [6] introduced a DEA-based approach for the multi-criteria assignment problem with precise data.

In the DEA literature, there are DEA models that determine the efficiencies of DMUs with interval input and output data. Despotis and Smirlis [7] proposed an approach for handling imprecise data in DEA. Afterwards, Wang, *et al.* [8] introduced a pair of interval DEA models to assess interval efficiency scores by using DEA. They expressed their models were more rational and more reliable in comparison with Despotis' models because of utilizing a fixed and unified production frontier (i.e., the same constraint set). Nevertheless, as far as we are aware, there is not any DEA-based technique for solving interval assignment problems where multiple attributes exist.

In this study for solving the assignment problem with various imprecise inputs and outputs we extend interval data envelopment analysis (IDEA) and then utilize formulations of Chen and Lu [2]. Actually, on the one hand, by using IDEA, the lower and upper bounds efficiency scores of an assignment of one individual to a particular job relative to the other jobs are measured, and on the other hand the lower and upper bounds efficiency scores of an assigned job to a particular individual relative to the other individuals are calculated. Then the composite efficiency indexes are defined for upper and lower bounds following Chen and Lu [2]. Generally, IDEA is extended to solve the interval assignment problem with multiple attributes.

The paper unfolds as follows. Section 2 briefly reviews the classical assignment problem and Wang, *et al.*'s models [8] that have been proposed to measure the efficiency scores of DMUs with interval inputs and outputs. The proposed approach to solve the assignment problem with inexact input and output data is presented in section 3. Section 4 describes an application of the introduced approach. Conclusions are provided in section 5.

2 An overview of the assignment problem and interval DEA

In this section we overview some models that are essential for clarifying the proposed approach herein. At first, the classical assignment problem is reviewed. Then, an interval DEA method is explained briefly.

2.1 The assignment problem

Consider c_{ij} as the cost of assigning individual i to job j. The aim of the classical assignment problem consists of finding an assignment of n individuals to n jobs that has the minimum total cost.

The mathematical model of the classical assignment problem is given as follows:

$$Min \quad \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$$

$$s.t.$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,...,n, \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,...,n,$$

$$x_{ij} = 0 \ or \ 1.$$

c_{ij} will be replaced with $[c_{lij}, c_{uij}]$ where the costs are uncertain. Moreover, if the problem has multiple attributes, we can set $\sum_{i=1}^{n}\sum_{j=1}^{n}[c_{lij}^{h}, c_{uij}^{h}]x_{ij}$, $h = 1,...,H$ instead of the mentioned objective functions. Notice that $x_{ij} = 1$ if i th individual is assigned to the j th job and $x_{ij} = 0$ if i th individual is not assigned to the j th job.

2.2 Interval Data Envelopment Analysis (IDEA)

As aforementioned, DEA is a powerful benchmarking technique to measure the relative performance of DMUs in the presence of multiple inputs and outputs. Traditional DEA models do not deal with imprecise data. Nevertheless, there are some approaches to handle imprecise data in the DEA literature (see, e.g. Despotis and Smirlis [7]; Wang, *et al.* [8]; Cooper, *et al.* [9], [10] and [11]). In this subsection Wang, *et al.*'s method [8] is illustrated in brief.

Suppose that there are n DMUs producing the same set of outputs at the consumption of the same set of inputs. Entity j is represented by DMU_j $(j = 1, 2, ..., n)$, whose i th input and r th output are denoted by x_{ij} $(i = 1,...,m)$ and y_{rj} $(r = 1,...,s)$ respectively. Furthermore, the levels of inputs and outputs are not known exactly, and they are only known to lie within the upper and lower bounds represented by the intervals $[x_{ij}^{L}, x_{ij}^{U}]$ and $[y_{ij}^{L}, y_{ij}^{U}]$ where $x_{ij}^{L} > 0$ and $y_{ij}^{L} > 0$. Wang, *et al.* [8] proposed the following models for measuring the upper and lower bounds of the efficiency scores of DMUs.

$$\begin{aligned} Max \quad & \theta_{jo}^{U} = \frac{\sum_{r=1}^{s} u_r y_{rj_0}^{U}}{\sum_{i=1}^{m} v_i x_{ij_0}^{L}} \\ s.t. \quad & \\ & \theta_{j}^{U} = \frac{\sum_{r=1}^{s} u_r y_{rj}^{U}}{\sum_{i=1}^{m} v_i x_{ij}^{L}} \le 1, \quad j = 1,\dots,n, \\ & u_r, v_i \ge \varepsilon \ \ \forall r, i. \end{aligned} \tag{2}$$

and

$$\begin{aligned} Max \quad & \theta_{jo}^{L} = \frac{\sum_{r=1}^{s} u_r y_{rj_0}^{L}}{\sum_{i=1}^{m} v_i x_{ij_0}^{U}} \\ s.t. \quad & \\ & \theta_{j}^{U} = \frac{\sum_{r=1}^{s} u_r y_{rj}^{U}}{\sum_{i=1}^{m} v_i x_{ij}^{L}} \le 1, \quad j = 1,\dots,n, \\ & u_r, v_i \ge \varepsilon \ \ \forall r, i. \end{aligned} \tag{3}$$

Then they transformed the above models to the following linear programming forms by using the Charnes-Cooper [12] transformation:

$$\begin{aligned} Max \quad & \theta_{o}^{U} = \sum_{r=1}^{s} u_r y_{ro}^{U} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{io}^{L} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj}^{U} - \sum_{i=1}^{m} v_i x_{ij}^{L} \le 0, \quad j = 1,\dots,n, \\ & u_r, v_i \ge \varepsilon \ \ \forall r, i. \end{aligned} \tag{4}$$

$$
\begin{aligned}
Max \quad & \theta_o^L = \sum_{r=1}^{s} u_r y_{ro}^L \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{io}^U = 1 \\
& \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0, \qquad j = 1,\dots,n, \\
& u_r, v_i \ge \varepsilon \ \ \forall r, i.
\end{aligned} \tag{5}
$$

So they claimed that a DMU is efficient if its best possible upper bound efficiency obtains 1.

3 The interval assignment problem with multiple attributes

In this section an approach based on DEA is introduced to solve the assignment problem considering multiple interval inputs and outputs for each possible assignment. Indeed, IDEA technique is utilized because of similarity of the mentioned discussion with IDEA technique.

Assume that n individuals and n jobs exist to be assigned, and each possible assignment has m inputs and k outputs. Suppose also the inputs and outputs are denoted by $X_{ij} = (X_{ij1}, X_{ij2}, \dots, X_{ijm})$ and $Y_{ij} = (Y_{ij1}, Y_{ij2}, \dots, Y_{ijk})$, $i, j = 1, \dots, n$, respectively, for the assignment of individual i to job j. Moreover, $X_{ij}^l = (X_{ij1}^l, X_{ij2}^l, \dots, X_{ijm}^l)$ and $X_{ij}^u = (X_{ij1}^u, X_{ij2}^u, \dots, X_{ijm}^u)$ are the lower and upper bounds of inputs, $Y_{ij}^l = (Y_{ij1}^l, Y_{ij2}^l, \dots, Y_{ijk}^l)$ and $Y_{ij}^u = (Y_{ij1}^u, Y_{ij2}^u, \dots, Y_{ijk}^u)$ are the lower and upper bounds of outputs, respectively.

Suppose that there are n DMUs. In other words, consider that all jobs consist of a set of decision making units (DMUs) for each individual. If we consider the j th job as the unit under evaluation, the upper and lower bounds efficiency of the assignment of the i th individual to the j th job can be determined, relative to that of their assignment to the remaining jobs.

Let the efficiency of the DMU_d (d th job) be as follows:

$$
\theta_d = \frac{\sum_{p=1}^{k} u_p y_{idp}}{v_o + \sum_{q=1}^{m} v_q x_{idq}}, \quad d = 1,\dots,n,
$$

Substituting interval inputs and outputs and using the rules of interval data we have:

$$
\theta_d = \frac{\sum_{p=1}^{k} u_p [y_{idp}^l, y_{idp}^u]}{v_o + \sum_{q=1}^{m} v_q [x_{idq}^l, x_{idq}^u]} = \frac{\left[\sum_{p=1}^{k} u_p y_{idp}^l, \sum_{p=1}^{k} u_p y_{idp}^u\right]}{v_o + \left[\sum_{q=1}^{m} v_q x_{idq}^l, \sum_{q=1}^{m} v_q x_{idq}^u\right]} = \left[\frac{\sum_{p=1}^{k} u_p y_{idp}^l}{v_o + \sum_{q=1}^{m} v_q x_{idq}^u}, \frac{\sum_{p=1}^{k} u_p y_{idp}^u}{v_o + \sum_{q=1}^{m} v_q x_{idq}^l}\right],
$$

$$
d = 1,\dots,n
$$

For measuring the upper and lower bounds of the efficiency of the assignment of the i th individual to the j th job, the following pair of fractional programming models is constructed:

$$\begin{aligned} Max \quad & \theta_{ij}^{U} = \frac{\sum_{p=1}^{k} u_p y_{ijp}^{U}}{v_o + \sum_{q=1}^{m} v_q x_{ijq}^{L}} \\ s.t. \quad & \\ & \theta_d^{U} = \frac{\sum_{p=1}^{k} u_p y_{idp}^{U}}{v_o + \sum_{q=1}^{m} v_q x_{idq}^{L}} \le 1, \quad d = 1,\dots,n, \\ & u_p, v_q \ge \varepsilon \quad \forall p,q. \end{aligned} \tag{6}$$

$$\begin{aligned} Max \quad & \theta_{ij}^{L} = \frac{\sum_{p=1}^{k} u_p y_{ijp}^{L}}{v_o + \sum_{q=1}^{m} v_q x_{ijq}^{U}} \\ s.t. \quad & \\ & \theta_d^{U} = \frac{\sum_{p=1}^{k} u_p y_{idp}^{U}}{v_o + \sum_{q=1}^{m} v_q x_{idq}^{L}} \le 1, \quad d = 1,\dots,n, \\ & u_p, v_q \ge \varepsilon \quad \forall p,q. \end{aligned} \tag{7}$$

The above models can be transformed to the linear programming models using the Charnes-Cooper [12] transformation as follows:

$$\begin{aligned} Max \quad & E_{ij}^{u(1)} = \theta_{ij}^{U} = \sum_{p=1}^{k} u_p y_{ijp}^{U} \\ s.t. \quad & \\ & v_o + \sum_{q=1}^{m} v_q x_{ijq}^{L} = 1 \\ & \theta_d^{U} = \sum_{p=1}^{k} u_p y_{idp}^{U} - v_o - \sum_{q=1}^{m} v_q x_{idq}^{L} \le 0, \quad d = 1,\dots,n, \\ & u_p, v_q \ge \varepsilon \quad \forall p,q. \end{aligned} \tag{8}$$

$$
\begin{aligned}
&Max \quad E_{ij}^{l(1)} = \theta_{ij}^{L} = \sum_{p=1}^{k} u_p y_{ijp}^{L} \\
&s.t. \\
&\qquad v_o + \sum_{q=1}^{m} v_q x_{ijq}^{U} = 1 \\
&\qquad \theta_d^U = \sum_{p=1}^{k} u_p y_{idp}^{U} - v_o - \sum_{q=1}^{m} v_q x_{idq}^{L} \le 0, \ \ d = 1,\dots,n, \\
&\qquad u_p, v_q \ge \varepsilon \ \ \forall p,q.
\end{aligned} \tag{9}
$$

Now, it is deemed that all individuals are considered as the set of decision making units (DMUs) for each job and the i th individual is the target unit. For evaluating the lower and upper bounds efficiency of assignment j th job to i th individual, relative to assigning it to the remaining individuals similarly we have:

$$
\begin{aligned}
&Max \quad E_{ij}^{u(2)} = \theta_{ij}^{U} = \sum_{p=1}^{k} u_p y_{ijp}^{U} \\
&s.t. \\
&\qquad v_o + \sum_{q=1}^{m} v_q x_{ijq}^{L} = 1 \\
&\qquad \theta_t^U = \sum_{p=1}^{k} u_p y_{tjp}^{U} - v_o - \sum_{q=1}^{m} v_q x_{tjq}^{L} \le 0, \ \ t = 1,\dots,n, \\
&\qquad u_p, v_q \ge \varepsilon \ \ \forall p,q.
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
&Max \quad E_{ij}^{l(2)} = \theta_{ij}^{L} = \sum_{p=1}^{k} u_p y_{ijp}^{L} \\
&s.t. \\
&\qquad v_o + \sum_{q=1}^{m} v_q x_{ijq}^{U} = 1, \\
&\qquad \theta_t^U = \sum_{p=1}^{k} u_p y_{tjp}^{U} - v_o - \sum_{q=1}^{m} v_q x_{tjq}^{L} \le 0, \ \ t = 1,\dots,n, \\
&\qquad u_p, v_q \ge \varepsilon \ \ \forall p,q.
\end{aligned} \tag{11}
$$

Similar to the Chen and Lu [2] formulation, we reconstruct the upper and lower bounds of a composite efficiency index to incorporate both the two kinds of the lower bound of relative efficiencies (i.e. models 9 and 11), and the two kinds of the upper bound of relative efficiencies (i.e. models 8 and 10). For this purpose, the upper and lower bounds of the composite efficiency index are defined respectively as follows:

$$E^u(i,j)=E_{ij}^{u(1)}\times E_{ij}^{u(2)}, \qquad i,j=1,\ldots,n,$$
$$E^l(i,j)=E_{ij}^{l(1)}\times E_{ij}^{l(2)}, \qquad i,j=1,\ldots,n.$$

Also, we have the following models to determine the lower and upper bounds of efficiency.

$$\begin{aligned} Max \quad & \prod_{i=1}^{n}\prod_{j=1}^{n} E(i,j)^{s_{ij}} \\ s.t. \quad & \\ & \sum_{j=1}^{n} s_{ij}=1, \quad i=1,\ldots,n, \\ & \sum_{i=1}^{n} s_{ij}=1, \quad j=1,\ldots,n, \\ & s_{ij}=0 \; or \; 1. \end{aligned} \tag{12}$$

The above model is transformed to an integer linear programming model by applying the logarithm function.

$$\begin{aligned} Max \quad & \prod_{i=1}^{n}\prod_{j=1}^{n} s_{ij}\times\log(E(i,j)) \\ s.t. \quad & \\ & \sum_{j=1}^{n} s_{ij}=1, \quad i=1,\ldots,n, \\ & \sum_{i=1}^{n} s_{ij}=1, \quad j=1,\ldots,n, \\ & s_{ij}=0 \; or \; 1. \end{aligned} \tag{13}$$

For evaluating the lower and upper bounds efficiency of the interval assignment problem, instead of $E(i,j)$, respectively, $E^l(i,j)$ (i.e., for evaluating the lower bound efficiency) and $E^u(i,j)$ (i.e., for evaluating the upper bound efficiency) are substituted in both models (12) and (13).Thus, the lower and upper bounds of the total maximal efficiency of the interval assignment problem with several attributes are determined, and two sets are obtained each having n variables equivalent to 1.

4 An application

We consider a company with seven crews. The skills of the crews differ from each other because of the difference in the composition of the crews. The company has seven different projects on hand. The manager considers two inputs, time and cost, and one output, profit. The data are all estimated and are thus imprecise and only known within bounds. The times (in days), costs and profits taken by different crews to complete different projects are summarized in Table 1 (as (time, cost, profit)). Aim consists of finding the best assignment of the crews to different projects such that the total time and the total cost taken to complete all the projects are minimized and the total profit is maximized.

For this purpose, models (8) and (9) are solved for evaluating the upper and lower bounds of the efficiency of the assignment of the i th crew to the j th project. The results of the models (8) and (9) are given respectively in Tables 2 and 3. Then models (10) and (11) are evaluated to estimate, respectively, the upper and lower bounds of the efficiency of the assignment of j th project to i th crew. Tables 3 and 4 reveal results.

Afterwards, the lower and upper bounds of the composite efficiency index (i.e. $E^{l}(i,j)$ and $E^{u}(i,j)$) are determined, then $\log(E^{l}(i,j))$ and $\log(E^{u}(i,j))$ are calculated and represented in Tables 5 and 6.

Finally, model (13) is utilized to assess the lower and upper bounds of the total maximal efficiency with considering respectively $\log(E^{l}(i,j))$ and $\log(E^{u}(i,j))$ instead of $\log(E(i,j))$. By solving model (13), the upper bound of the total maximal efficiency obtains 1 with solution variables $x_{14}, x_{25}, x_{37}, x_{42}, x_{56}, x_{63}, x_{71}$ equal to 1 and the lower bound of the total maximal efficiency is 0.00248 with optimal solution $x_{15}, x_{24}, x_{37}, x_{42}, x_{53}, x_{61}, x_{76}$ equal to 1.

Table 1 Data of an assignment problem

Crew ↓	Project→ 1	2	3	4	5	6	7
1	([16, 24],[65.2, 72.3],[82, 88])	([20,31],[29.3,34.2],[75,86])	([23,26],[47.2,57.2],[73.2,82])	([13,15],[62.8,66.1],[73.1,82])	([29,33],[60.3, 63.8],[80.3,85.6])	([23,24],[45.2,48.3],[71.2, 75.8])	([17,20], [40.2, 43.8], [50.2, 54.7])
2	([20, 26],[35.1, 45.2],[78, 83])	([8,13],[14.7,24],[65.2,87])	([33,45],[32.2,40.2],[83,86])	([11,16],[41.8,48.2],[71.3,76.2])	([23,29],[58.2, 64.8],[84.7,92.3])	([24,27],[47.2,50.3],[73.1, 76.7])	([19,23], [45.6, 49.3], [53.7, 56.7])
3	([17, 25],[13.1, 18.6],[42, 63])	([14,22],[22,27],[54.1,61.1])	([19,22],[52,58],[72,78])	([22,31],[52.1,60.2],[82.8,90.7])	([22,26],[47.2, 52.3],[68.3,73.4])	([30,35],[50.8,55.1],[85.3, 88.8])	([14, 18]), [38.2, 41.2], [60.3, 65.4])
4	([24, 34],[64.2, 73.4],[90, 96.2])	([9,15],[12.7,25.3],[90.2,103])	([32,34],[62,68],[82,86])	([6,10],[32.3,40.2],[47.3,53.2])	([33,41],[53.4, 58.2],[73.5,78.4])	([36,40],[70.2,75.8],[90.3, 102])	([19, 24], [47.3, 49.8], [62.8, 64.9])
5	([28,35],[80.6, 88.3],[97.1,108.2])	([21,27],[24,35],[39.8,47.1])	([14,20],[34.1,40],[63,67])	([22,29],[41,45.2],[65.2,71.2])	([41,47],[62.5, 68.4],[92.1,98.7])	([29,32],[46.2,50.3],[79.3, 83.1])	([22, 26], [50.2, 53.4], [70.3, 73.4])
6	([19, 23], [31.2, 36.2], [69, 73])	([12,20], [24, 30], [56.2, 63.2])	([29, 32], [63, 67], [90, 101])	([19,23], [46.2, 52.3], [79.5, 83.2])	([19,23], [42.8, 48.5], [62.3, 68.7])	([30,33], [48.3,52.8], [78.9, 85.2])	([30, 32], [51.3, 54.2], [72.8, 73.7]
7	([26, 32], [70.2, 78.4], [101.2, 106.3])	([10,14], [13.8, 20.2], [82.1, 90.3])	([33,36], [65, 70], [83, 89])	([8,13], [29.8, 35.1], [50.1, 55.6])	([31, 35], [49.6, 52.3], [68.7, 70.3])	([17, 20], [41.5, 44.7], [69.1, 74.2])	([28, 34], [61.4, 62.3], [80.1, 82.3])

Table 2 Results of the model (8)

Crew	Project						
	1	2	3	4	5	6	7
1	1.000000	1.000000	0.942512	1.000000	0.975633	0.872368	1.000000
2	0.927478	1.000000	0.964676	0.865160	1.000000	0.843190	0.624689
3	1.000000	1.000000	0.960394	1.000000	0.841547	0.989040	1.000000
4	0.933359	1.000000	0.834348	1.000000	0.760673	0.989454	0.629816
5	1.000000	1.000000	1.000000	0.937845	1.000000	1.000000	0.886921
6	1.000000	1.000000	1.000000	1.000000	0.849323	0.967505	0.812548
7	1.000000	1.000000	0.848964	1.000000	0.699733	0.762567	0.792808

Table 3 Results of the model (9)

Crew	Project						
	1	2	3	4	5	6	7
1	0.654147	0.747144	0.59416	0.804883	0.531369	0.641222	0.530379
2	0.291578	0.461185	0.348859	0.40977	0.268569	0.248957	0.214693
3	0.469534	0.60252	0.704274	0.598519	0.586222	0.558332	0.7355
4	0.231296	0.525437	0.210737	0.413301	0.156642	0.197257	0.228641
5	0.579701	0.578755	0.801604	0.734157	0.685304	0.802389	0.670029
6	0.723827	0.711392	0.534019	0.656302	0.514309	0.567463	0.510066
7	0.350221	0.649423	0.255322	0.426783	0.217371	0.382614	0.260895

Table 4 Results of the model (10)

Crew	Project						
	1	2	3	4	5	6	7
1	1.000000	0.834721	0.967552	1.000000	0.906454	0.944659	0.818151
2	1.000000	1.000000	1.000000	1.000000	1.000000	0.931304	0.800950
3	1.000000	0.593119	0.995662	1.000000	0.972904	1.000000	1.000000
4	0.955114	1.000000	0.855586	1.000000	0.922856	1.000000	0.908502
5	1.000000	0.457175	1.000000	0.951426	1.000000	1.000000	0.989993
6	0.933950	0.613504	1.000000	1.000000	1.000000	0.994135	0.983339
7	1.000000	0.876681	0.881135	1.000000	0.888424	1.000000	1.000000

Table 5 Results of the model (11)

Crew	Project						
	1	2	3	4	5	6	7
1	0.694004	0.270397	0.634982	0.629804	0.784119	0.822496	0.669445
2	0.708988	0.438237	0.773053	0.792837	0.814321	0.809979	0.636228
3	0.469534	0.247059	0.683853	0.737183	0.813591	0.860673	0.854881
4	0.593562	0.525437	0.583226	0.648749	0.786777	0.664127	0.736573
5	0.595689	0.140211	0.761751	0.773127	0.838862	0.876488	0.768953
6	0.724280	0.245534	0.657868	0.814717	0.800264	0.830838	0.784545
7	0.684863	0.512413	0.568954	0.765019	0.818356	0.863699	0.750983

Table 6 Logarithm of the upper bound of the composite efficiency

$\log E^{u}(i,j)$	1	2	3	4	5	6	7
1	0	-0.07846	-0.04004	0	-0.05337	-0.08403	-0.08717
2	-0.0327	0	-0.01562	-0.0629	0	-0.10498	-0.30073
3	0	-0.22686	-0.01944	0	-0.08685	-0.00479	0
4	-0.0499	0	-0.14639	0	-0.15367	-0.0046	-0.24246
5	0	-0.33992	0	-0.04949	0	0	-0.05648
6	-0.02968	-0.21218	0	0	-0.07093	-0.0169	-0.09745
7	0	-0.05716	-0.12607	0	-0.20645	-0.11772	-0.10083

Table 7 Logarithm of the lower bound of the composite efficiency

$\log E^{l}(i,j)$	1	2	3	4	5	6	7
1	-0.34296	-0.69459	-0.42334	-0.29506	-0.38022	-0.27786	-0.4497
2	-0.68461	-0.69442	-0.56914	-0.48828	-0.66015	-0.6954	-0.86457
3	-0.65667	-0.82723	-0.3173	-0.35535	-0.32153	-0.31827	-0.20151
4	-0.86237	-0.55896	-0.91042	-0.57166	-0.90924	-0.88272	-0.77363
5	-0.46178	-1.09072	-0.21423	-0.24596	-0.24043	-0.15287	-0.28801
6	-0.28046	-0.75778	-0.4543	-0.27189	-0.38554	-0.32655	-0.39776
7	-0.62005	-0.47785	-0.83783	-0.48612	-0.74986	-0.48088	-0.7079

5 Conclusion

In this paper, a DEA-based procedure has been proposed to estimate the maximum efficiency of the interval assignment problem with various attributes (costs and profits and so on). Indeed, the efficiency scores of assignment problems with multiple attributes have been evaluated while interval factors exist. To illustrate, the IDEA technique has been utilized to obtain the lower and upper bound of efficiency of the assignment problem. Then the lower and upper bounds composite efficiencies have been calculated. An application has illustrated the approach and indicated an application of the models.

It seems that incorporating undesirable factors and non-discretionary measures in the assignment problem with multiple attributes will be interesting topics for future research.

Acknowledgement

The second author is grateful for financial support by Lahijan Branch, Islamic Azad University Grant No. 1235, 17-20-5/3507.

References

1. Zarafat Angiz, L.M., Saati, S., Mokhtaran, M., (2003). An alternative approach to assignment problem with non-homogeneous costs using common set of weights in DEA, Far East Journal of Applied Mathematics, 10, 29–39.
2. Chen, L. H., Lu, H. W., (2007). An extended assignment problem considering multiple inputs and outputs, Applied Mathematical Modelling, 31, 2239–2248.

3. Emrouznejad, A., Zarafat Angiz, L. M., Ho, W., (2012). An alternative formulation for the fuzzy assignment problem, Journal of the Operational Research Society, 63, 59–63.
4. Pereira, J., Averbakh, I., (2011). Exact and heuristic algorithms for the interval data robust assignment problem, Computers & Operations Research, 38, 1153-1163.
5. Tapkan, P., Ozbakır, L., Baykasoglu, A., (2013). Solving fuzzy multiple objective generalized assignment problems directly via bees algorithm and fuzzy ranking, Expert Systems with Applications, 40, 892-898.
6. Shirdel G. H., Mortezaee, A., (2015). ADEA-based approach for the multi-criteria assignment problem, Croatian Operational Research Review, 6, 145-154.
7. Despotis, D. K., Smirlis, Y.G., (2002). Data envelopment analysis with imprecise data, European Journal of Operational Research, 140, 24–36.
8. Wang, Y. M., Greatbanks, R. Yanga, J.B., (2005). Interval efficiency assessment using data envelopment analysis, Fuzzy Sets and Systems, 153, 347-370.
9. Cooper, W. W., Park, K.S., Yu, G., (1999). IDEA and AR-IDEA: models for dealing with imprecise data in DEA, Management Science, 45, 597-607.
10. Cooper, W. W., Park, K. S., Yu, G., (2001). An illustrative application of IDEA (imprecise data envelopment analysis) to a Korean mobile telecommunication company, Operations Research, 49, 807-820.
11. Cooper, W. W., Park, K. S., Yu, G., (2001). IDEA (imprecise data envelopment analysis) with CMDs (column maximum decision making units), Journal of the Operational Research Society, 52, 176-181.
12. Charnes, A., Cooper, W. W., (1962). Programming with linear fractional functions, Naval Research Logistics, 9, 181-186.