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A two- phase approach for solving fuzzy flexible linear programming problems with fuzzy objective coefficients

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Abstract In this paper, new concepts of $\bar{\alpha}$-feasibility and $\bar{\alpha}$-efficiency of solutions for fuzzy mathematical programming problems are used, where $\bar{\alpha}$ is a vector of distinct satisfaction degrees. Recently, a special kind of fuzzy mathematical programming entitled Fuzzy Flexible Linear programming (FFLP) is attracted much interest. Using the mentioned concepts, we propose a two-phase approach to solve FFLP. In the first phase, the original FFLP problem converts to a Multi-Objective Linear Programming (MOLP) problem, and then in phase II a weighting technique for the reduced program is introduced. We saw that it was observed that using this concept as a generalization of the parametric approach in linear programming provides a more appropriate tool for modeling real problems and improving the solving process. Thus, by using this two-phase approach, we achieve better utilization of available resources. Further, the solution resulting from these two approaches is always an $\bar{\alpha}$-efficient solution. Finally, an example in the real world is described to express this approach.

Keyword: Fuzzy Linear Programming, Triangular Fuzzy Numbers, Multi-Objective Linear Programming, Feasibility and Efficiency.

1 Introduction

As an important part of mathematical programming, the linear programming is one of the most frequently applied operation research techniques. In the real-world situations, the decision marker might not really want to actually maximize or minimize the objective function. Rather, he or she might want to reach some aspiration levels that might not even be definable crispy. Thus, he or she might want to improve the present cost situation considerably and so on. Also, the role of the constraints can be different from that in the classical one, where the violation of any single constraint by any amount renders the solution infeasible [7]. The decision maker might accept small violations of constraints, but might also attach different (crisp or fuzzy) degrees of importance to violations of different constraints. Fuzzy mathematical programming offers a number of ways to allow for these types of imprecisions. It is necessary to distinguish between flexibility in the constraints and goals and

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uncertainty of the data. Flexibility is modeled by fuzzy sets and may reflect the fact that constraints or goals are linguistically formulated. Their satisfaction is a matter of tolerance and degrees or fuzziness [2]. Ramik and Rimanek [6] also dealt with and LP problem with fuzzy parameters in the constraints. Later, also Verdegay [8] and Chanas [3] have shown an application of parametric programming techniques in the fuzzy LP. In [8], Verdegay proposed a parametric linear programming model with single parameter using $\alpha-$cuts to achieve an equivalent model for the fuzzy linear programming with flexible constraints. Werner's in [2] introduced an interactive multiple objective programming model subject to its constraint is flexible and proposed a special approach for solving multiple objective programming model based on fuzzy set theory. In the mentioned work, the classical model is extended by integration flexible constraints. After that, Delgado and et al. in [4] proposed a general model for fuzzy linear programming problem. In particular, they suggested a resolution method for the mentioned problem. Recently, Attari and Nasseri in [1] introduced a concept of feasibility and efficiency of the solution for the fuzzy mathematical programming problems. The suggested algorithm needs to solve two classical associated linear programing problems to achieve an optimal flexible solution. Now, we are going to improve their method and propose a new approach, which is more flexible in order to overcome the mentioned shortage. The new approach can determine the optimal solution by solving an associated auxiliary problem in just one phase. And hence, our method can obtain the flexible optimal solution with the higher satisfaction degree in comparison with the earlier approach, which was introduced by Attari and Nasseri in [1]. Recently, Ramzannia and Nasseri in [7] Solving Flexible Fuzzy Multi Objective Linear programming problems. The rest of the paper is organized as follows. In Section 2, we demonstrate some preliminaries of fuzzy set theory. We introduce the concepts of $\bar{\alpha}$-feasible and $\bar{\alpha}$-efficient solutions which contain triangular fuzzy numbers in the coefficients objective function in Section 3. An example in the real world of the methods is described in fuzzy linear programming problems in Section 4. We will allocate Section 5 to conclusions.

2 Preliminaries and fundamental definitions

In this section, some basic concepts of fuzzy set theory and concept of feasible solution to the fuzzy programming problem is given. Furthermore, consider a decision maker faced with a linear programming problem in which s/he can endure violation in completing the constraints, that is, s/he allows the constraints to be held as well as possible. For each constraint in the constraints set this assumption can be denoted by $a_i x \preceq \tilde{b}_i$, $i=1,...,m$ and for every, modeled by the use of a membership function

$$\mu_i(x)=\begin{cases} 1, & a_i x \le b_i \\ f_i(a_i x), & b_i \le a_i x \le b_i + p_i \\ 0, & a_i x \ge b_i + p_i \end{cases} \tag{1}$$

$$\begin{aligned} &\max \tilde{z} = \tilde{c}x \\ (2) \quad &s.t. \quad Ax \preceq \tilde{b} \\ &\qquad x \ge 0 \end{aligned}$$

where $f_i(0)$ is strictly decreasing and continuous for $a_i x$, $f_i(b_i)=1$ and $f_i(b_i+p_i)=0$ .This

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membership function expresses that the decision maker tolerates violation in the accomplishment of the constraints i up the value $b_i + d_i$. The function $\mu_i(x)$ gives the degree of satisfaction of the i - th constrains for $x \in \mathbb{R}^n$, but this value is obtained by means of the function f_i which is defined over $\mathbb{R}$. Based on the above assumption the associated FFLP Problem can be presented as:

$$\max \tilde{z} = \tilde{c}x$$
$$s.t.\ a_i x \le b_i + p_i(1-\alpha_i) \tag{3}$$
$$x \ge 0, \alpha_i \ge \alpha_i^D,\ 0 \le \alpha_i \le 1,\ i = 1,\dots,m.$$

We name the above problem as Multi-Parametric Linear Programming (MPLP) problem [1,3,10]. Now, we are going to give the fundamental concept of feasible solution to the fuzzy linear programming problem, which is defined in (3).

Definition 2.2 The α-cut or α-level set of a fuzzy set $\tilde{a}$ is a crisp set defined by $A_\alpha = \{x \epsilon \mathbb{R} \mid \mu_{\tilde{a}}(x) > 0\}$.

Definition 2.3 Let $\bar{\alpha} = (\alpha_1, \cdots, \alpha_m) \epsilon (0.1]^m$ be a vector, and $x_{\tilde{\alpha}} = \{x \epsilon \mathbb{R}^n \mid x \ge 0,\ \mu_i\{g_i(x) \preceq 0\}\} \ge \alpha_i \quad i = 1, \cdots, m$. A vector $x \epsilon X_{\bar{\alpha}}$ is called the $\bar{\alpha}$-feasible solution to problem.

Definition 2.4 Let $\preceq$ be a fuzzy extension of binary relation $\le$ and let $x = (x_1, \cdots, x_n)^T \epsilon \mathbb{R}^n$ be an $\bar{\alpha}$-feasible solution to (3), where $\bar{\alpha} = (\alpha_1, \cdots, \alpha_m)\ \epsilon\ (0.1)^m$ and let $Z(\tilde{c}, x)$ be a fuzzy objective. The vector $x \in \mathbb{R}^n$ is an $\bar{\alpha}-$efficient solution to (3) with the maximization of the objective function, if there is no any $x' \in X_{\bar{\alpha}}$ such that. Similarly, an $\bar{\alpha}-$efficient solution with minimization of the objective function can be defined. Pay attention that any $\bar{\alpha}-$efficient solution to the FFLP problem is an $\bar{\alpha}-$feasible solution to the FFLP problem with some extra properties. In the following theorem, we represent the necessary and sufficient condition for an $\bar{\alpha}-$efficient solution to (3).

Theorem 2.1 Let $\bar{\alpha} = (\alpha_1, \dots, \alpha_m) \in (0,1]^m$ and $x^* = (x_1^*, \dots, x_n^*)^T$, $x_j^* \ge 0,\ j = 1, \dots, n$ be an $\bar{\alpha}-$feasible solution to (3). Then a vector $x^* \in \mathbb{R}^n$ is an $\bar{\alpha}-$efficient solution to Problem (3) with the maximization of the objective function, if and only if x^* is an optimal to the following problem:

$$\max \quad \tilde{z}(x) = z(\tilde{c}, x)$$
$$s.t. \quad a_i x \le b_i + p_i(1-\alpha_i),\ i = 1, \dots, m\ , \tag{4}$$
$$x_j \ge 0,\ \alpha_i \ge \alpha_i^D, 0 \le \alpha_i \le 1,\ j = 1, \dots, n,$$

where p_i is the predefined maximum tolerance.

Proof Let $\bar{\alpha} = (\alpha_1, \dots, \alpha_m) \in [0,1]^m$ and $x^* = (x_1^*, \dots, x_n^*)^T$, $x_j^* \ge 0,\ j = 1, \dots, n$ be an $\bar{\alpha}-$ efficient solution to Problem (7) with the maximization of the objective function. By Definition 2.3 and equation (1), we have $a_i x^* \le b_i + p_i(1-\alpha_i), \alpha_i \ge \alpha_i^D$ for $i = 1, \dots, m$. Therefore, x^* is a feasible solution to (4). Also by Definition 2.3, there is no any $x' \in X_{\bar{\alpha}}$ such that $Z(\tilde{c}, x^*) < Z(\tilde{c}, x')$, it means that x^* is an optimal solution to (4), and in this case x^* is obviously an $\bar{\alpha}-$feasible solution to Problem (3). Thus, by Definition 2.4, the optimality of x^* implies the $\bar{\alpha}-$efficiency of x^*.■

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Proposition 2.1 Let $\bar{\alpha}=(\alpha_1,...,\alpha_m)\in(0,1]^m$, then $X_{\bar{\alpha}}=\bigcap_{i=1}^{m}X_{\alpha_i}^{i}$, where

$$X_{\alpha_i}^{i}=\left\{x\in\mathbb{R}^n \setminus x\geq 0, \alpha_i\geq\alpha_i^D, a_i x\leq b_i+p_i(1-\alpha_i)\right\} \quad (5)$$

For $i=1,...,m$ (namely, X_{α}^{i} is the α-cut of the i - th fuzzy constraint).

Proof For any $\bar{\alpha}=(\alpha_1,...,\alpha_m)\in(0,1]^m$, let $x\in X_{\bar{\alpha}}$, therefore $\alpha_i\geq\alpha_i^D$, $a_i x\leq b_i+p_i(1-\alpha_i)$. Now and from (5) we have $x\in X_{\alpha_i}^{i}, i=1,...,m$, and therefore $x\in\bigcap_{i=1}^{m}X_{\alpha_i}^{i}$. On the other hand, if $x\in\bigcap_{i=1}^{m}X_{\alpha_i}^{i}$, we have $x\in X_{\alpha_i}^{i}$, for all $i=1,...,m$. Therefore $\alpha_i\geq\alpha_i^D$, $a_i x\leq b_i+p_i(1-\alpha_i)$ and hence $x\in X_{\bar{\alpha}}$. This completes the proof. ■

Proposition 2.2 Let $\bar{\alpha}'=(\alpha_1',...,\alpha_m')$ and $\bar{\alpha}''=(\alpha_1'',...,\alpha_m'')$, where $\alpha_i'\leq\alpha_i''$ for all i, then $\bar{\alpha}''-$ feasibility of x implies the $\bar{\alpha}'-$feasibility of it.

Proof The proof is straightforward. ■

For a given $\alpha\in(0,1]$, let $x\in\mathbb{R}^n$ be a usual $\alpha-$feasible solution to (3) (a solution with the same degrees of satisfaction in all constraints). It has the meaning of $a_i x\leq b_i+p_i(1-\alpha_i)$, $\alpha_i\geq\alpha_i^D$ or equivalently $x\in X_{\alpha}^{i}$, for all $i=1,...,m$.

If $\bar{\alpha}=(\alpha,...,\alpha)\in(0,1]^m$, then $x\in X_{\alpha}$ which implies that the $\alpha-$feasibility of (3) can be understood as a special case of the $\bar{\alpha}-$feasibility. Thus, the following result can be obtained.

Remark 2.1 If the problem (3) is not infeasible, then X_{α} is not empty.

Proof The proof is straightforward. ■

3 Flexible Fuzzy Linear Programming

Let us consider the following fuzzy mathematical programming problem,

$$\begin{aligned} & max f(x,\tilde{c}) \\ & s.t. \quad g_i(x)\preceq 0, \quad i=1,\cdots,m \\ & \mathrm{x}\geq 0, \end{aligned} \quad (6)$$

where $x=(x_1,\cdots,x_n)^T$ is an n-dimensional real decision vector $\tilde{c}=(\tilde{c}_1,\tilde{c}_2,...,\tilde{c}_n)$ is an n-dimensional fuzzy vector of fuzzy parameters involved in the objective function f.

where $f(x,\tilde{c})\approx\tilde{c}x$, $g_i(x)\leq 0 \approx a_i^T x\leq b_i$.

Unfortunately, the model (6) is not well-defined because:

i. We cannot maximize the fuzzy quantity $f(x,\tilde{c})$

ii. the constraints $g_i(x)\preceq 0$, $i=1,\cdots,m$, do not produce a crisp feasible set.

Therefore, in order to obviate those mentioned restrictions, we introduce the following problem,

$$\begin{aligned} & max \langle\widetilde{c},x_F\rangle=\textstyle\sum_{j=1}^{n}\tilde{c}_j x_j \\ & s\cdot t. \quad \mu_i\left\{g_i(x)\preceq 0\right\}\geq\alpha, \\ & x\geq 0, \\ & 0<\alpha_i\leq 1, \quad i=1,\cdots,m, \end{aligned} \quad (7)$$

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To motivate for a meaningful choice of membership function for each fuzzy constraints, it is argued that if $g_i(x) \leq 0$, then the i-th constraint is absolutely satisfied, whereas if $g_i(x) \geq p_i$, where p_i the predefined maximum tolerance from zero, as determined by the decision marker, then the i-th constraint is absolutely violated. for $g_i(x) \epsilon (0, p_i)$, the membership function is monotonically decreasing. If this decrease is along a linear function, then it makes sense to choose the membership function of the i-th constraint $(i = 1, 2, \cdots, m)$ as

$$\mu_i\{g_i(x) \leq 0\} = \begin{cases} 1, & g_i(x) \leq 0 \, , \\ 1 - \frac{g_i(x)}{p_i} \, , & 0 \leq g_i(x) \leq p_i \, , \\ 0 \, , & g_i(x) \geq p_i \, , \end{cases}$$

Also, in the objective function $\tilde{c}_j$ is fuzzy number. Here for the rest of the paper, we assume the fuzzy number is triangular. Any triangular fuzzy number $\tilde{c}_j$ can be represented by three real numbers c_0^L, c_1^m and c_0^R. Using this representation, we write $\tilde{c} = \left\langle c_0^L, c_1^m, c_0^R \right\rangle$. Problem (7) can then be rewritten as

$$\begin{aligned} & max \, \langle \tilde{c}. x_F \rangle = \textstyle\sum_{j=1}^{n} \left\langle c_0^L, c_1^m, c_0^R \right\rangle \; x_j \\ & s.t. \qquad \mu_i\{g_i(x) \preceq 0\} \geq \alpha \, . \\ & \qquad\quad x \geq 0 \, , \\ & \qquad\quad 0 < \alpha_i \leq 1 \, , \; i = 1, \cdots, m \end{aligned} \tag{8}$$

where $\tilde{c}_j = \left\langle c_0^L, c_1^m, c_0^R \right\rangle$ is the triangular fuzzy number. Now, we consider the following MOLP problem which is associated to the original fuzzy LP:

$$\begin{aligned} & max \;\; \left(\langle c_0{}^l, x\rangle, \langle c_1{}^m, x\rangle\langle c_0{}^R, x\rangle\right)^T \\ & s.t. \qquad \mu_i\{g_i(x) \preceq 0\} \geq \alpha \;\; , \\ & \qquad\quad x \geq 0 \, , \\ & \qquad\quad 0 < \alpha_i \leq 1 \, , \; i = 1, \cdots, m \end{aligned} \tag{9}$$

where $c_0{}^L = \left(c_{01}{}^L, C_{02}{}^L, \cdots, C_{0n}{}^L\right)^T, c_0{}^R = \left(c_{01}{}^R, C_{02}{}^R, \cdots, C_{0n}{}^R\right)^T, \; c_1{}^m = \left(c_{11}{}^m, C_{12}{}^m, \cdots, C_{1n}{}^m\right)^T \epsilon \, \mathbb{R}^{\,n}$.

Also, from Theorem 4.3 in [5] we consider the following weighting LP problem defined by:

$$\begin{aligned} & max \, \langle w. \tilde{c}. x\rangle = w_0{}^L\langle c_0{}^L, x\rangle + w_1{}^m\langle c_1{}^m, x\rangle + w_0{}^R\langle c_0{}^R, x\rangle \\ & s.t. \qquad \mu_i\{g_i(x) \preceq 0\} \geq \alpha \;\; , \quad x \geq 0 \, , \; i = 1, \cdots, m \; , \end{aligned} \tag{10}$$

where $c_i{}^L = \left(c_{01}{}^L, C_{02}{}^L, \cdots, C_{0n}{}^L\right)^T, \; c_i{}^m = \left(c_{11}{}^m, C_{12}{}^m, ..., C_{1n}{}^m\right)^T, \; c_i{}^R = \left(c_{01}{}^R, C_{02}{}^R, \cdots, C_{0n}{}^R\right)^T \epsilon \, \mathbb{R}^{\,n}$ and $w = \left(w_0{}^L, w_1{}^m, w_0{}^R\right) \geq 0$. Also, using Theorem 2.1, we have the following problem:

$$\begin{aligned} & max \, \langle w, \widetilde{c}, x\rangle = w_0{}^L\langle c_0{}^L, x\rangle + w_1{}^m\langle c_1{}^m, x\rangle + w_0{}^R\langle c_0{}^R, x\rangle \\ & s \cdot t. \qquad g_i(x) \leq (1 - \alpha_i)p_i \; , \qquad i = 1, \cdots, m, \\ & \qquad\quad x_j \geq 0, \quad j = 1, \cdots, n. \end{aligned} \tag{11}$$

In order to find a maximum efficient solution, i.e., an $\bar{\alpha}'$-efficient solution with $\bar{\alpha}' \geq \alpha, i = 1. \cdots. m$, we perform the following two-phase approach. In the two-phase approach, Eq

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(11) is solved in phase 1, while in phase 2, a solution is obtained which has higher satisfaction degrees than the previous solution. Thus, by using this two-phase approach, we achieve a better utilization of available resources. Further the solution resulting by these two approaches is always an $\bar{\alpha}$-efficient solution. Let us call the problem (11) as the phase 1 problem.

Let $\bar{\alpha}^0 = (\alpha_1{}^0, \cdots, \alpha_m{}^0)$, and $\left(x^*, f_w\left(x^*, c\right)\right)$ be the optimal solution of phase 1 with $\bar{\alpha}^0$ degree of efficiency. Set $\alpha_i{}^* = \mu_i\{g_i(x^*) \preceq 0\} \geq \alpha_i{}^0,\ i = 1, \cdots, m.$ in the phase 2, we solve the following problem,

$$\begin{aligned} &\max \quad \textstyle\sum_{i=1}^{m} \alpha_i \\ &s.t.\ f_w\left(x, c\right) \geq f_w\left(x^*, c\right) \\ &g_i(x) \leq (1-\alpha_i) p_{i.} \\ &\alpha_i{}^* \leq \alpha_i \leq 1, \quad i = 1, \cdots, m \\ &x \geq 0. \end{aligned} \tag{12}$$

In the below, we give an illustrative example.

4 An example in the real world

The factory produces three types of oils with three different combinations. The ratios of this composition, together with the total raw material available and the income derived from each kilo of oil, are shown in Table 1. The goal of the factory is to know how much each oil should be produced to maximize the revenue generated by its sale.

Table 1 Data

	Raw materials of the first type	Raw materials of the second type	Raw materials of the third type	Revenue per kg of oil (toman)
First type oil	25	50	25	350
Second type oil	40	30	30	300
Third type oil	40	40	20	320
Total raw material in kg of any type	1600	2200	1300	

In addition, the available materials from the raw materials required by this plant, according to the expert's opinion, will be added to the following amounts (amount of tolerance):

Table 2 Materials

Raw materials of the first type	Raw materials of the second type	Raw materials of the third type	
300	500	170	Amount of tolerance of Total raw material in kg of any type

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Given the assumptions of the problem, the coefficients of the objective function (revenues) are the following triangular fuzzy numbers: $c_1 = (330, 350, 380)$, $c_2 = (290, 300, 320)$ and $c_3 = (305, 320, 325)$

Solving: We first model the problem.
x_1: the amount of kilogram produced from the first type oil
x_2: the amount of kilogram produced from the second type oil
x_3: the amount of kilogram produced from the third type oil

$$\begin{aligned} &\max z = 350x_1 + 300x_2 + 320x_3 \\ &s.t. \quad 25x_1 + 40x_2 + 40x_3 \preceq 1600, \\ &\quad 50x_1 + 30x_2 + 40x_3 \preceq 2200, \\ &\quad 25x_1 + 30x_2 + 20x_3 \preceq 1300, \\ &\quad x_1, x_2, x_3 \geq 0. \end{aligned} \tag{13}$$

We consider the following membership function:

$$\mu_i(A_i x, b_i) = \begin{cases} 1, & A_i x < b_i \\ 1 - (A_i x - b_i)/p_i, & b_i \leq A_i x \leq b_i + p_i, \quad i = 1,2,3 \\ 0, & A_i x > b_i + p_i \end{cases}$$

Where $p_1 = 300$, $p_2 = 500$ and $p_3 = 170$ are predefined maximum tolerance from b_i, $i = 1.2.3$.

Now, by considering the weights as $w_1 = 1/4. w_2$=1/2 and $w_3 = 1/4$ for the objective function. we can rewrite (13) as follows:

First stage problem:

$$\begin{aligned} &\max z = 352.5x_1 + 302.5x_2 + 317.5x_3 \\ &s.t. \quad 25x_1 + 40x_2 + 40x_3 \leq 1600 + 300(1-\alpha_1), \\ &\quad 50x_1 + 30x_2 + 40x_3 \leq 2200 + 500(1-\alpha_2), \\ &\quad 25x_1 + 30x_2 + 20x_3 \leq 1300 + 170(1-\alpha_3), \\ &\quad 0 < \alpha_i \leq 1, i = 1,\ldots,m, \\ &\quad x_j \geq 0, j = 1,2,3. \end{aligned} \tag{14}$$

Some $\bar{\alpha}$-efficient solution with satisfaction degrees, which decision maker's desire can be found in the following table (3):

Table 3 Some typical α -feasibility solution

a	***b***	***c***	***d***	***e***	***f***
$\bar{\alpha}$	**(0.5.0.5,0.5)**	(0.5,0.5,0.3)	(0.7,0.5,0.5)	(0.5,0.7,0.5)	(0.5,0.5,0.7)
$\mathbf{c^T x}$	18701.8	18807.4	18595.2	18240.9	18135.2
x_1	**32.27**	33.17	34.67	29.60	28.69
x_2	**10.67**	12.93	10.67	14.00	11.73

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x_3	**12.92**	10.08	9.92	11.25	14.08
α_1	**0.5**	0.5	0.7	0.5	0.5
α_2	**0.5**	0.5	0.5	0.7	0.7
α_3	**0.5**	0.3	0.5	0.5	0.7

If all of the satisfaction degrees are equal, then the $\bar{\alpha}$-feasibility and $\bar{\alpha}$-efficiency reduce to classic α- feasibility and α-optimality (see table 3, column b). Let x^*be $(0.7, 0.5, 0.5)$ -efficient solution with $c^T x^* = 18595.2$ an optimal objective value (see table3, column d). In Phase II, we need to solve the following linear programming,

Second stage problem:

$$\begin{aligned} \max \quad & \alpha_1 + \alpha_2 + \alpha_3 \\ s.t. \quad & 352.5x_1 + 302.5x_2 + 317.5x_3 \geq 18595.2, \\ & 25x_1 + 40x_2 + 40x_3 \leq 1600 + 300(1-\alpha_1), \\ & 50x_1 + 30x_2 + 40x_3 \leq 2200 + 500(1-\alpha_2), \\ & 25x_1 + 30x_2 + 20x_3 \leq 1300 + 170(1-\alpha_3), \\ & 0.7 \leq \alpha_1 \leq 1,\ 0.5 \leq \alpha_2 \leq 1,\ 0.5 \leq \alpha_3 \leq 1, \\ & x_j \geq 0,\ j = 1,2,3. \end{aligned} \tag{15}$$

Table 4 Comparison of the solutions of the first and second stage problems

	Optimal solution Phase I of column d	Optimal solution phase II
$\bar{\alpha}$	$(0.7, .05, .05)$	$(0.7, .0.5, .0.5)$
$c^T x$	18595.2	18595.2
x_1	34.67	34.66
x_2	10.67	10.66
x_3	9.92	9.91
α_1	0.7	0.7
α_2	0.5	1
α_3	0.5	1

An optimal solution to the above problem is $x^{**} = (34.66, 10.66, 9.91)$. Also $c^T x^{**} = c^T x^* = 18595.2$. We have $\mu_1(A_1 x^{**}, b_1) = 1$, $\mu_2(A_2 x^{**}, b_2) = \mu_3(A_3 x^{**}, b_3) = 0.5$. Thus, using the two-phase approach, we can get an optimal solution x^{**}which not only achieves the optimal objective value, but also gives a higher membership value in μ_1.

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5. Conclusion

In this study, a two- phase approach for solving fuzzy flexible linear programming as one of the comfortable models which is formulated in some real situations proposed. The method based on extending $\bar{\alpha}-$feasibility solution to $\bar{\alpha}-$efficiency solution is established. In the illustrative example, we saw that the defined method in Phase II suitably can improve the satisfaction degree of the solution based on the new proposed concept. In particular, unlike of the existing approach, the proposed method without using any ranking function. Hence, we saw that in the solving process it is necessary to apply a kind of multi-objective programming techniques. Here, we used the weighted method for this aim.

We saw that it was observed that using this concept as a generalization of the parametric approach in linear programming provides a more appropriate tool for modeling real problems and improving the solving process. This approach will be useful in obtaining flexible responses with a degree of satisfaction determined by the decision maker for fuzzy mathematical programming. We emphasize that this approach can be extended for the other generalized form of fuzzy flexible linear programming models.

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