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# Sustainability evaluation of supply chain by inverse data envelopment analysis model: a case study of the power industry

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**Abstract** The Pollutant emissions control and management of greenhouse gas play a fundamental role in wasted energy mitigation in the energy and power plant sectors and transmission and distribution networks. The majority of the energy consumption mostly derived from Fossil fuels. This results in extensive pollution, which endangers human health and other organisms while also reduces the economic return on industrial activities. The purpose of this study is to evaluate the sustainability of the electricity supply chain by the inverse output-oriented data envelopment analysis (DEA) model. The inverse output-oriented (DEA) model provides optimal amount of economic return order to desirable products and undesirable outputs while other factors are kept unchanged. An empirical conclusion yielded on the model's performance in the electrical supply chains and their divisions. According to the results of the inverse output-oriented DEA model, a supply chain' first power plan and first transmission line require an influential investment in flare gas inhibition and economic return enhancement. Also, the distribution lines confront fluctuations of power loss hence; it is recommended that specialized workforce employed to avoid power loss.

**Keyword:** Optimal Allocation, Environmental Efficiency, Inverse DEA, Optimal Resources, Economical Return.

## 1 Introduction

The daily increase in the consumption of fossil fuels such as oil, natural gas, and coal has resulted to an increase in the concentration of $CO_2$ and a shift in the energy balance of the Earth's atmosphere. Furthermore, we must reduce GHG emissions by improving system efficiency. The current paper contributes to this line of research by introducing inverse data envelopment analysis (DEA) model when two categories of inputs, desirable and undesirable outputs and dual-role factors are present for electricity supply chain. In this study, inputs are divided into two categories natural and managerial disposability. In managerial disposability, a firm increases a directional vector of inputs to decrease a directional vector of undesirable outputs by utilizing technology innovation on undesirable outputs or managerial effort such as using high quality fuel with less CO2 emissions. The proposed inverse output-oriented (DEA) model determines the value optimal of desirable output or produced energy of supply chain divisions to optimal changes of undesirable outputs while resources and applied investment

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capacities of supply chain divisions and dual-role factors as well as efficiency score of under evaluation supply chain remained unchanged.

In this case, supply chain management should be able to identify the divisions of the supply chain in which the optimal value of desirable outputs leads to changes in undesirable outputs. On the other hand, the proposed inverse output-oriented DEA model identifies a fundamental policy for harmful emissions prevention and waste energy inhibition. A relevant but different question is how can manage greenhouse gases and pollutant emission of supply chain divisions to optimal produced energy while other production factors and output-oriented efficiency remained unchanged. The changes in GHGs and the fluctuation of power losses based on optimal economic return or optimal production of energy have a fundamental role in the sustainability and effectiveness of the electricity supply chain. It is critical to have data on optimal desirable outputs impacts on harmful emissions like flare gas in energy sections, pollutant emissions and GHGs in power plant sectors, and waste energy in transmission and distribution networks.

DEA is a profitable method for operationalizing new ideas in sustainability assessment of decision-making units. DEA was developed through the CCR model by Charnes et al. [1] to evaluate the relative efficiency of decision-making units (DMUs). The proposed model was then expanded by Banker et al. [2] for measuring the variable return scale (VRS).The initial mention of inverse models was found in Zhang et al. [3]. They suggested a model that could solve a DMU's input increments and provide output increments under a constant return scale (CRS).

The remainder of this paper is as follows: Section 2 is an overview of the literature on how DEA has been used to investigate the inverse DEA model. After, this, another section devoted to introducing the inverse output-oriented DEA model to achieve optimal of desirable outputs to emission management of supply chain divisions in the presence of two categories of inputs, desirable and undesirable products, and dual-role factors, and the two sets of intermediate measures. In section 4, a case study presented to demonstrate the applicability of the proposed method in Iran's power industry. Finally, Section 5 presents conclusions.

## 2 Literature Review

The subsequent, subsection provides a summary of several studies on inverse DEA model, supply chain sustainability, environmental, and operational assessment.

### 2.1 Inverse DEA Model

The first inverse DEA model was developed by Wei et al. [4]. They raised the question of how much more output or input must be produced if input or output for a specific DMU within a group increased and if it assumed that the DMU maintains current efficiency level in comparison to other units how much more output or input must be produced by a particular unit among a group of DMUs. Yan et al. [5] extended the inverse model for resource reallocation and input/output production estimation. An inverse DEA model of the additive model was proposed by Amin and Emrouznejad. [6] by accounting for inverse linear programming. Also, an inverse DEA model with fuzzy data for output estimation was presented by Rad et al. [7].

An inverse Banker–Charnes-Cooper (BCC) model presented by Lertworasirikul et al. [8] to preserve relative efficiency values with the VRS, which could deal with data positive. In their



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model, resource allocation computed while the efficiency scores remained unchanged. Jahanshaloo et al. [9] extended the inverse DEA model proposed by Yan et al. [5]. Also, Jahanshaloo et al. [10] developed a time-based inverse DEA model by assuming temporal dependence of the dataset.

A generalized DEA model for input/output estimation was presented by Hadi-Vencheh et al. [11]. Han et al. [12] presented an inverse DEA model with stochastic factors. Zhang and Cui [13] developed an extension and integration of the inverse DEA method.

Hassenzadeh et al. [14] proposed an inverse DEA model for the sustainability assessment of countries via inverse DEA environmental and operational assessment. Ghiyasi et al. [15] focused on this subject and tried to formulate some other relevant inverse DEA models from different viewpoints. Recently, a similar model was formulated by Nasrabadi et al. [16] based on the additive model. The primary aim of inverse DEA models is to estimate the level of inputs (outputs) required for the unit under evaluation while keep its efficiency score unchanged, assuming that its level of outputs (inputs) is changed.

Gerami et al. [17] proposed a generalized inverse DEA model for a firm restructuring based on value efficiency. A review of inverse DEA was proposed by Emrouznejad et al. [18].

### 2.2 Environmental and Operational Assessment

To incorporate the two concepts of natural and managerial disposability into environmental assessment in technology and manage harmful substances' prevention and negative impacts on productivity, Glover and Sueyoshi [19] discussed the history of DEA, beginning with the contributions of Banker [2], who proposed DEA in the nineteenth century. The concept of natural and managerial disposability was then used as a conceptual foundation for previous research efforts see [20, 21].

Sueyoshi et al. [22] proposed a stage-DEA model for the operational and environmental assessment of Japan's industrial sectors. They calculated a unified efficiency score under the natural and managerial disposability of the DMU by resource utilization and technology innovation. Pouran Manjily et al. [23] proposed a technology transfer strategy for field oil development of Iran.

### 2.3 Sustainability of the Supply Chains

Tone and Tustusi [24] proposed a slacks-based measure network DEA model called network SBM. Tavana et al. [25] extended the EBM model proposed by Tone and Tustusi [24] and suggested a network epsilon best measure (NEBM).

Tajbakhsh et al. [26] proposed a multi-stage DEA model to evaluate the sustainability of a chain of business partners. They assessed supply chain sustainability in the banking and beverage sectors.

Khodakerami et al. [27] proposed a new two-stage DEA model of supply chain sustainability in resin-producing companies. The authors considered performance evaluation of some real-life imprecise and uncertain problems since they needed to be solved by fuzzy sets in the DEA model.

Babazadeh et al. [28] used DEA to evaluate the social and climate criteria in cultivation area. They evaluated the strategic design of the biodiesel supply chain network through the integration of DEA and mathematical programming. Besides, the authors believed there had


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been a gap in previous studies (not focusing on climatic and social criteria). They proposed a new DEA model related to biodiesel supply chain planning.

Nikfarjam et al. [29] proposed a new DEA method for evaluating supply chains with integrated approaches. They showed that the proposed model could be used for evaluating performance to identify the benchmarking units for the inefficient supply chain.

Farzipoor Saen [30] proposed a model for selecting third-party reverse logistics providers in the presence of multiple dual-role factors.

Pouralizadeh et al. [31] proposed a new DEA-based model to evaluate the sustainability of an electricity supply chain in the presence of undesirable outputs. They planned a supply chain with five stages and fifteen divisions from different districts of Iran. Also, the weak disposability assumption adopted for activity level control in production activities. The proposed model enabled the authors to determine the type and size of inputs to control the undesirable outputs.

Pouralizadeh [32] presented a new DEA model for sustainability improvement of the electricity supply chain in the presence of dual-role factors and undesirable outputs. This model identified whether increasing inputs under managerial disposability to new technology innovation could reduce undesirable production in the electricity supply chain divisions or whether the increased inputs for investment were ineffective in decreasing the number of undesirable outputs.

Pouralizadeh [33] suggested two models for managing pollution emissions and reducing resource waste for the sustainability evaluation of the electrical supply chain.

Mirhedayrian et al. [34] presented a DEA-based model in the presence of undesirable outputs, dual-role factors, and fuzzy data in a supply chain. They proposed a method to improve environmental performance through green supply chain management and incorporated dual-role factors and undesirable output into the NSBM model proposed by Tone and Tsutsui [24].

In summary, none of the abovementioned references for sustainability assessment of the supply chain considered the inverse DEA model based on the dual-role factors in the presence of undesirable outputs.

## 2.4 Fundamental Concepts

In this section, fundamental concepts for the approach to calculating the unified efficiency (operational and environmental) of supply chain divisions reported.

Let us suppose $X_j=(x_{1j},x_{2j},...,x_{mj})^T>0,\ Y_j=(y_{1j},y_{2j},...,y_{sj})^T>0,\ B_j=(b_1,b_2,...,b_{hj})^T>0$ denote column vectors of inputs and desirable and undesirable outputs in the j$^{th}$ DMU. The unified efficiency (operational and environmental) of the k$^{th}$ DMU under natural and managerial disposability of inputs is calculated by a radial model under VRS as follows:



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$$
\begin{aligned}
&\theta = \min \xi \\
&\sum_{j=1}^{n} \bar{x}_{ij}\lambda_j && \le \xi \bar{x}_{ik} && i = 1,\dots,m^- \\
&\sum_{j=1}^{n} \tilde{x}_{qj}\lambda_j && \ge \tilde{x}_{qk} && q = 1,\dots,m^+ \\
&\sum_{j=1}^{n} y_{rj}\lambda_j && \ge y_{rk} && r = 1,\dots,s \\
&\sum_{j=1}^{n} b_{fj}\lambda_j && \le b_{fk} && f = 1,\dots,h \\
&\sum_{j=1}^{n} \lambda_j && = 1 && j = 1,\dots,n \\
&\xi URS,\ i = 1,\dots,m,\ f = 1,\dots,h
\end{aligned}
\tag{1}
$$

In this model, the number of original $m$ inputs is separated into two categories $m^-$ (under natural disposability) and $m^+$ (under managerial disposability), respectively. Also, $\bar{X}_j^h = (\bar{x}_{1j}, \bar{x}_{2j}, \dots, \bar{x}_{m^- j})^T > 0$ , $\tilde{X}_j^h = (\tilde{x}_{1j}, \tilde{x}_{2j}, \dots, \tilde{x}_{m^+ j})^T > 0$ indicate column vectors of the original $m$ inputs from the j[th] DMU are divided into two categories $m^-$ and $m^+$, as $M = m^- + m^+$.

### 2.4.1 A inverse DEA output-oriented model

In the inverse output-oriented model, we wish to increase efficiency by increasing outputs. Therefore, objective function maximized and the objective of inverse output-oriented model is to determine investments while efficiency score is unchanged. Assume that output is changed from $y_{rk}$ to $y_{rk} + \Delta y_{rk}$ while the efficiency of $DMU_k$ is unchanged and $\Delta y_{rk}$ is optimal variations of outputs for $DMU_k$ and $\Delta$ can be positive or negative. The proposed the output-oriented inverse DEA model determines the maximal variation of desirable outputs based on the variation of undesirable outputs as other production factors and efficiency keep constant under VRS.The value maximization of desirable output variation while other production factors remained unchanged is computed as follows:

$$
\begin{aligned}
&\phi = Max\ \Delta y_{rk} \\
&\sum_{j=1}^{n} \bar{x}_{ij}\lambda_j && \le \bar{x}_{ik} && i = 1,\dots,m^- \\
&\sum_{j=1}^{n} \tilde{x}_{qj}\lambda_j && \ge \tilde{x}_{qk} && q = 1,\dots,m^+ \\
&\sum_{j=1}^{n} y_{rj}\lambda_j && \ge \varphi\,(y_{rk} + \Delta y_{rk}) && r = 1,\dots,s \\
&\sum_{j=1}^{n} b_{fj}\lambda_j && \le b_{fk} && f = 1,\dots,h \\
&\sum_{j=1}^{n} \lambda_j && = 1 && j = 1,\dots,n \\
&\xi URS,\ i = 1,\dots,m,\ f = 1,\dots,h
\end{aligned}
\tag{2}
$$



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## 3 Modeling of Input-oriented Efficiency of Supply Chain Divisions

In this section, we propose a DEA model for the sustainability assessment of supply chains. We suppose a supply chain contains an arbitrary number of suppliers, manufacturers, transmitters, distributors, and customers.

Suppose a supply chain (or DMU) concluded with five stages: supplier, manufacturer, transmitter, distributor, and customer. We treat each supply chain as a DMU. Let us consider $h_s, h_m, h_t, h_d, h_c$ the number of divisions in the supplier, manufacturer, transmitter, distributor, and customer. These entities collaborate on power production and management in economic business. Model (1) can be further developed as a network model by incorporating the two categories of intermediate measures and dual-role factors for each supply chain division into an efficiency assessment of the overall supply chain. In this study, we considered the different weights for partners of a particular stage of the network supply chain as $W_h, (h=1,\dots,H)$ weights for H divisions that were defined by decision makers in production activities. The following is a summary of the n^th^ supply chain's production factors.

$\bar{X}_j^h = (\bar{x}_{1j}^h, \bar{x}_{2j}^h, \dots, \bar{x}_{m^-j}^h)^T > 0$: A column vector of $m^-$ inputs under natural disposability from the h$^{th}$ division in the j$^{th}$ supply chain $h=1,\dots,H$ , $j=1,\dots,n$ .

$\tilde{X}_j^h = (\tilde{x}_{1j}^h, \tilde{x}_{2j}^h, \dots, \tilde{x}_{m^+j}^h)^T > 0$: A column vector of $m^+$ inputs under managerial disposability from the h$^{th}$ division in the j$^{th}$ supply chain $h=1,\dots,H$ , $j=1,\dots,n$ .

$Y_j^h = (y_{1j}^h, y_{2j}^h, \dots, y_{sj}^h)^T > 0$: A column vector of $s$ desirable outputs from the h$^{th}$ division in the j$^{th}$ supply chain $h=1,\dots,H$, $j=1,\dots,n$ .

$B_j^h = (b_{ij}^h, b_{2j}^h, \dots, b_{Fj}^h)^T > 0$: A column vector of $F$ undesirable outputs from the h$^{th}$ division in the jth supply chain $h=1,\dots,H$, $j=1,\dots,n$ .

$W_j^h = (w_{1j}^h, w_{2j}^h, \dots, w_{Ej}^h)^T > 0$: A column vector of $E$ dual-role factors from the h$^{th}$ division in the j$^{th}$ supply chain $h=1,\dots,H$, $j=1,\dots,n$ .

$V_j^{(h,h')} = (v_{1j}^{(h,h')}, v_{2j}^{(h,h')}, \dots, v_{Pj}^{(h,h')})^T > 0$: A column vector of $P$ material flows or intermediate measures sent from the division $h$ to the division $h'$ in the j$^{th}$ supply chain $h=1,\dots,H$, $j=1,\dots,n$

$Z_j^{(h',h)} = (z_{1j}^{(h',h)}, z_{2j}^{(h',h)}, \dots, z_{Aj}^{(h',h)})^T > 0$: A column vector of A inverse intermediate measures sent from the division $h'$ to the division $h$ in the j$^{th}$ supply chain, $h=1,\dots,H$, $j=1,\dots,n$ .

$s_{pj}^{(h,h')}$: The slack variables of the p$^{th}$ intermediate measure from the division $h$ to division $h'$ in the jth supply chain (p = 1,…,P), (j = 1,…,n).

$s_{aj}^{-(h',h)} \geq 0$: The input slack variables of the a$^{th}$ inverse intermediate measure from the division $h'$ to the division $h$ in the j$^{th}$ supply chain (a = 1,…,A), (j = 1,…,n).

$s_{aj}^{+(h',h)} \geq 0$: The output slack variables of the a$^{th}$ intermediate measure or inverse flow from the division. $h'$ to the division $h$ in the j$^{th}$ supply chain (a = 1,…,A), (j = 1,…,n).

$\Lambda^h = (\lambda_1^h, \lambda_2^h, \dots, \lambda_n^h)^T$ : An unknown column vector.

$\varphi_r^h$ : Efficiency score of rth output from the h$^{th}$ division



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Consequently, a weighted average of the input efficiency scores of the supply chain divisions in production processes used to calculate the overall supply chain's efficiency, as shown by the model (2).

$$
\begin{array}{lll}
\theta = Min\ \ \omega_h \left(\frac{1}{m_h^-}\sum_{i=1}^{m^-} \xi_i^h\right) & & \\
\sum_{j=1}^{n} \bar{x}_{ij}^{h}\lambda_j^h & \le \xi_i^h\, \bar{x}_{ik}^{h} & i=1,\dots,m_h^-,\ h=1,\dots,H \\
\sum_{j=1}^{n} \tilde{x}_{qj}^{h}\lambda_j^h & \ge \tilde{x}_{qk}^{h} & q=1,\dots,m_h^+,\ h=1,\dots,H \\
\sum_{j=1}^{n} y_{rj}^{h}\lambda_j^h & \ge y_{rk}^{h} & r=1,\dots,S_h\ ,h=1,\dots,H \\
\sum_{j=1}^{n} b_{fj}\lambda_j^h & \le b_{fk}^{h} & f=1,\dots,F_h\ ,h=1,\dots,H \\
\sum_{j=1}^{n} w_{ej}^{h}\lambda_j^h & = w_{ek}^{h} & e=1,\dots,E_h\ ,h=1,\dots,H \\
\sum_{j=1}^{n} \lambda_j^h v_{pj}^{(h,h')} + s_p^{(h,h')} & = \sum_{j=1}^{n} \lambda_j^{h'} v_{pj}^{(h,h')} & h=1,\dots,h_s\ ,p=1,\dots,P_s\ ,h'=1,\dots,h_m \\
\sum_{j=1}^{n} \lambda_j^h v_{pj}^{(h,h')} + s_p^{(h,h')} & = \sum_{j=1}^{J} \lambda_j^{h'} v_{pj}^{(h,h')} & h=1,\dots,h_m\ ,p=1,\dots,P_m\ ,h'=1,\dots,h_t \\
\sum_{j=1}^{n} \lambda_j^h v_{pj}^{(h,h')} + s_p^{(h,h')} & = \sum_{j=1}^{n} \lambda_j^{h'} v_{pj}^{(h,h')} & h=1,\dots,h_t\ ,p=1,\dots,P_t\ ,h'=1,\dots,h_d \\
\sum_{j=1}^{n} \lambda_j^h v_{pj}^{(h,h')} + s_p^{(h,h')} & = \sum_{j=1}^{n} \lambda_j^{h'} v_{pj}^{(h,h')} & h=1,\dots,h_d\ ,p=1,\dots,P_d\ ,h'=1,\dots,h_c \\
\sum_{j=1}^{n} \lambda_j^h z_{aj}^{(h',h)} - s_a^{+(h',h)} = z_{ak}^{(h',h)} & & h=1,\dots,h_s\ ,h'=1,\dots,h_m\ ,a=1,\dots,A_s \\
\sum_{j=1}^{n} \lambda_j^h z_{aj}^{(h',h)} + s_a^{+(h',h)} = z_{ak}^{(h',h)} & & h=1,\dots,h_s\ ,h'=1,\dots,h_m\ ,a=1,\dots,A_m \\
\sum_{j=1}^{n} \lambda_j^{h'} z_{aj}^{(h',h)} - s_a^{-(h',h)} = z_{ak}^{(h',h)} & & h=1,\dots,h_m\ ,h'=1,\dots,h_t\ ,a=1,\dots,A_m \\
\sum_{j=1}^{n} \lambda_j^{h'} z_{aj}^{(h',h)} + s_a^{-(h',h)} = z_{ak}^{(h',h)} & & h=1,\dots,h_m\ ,h'=1,\dots,h_t\ ,a=1,\dots,A_t \\
\sum_{j=1}^{n} \lambda_j^h & = 1 & j=1,\dots,n\ ,\ h=1,\dots,H \\
\multicolumn{3}{l}{\lambda_j \ge 0\ ,\ s_a^{+(h,h')} \ge 0,\ s_a^{-(h,h')} \ge 0,\ s_p^{(h,h')},\ \xi\ UR\ ,\quad j=1,\dots,n\ ,\ h=1,\dots,H}
\end{array}
\tag{3}
$$

The first and second category constraints correspond to inputs set under natural and managerial disposability. Furthermore, the third and fourth category constraints are related to desirable and undesirable outputs, and the fifth category constraints are related to dual-role factors of the supplier, manufacturer, and transmitter divisions, respectively. The sixth, seventh, eighth, and ninth category constraints correspond to intermediate measures sent from the supplier divisions to manufacturer divisions, from manufacturer divisions to transmitter divisions, from transmitter divisions to distributor divisions, and from them to customer divisions, respectively. The tenth and eleventh category constraints are related to intermediate measures that exit manufacturer divisions and enter supplier divisions. Also, the twelfth and



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thirteenth category constraints correspond to intermediate measures that exit transmitter divisions and enter manufacturing divisions. The production return to scale is the subject of the final category of constraints.

### 3.3 The inverse output-oriented DEA Model of supply chain

In this section, we focus on the inverse output-oriented models in the presence of two categories of inputs under natural and managerial disposability, desirable and undesirable outputs, dual-role factors, and intermediate measures that maintain efficiency under variables to return to scale. Assume, that output of h$^{th}$ division is changed from $y_{rk}^{h}$ to $y_{rk}^{h}+\Delta y_{irk}^{h}$ while efficiency of $DMU_K$ is unchanged. $\Delta y_{irk}^{h}$ is optimal variations of outputs for $DMU_K$ , and $\Delta$ can be positive or negative. To formulate inverse model, suppose the rate of the variations in production factors is defined as follows:

$\Delta Y_{j}^{h}=(\Delta y_{1j}^{h},\Delta y_{2j}^{h},...,\Delta y_{sj}^{h})^{T}>0$ : A column vector of the changes of $s$ desirable outputs from the h$^{th}$ division in the j$^{th}$ supply chain $j=1,...,n \quad h=1,...,H$ .

$\Delta B_{f}^{h}=(\Delta b_{1j}^{h},\Delta b_{2j}^{h},...,\Delta b_{fj}^{h})^{T}>0$ : A column vector of the changes of $f$ indesirable outputs from the h$^{th}$ division in the j$^{th}$ supply chain $j=1,...,n \quad h=1,...,H$ .

Following the assessment of the supply chain's optimal efficiency using Model (3), the efficiency scores now incorporated into Model (4). The changes of the two categories of outputs examined for supply chain divisions by the inverse DEA output-oriented model (4). In addition, the suggested inverse DEA output-oriented model can determine the optimal variation of outputs for DMU while assuming that the DMU keeps its current efficiency level compared to other DMUs. Additionally, the weighted average of the optimal output changes of each division of the supply chain used to calculate the optimal value of objective function of the inverse output-oriented model.

The inverse DEA output-oriented model calculates the maximum rate of desirable output changes based on undesirable output variations for supply chain as other production factors remain constant and efficiency keep unchanged.

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$$
\begin{array}{lll}
Max \sum_{h=1}^{H} \omega_h \, (\Delta y_{rk}^h) & & \\
\sum_{j=1}^{n} \bar{x}_{ij}^h \lambda_j^h & \leq \bar{x}_{ik}^h & i = 1,\ldots, m_h^-,\ h = 1,\ldots, H \\
\sum_{j=1}^{n} \tilde{x}_{qj}^h \lambda_j^h & \geq \tilde{x}_{qk}^h & q = 1,\ldots, m_h^+,\ h = 1,\ldots, H \\
\sum_{j=1}^{n} y_{rj}^h \lambda_j^h & \geq \varphi^h (y_{rk}^h + \Delta y_{rk}^h) & r = 1,\ldots, S_h\ , h = 1,\ldots, H \\
\sum_{j=1}^{n} b_{fj} \lambda_j^h & \leq (b_{fk}^h + \Delta b_{fk}^h) & f = 1,\ldots, F_h\ , h = 1,\ldots, H \\
\sum_{j=1}^{n} w_{ej}^h \lambda_j^h & = w_{ek}^h & e = 1,\ldots, E_h\ , h = 1,\ldots, H \\
\sum_{j=1}^{n} \lambda_j^h v_{pj}^{(h,h')} + s_p^{(h,h')} & = \sum_{j=1}^{n} \lambda_j^{h'} v_{pj}^{(h,h')} & h = 1,\ldots, h_s\ , p = 1,\ldots, P_s\ , h' = 1,\ldots, h_m \\
\sum_{j=1}^{n} \lambda_j^h v_{pj}^{(h,h')} + s_p^{(h,h')} & = \sum_{j=1}^{J} \lambda_j^{h'} v_{pj}^{(h,h')} & h = 1,\ldots, h_m\ , p = 1,\ldots, P_m\ , h' = 1,\ldots, h_t \\
\sum_{j=1}^{n} \lambda_j^h v_{pj}^{(h,h')} + s_p^{(h,h')} & = \sum_{j=1}^{n} \lambda_j^{h'} v_{pj}^{(h,h')} & h = 1,\ldots, h_t\ , p = 1,\ldots, P_t\ , h' = 1,\ldots, h_d \\
\sum_{j=1}^{n} \lambda_j^h v_{pj}^{(h,h')} + s_p^{(h,h')} & = \sum_{j=1}^{n} \lambda_j^{h'} v_{pj}^{(h,h')} & h = 1,\ldots, h_d\ , p = 1,\ldots, P_d\ , h' = 1,\ldots, h_c \\
\sum_{j=1}^{n} \lambda_j^h z_{aj}^{(h',h)} - s_a^{+(h',h)} & = z_{ak}^{(h',h)} & h = 1,\ldots, h_s\ , h' = 1,\ldots, h_m\ , a = 1,\ldots, A_s \\
\sum_{j=1}^{n} \lambda_j^h z_{aj}^{(h',h)} + s_a^{+(h',h)} & = z_{ak}^{(h',h)} & h = 1,\ldots, h_s\ , h' = 1,\ldots, h_m\ , a = 1,\ldots, A_m \\
\sum_{j=1}^{n} \lambda_j^{h'} z_{aj}^{(h',h)} - s_a^{-(h',h)} & = z_{ak}^{(h',h)} & h = 1,\ldots, h_m\ , h' = 1,\ldots, h_t\ , a = 1,\ldots, A_m \\
\sum_{j=1}^{n} \lambda_j^{h'} z_{aj}^{(h',h)} + s_a^{-(h',h)} & = z_{ak}^{(h',h)} & h = 1,\ldots, h_m\ , h' = 1,\ldots, h_t\ , a = 1,\ldots, A_t \quad * \\
\sum_{j=1}^{n} \lambda_j^h & = 1 & j = 1,\ldots, n,\ h = 1,\ldots, H \\
\end{array}
$$

$$\lambda_j \geq 0\ ,\ s_a^{+(h,h')} \geq 0,\ \ s_a^{-(h,h')} \geq 0,\ \ s_p^{(h,h')}\ ,\ \varphi\ UR,\quad j = 1,\ldots, n,\ \ h = 1,\ldots, H,\quad \Delta g_{rk}^h,\ \ \Delta b_{fk}^h\ UR \tag{4}$$

Given the output-oriented model, outputs are classified into desirable and undesirable outputs.

Constraints in the first and second categories relate to outputs that fall under natural and managerial disposability. Variations of the r[th] input under natural disposability from $y_{rk}^h$ to $y_{rk}^h + \Delta y_{rk}^h$ and variations of the f[th] undesirable output from $b_{fk}^h$ to $b_{fk}^h + \Delta b_{fk}^h$ for the h[th] division in the k[th] supply chain are indicated by right-hand side. Since $\Delta$ is free under signal, an optimal solution may contain values of either positivity or negativity.



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## 4 A Real Case of the Power Industry

In this section, we analyze the Iran power industry using the suggested model. The next subsection will describe the dataset, and the results indicated in the following subsection.

### 4.1 Dataset

Each of the DMUs or the supply chain consist of five stages, and each stage includes a set of partners connected to the preceding stages' members by some sustainable intermediate measures. In the application phase, ten supply chains (DMUs), including oil and gas fields (suppliers) that provide different fuels to power stations, power plants (manufacturers), regional power companies (transmitters), distribution companies (distributors), and customers, were considered. Two suppliers assumed per supply chain: oil and gas companies that satisfied the fuel demand of power plants (intermediate products) and sold fuel as the final output.

In the proposed model, suppliers used one input (capital) under natural disposability and one input under managerial disposability (labor). The suppliers produced one desirable (oil or gas) and one undesirable output (flaring gas). The dual-role factor considered to be the cost of cleanup of flare gas pollution. Each manufacturer included at least three power plants with different technologies (e.g., thermal, combined cycle, gas, hydro, wind, and solar) that used fuels, capital, and labor under natural and labor of hydropower plant under managerial disposability to produce electricity and sell it to the regional power companies.

Three undesirable outputs were considered for manufacturers: $CO_2$, Nitrogen Oxides (NOx), and Sulfur Oxides (SOx) emissions. Also, the dual-role factor was the inner electricity consumption of power plants as technical and non-technical consumption. The transmitters were transferring electricity from manufacturers to distributing companies, and the capacity and length of the lines considered as the inputs under natural and the number employees of the department of programming and researches used as input under management disposability. The dual-role factor was the specialist workforce in programming. The transmission lines' loss considered an undesirable output, while the construction of new lines was a desirable output.

Distribution companies receiving electricity from transmitters and dispatching it to the final consumers. They were using two additional capital inputs estimated as the capacity and length of the distribution lines under natural disposability and the number of employees of the engineering assistance department and programming as input under managerial disposability, one final desirable output as the meter of electricity, and one undesirable output as losses in the distribution lines. Finally, customers classified as residential, agricultural, public, and industrial. They were using one input under natural disposability and one input under managerial disposability and producing two desirable and one undesirable output.

More details concerning the parameters used to characterize this supply chain are as follows:

$h_s$ : Numerator of divisions in the supplier level ( $h_s$ : 1, 2).

$\bar{x}_{1j}^{h(s)}$: Capacity of oil ($10^3$ Barrels) and gas ($10^6$ $m^3$) fields of the $h_s$ th supplier in the j th supply chain.

$\tilde{x}_{1j}^{h(s)}$ : Number of employees from $h_s$ th supplier in jth supply chain.



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$y_{1j}^{h(s)}$: Oil ($10^3$ Barrels) and gas ($10^6$ $m^3$) sold to other companies from the $h_s$ th supplier in the j th supply chain.
$b_{1j}^{h(s)}$: Flaring gas of oil field ($10^3$ barrels) and gas field ($10^6 m^3$) of the $h_s$ th supplier in the j th supply chain.
$w_{1j}^{h(s)}$: The cost of cleanup of burned gas (flaring gas) of the $h_s$ th supplier in the j th supply chain.
$h_m$: Numerator of division in the manufacturer level ($h_m$: 3, 4, and 5).
$\bar{x}_{1j}^{h(m)}$: Power nominal of the $h_m$ th manufacturer in the j th supply chain ($10^6$ kWh).
$\bar{x}_{2j}^{h(m)}$: Number of employees of the $h_m$ th manufacturer in the j th supply chain.
$\tilde{x}_{1j}^{h(m)}$: Number of hydropower employees of $h_m$ th manufacturer in the jth supply chain.
$y_{1j}^{h(m)}$: Percentage of new construction of power plant of the $h_m$ th manufacturer in the j th supply chain.
$b_{1j}^{h(m)}$: Emissions of $NO_X$ of the $h_m$ th manufacturer in the j th supply chain ($10^3$kg/$10^6$kWh).
$b_{2j}^{h(m)}$: Emissions of $SO_X$ of the $h_m$ th manufacturer in the j th supply chain ($10^3$kg/$10^6$kWh).
$b_{3j}^{h(m)}$: Emission of $CO_2$ of the $h_m$ th manufacturer in the j th supply chain ($10^3$ kg/$10^6$ kWh).
$w_{1j}^{h(m)}$: Inner consumption of power plants (technical and non-technical consumptions) of the $h_m$ th manufacturer in the j th supply chain ($10^6$ kWh).
$h_t$: Numerator of the divisions in the level of the transmitters ($h_t$: 6, 7).
$\bar{x}_{1j}^{h(t)}$: Capacity of transmission lines of the $h_t$ th transmitter in the j th supply chain (MWa).
$\bar{x}_{2j}^{h(t)}$: Length of transmission line of the $h_t$ th transmitter in the j th supply chain (km circuit).
$\tilde{x}_{1j}^{h(t)}$: Number of employees department of programing and researches of the $h_t$ th transmitter in the jth supply chain.
$y_{1j}^{h(t)}$: New construction of transmission lines of the $h_t$ th transmitter in the j th supply chain (km circuit).
$b_{1j}^{h(t)}$: Loss of transmission line of the $h_t$ th transmitter in the j th supply chain ($10^6$ kWh).
$w_{1j}^{h(t)}$: Number of the deputy employees of transfer and exploitation of the $h_t$ th transmitter in the j th supply chain.
$h_d$: Numerator of the division in the distributer level ($h_d$: 8, 9, 10, and 11).
$\bar{x}_{1j}^{h(d)}$: Capacity of the distribution lines of the $h_d$ th distributer in the j th supply chain (MVa).
$\bar{x}_{2j}^{h(d)}$: Length of distribution line of the $h_d$ th distributer in the j th supply chain (km).
$\tilde{x}_{1j}^{h(d)}$: Number of employees of engineering assistance department and programming of the $h_d$ th distributer in the jth supply chain.
$y_{1j}^{h(d)}$: Meter of electricity of the $h_d$ th distributer in the j th supply chain.
$b_{1j}^{h(d)}$: Percentage of losses of the distribution line of the $h_d$ th distributer in the j th supply chain (%).


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$h_c$ : Numerator of the division in the customer level ( $h_c$ : 12, 13, 14, and 15).
$\overline{x}_{1j}^{h(c)}$ : Average cost with fuel subsidy of the $h_c$ th customer in the j$^{th}$ supply chain (USD).
$\tilde{x}_{1j}^{h(c)}$ : Direct selling of electricity from transmitter Company to the $h_c$ th customer in the jth supply chain ($10^6$ Kwh).
$y_{1j}^{h(c)}$ : Number of customers of the $h_c$ th customer in the j$^{th}$ supply chain.
$y_{2j}^{h(c)}$ : Sales of electricity of the $h_c$ th customer in the j$^{th}$ supply chain ($10^6$ kWh).
$b_{2j}^{h(c)}$ : Cut off the power of the $h_c$ th customer in the j$^{th}$ supply chain (minute/year).
$v_{pj}^{(h,h')}$ : Material flow from the division $h$ to division $h'$ ($10^6$ kVA).
$z_{aj}^{(h,h')}$ : Power flow sent from the division $h$ to division $h'$ ($10^6$ kVA).

All data from the two oil and gas fields (suppliers), power plants (manufacturers), regional power companies (transmitters), distribution companies (distributors), and customers (residential, public, agricultural, industrial) is available on the Iran Power Generation and Transmission Company's TAVANIR) website [35]. The dataset has collected from the power industry companies in Iran, and the reference year is 2015 (see TAVANIR's website for the detailed data.

### 4.2 Results

We now describe the results obtained by the inverse DEA model. First, model (3) applied to estimates the output-oriented efficiency score of 10 supply chains (DMUS) and 15 divisions under VRS. The results listed in Table 1. The first column of Tables 1 represents the global efficiency score of the supply chains based on variable returns to scale.

According to Table 1, supply chain number 4 obtained the highest efficiency score (1) under VRS. Moreover, 15 divisions of supply chain 4 were efficient under VRS, while DMU7 with efficiency score (0.946) had 13 efficient divisions.

Looking vertically across the table reveals that the second and third power plants, the first transmitter and power customers of residential, public, agricultural, and industrial divisions were efficient under VRS in 10 supply chains.

**Table1** The input-oriented efficiency scores of supply chains (DMUs) under VRS

| DMU | $\theta_o$ | $\xi_k^{S1}$ | $\xi_k^{S2}$ | $\xi_k^{M1}$ | $\xi_k^{M2}$ | $\xi_k^{M3}$ | $\xi_k^{T1}$ | $\xi_k^{T2}$ | $\xi_k^{D1}$ | $\xi_k^{D2}$ | $\xi_k^{D3}$ | $\xi_k^{D4}$ | $\xi_k^{C1}$ | $\xi_k^{C2}$ | $\xi_k^{C3}$ | $\xi_k^{C4}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.977 | 1 | 0.99 | 1 | 1 | 1 | 1 | 1 | 0.71 | 1 | 0.64 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0.962 | 1 | 0.84 | 0.74 | 1 | 1 | 1 | 1 | 1 | 0.85 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 0.986 | 1 | 0.83 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1.000 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 0.999 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.96 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0.986 | 1 | 0.83 | 1 | 1 | 1 | 1 | 1 | 0.87 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | 0.946 | 0.56 | 0.77 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 0.965 | 1 | 0.84 | 1 | 1 | 1 | 1 | 0.88 | 1 | 0.69 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | 0.978 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.71 | 1 | 0.64 | 1 | 1 | 1 | 1 | 1 |
| 10 | 0.999 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.97 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Now, efficiency scores of supply chain divisions incorporated into the model (4) to determine the simultaneous variation of desirable and undesirable outputs. The proposed


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inverse DEA model determines the minimal variation of applied sources based on the variation of two categories of inputs as other production factors keep constant under VRS.

### 4.2.1 Results of output-oriented Invers model

We now describe the results obtained by the output-oriented inverse DEA model. First, model (3) is applied to estimate the input-oriented efficiency score of 10 supply chains (DMUS) and 15 divisions under two categories of inputs and desirable and undesirable outputs and dual-role factor based on variable return to scale. Now, efficiency scores of supply chain divisions incorporated into the model (4) to determine optimal output variations of supply chain divisions without changing efficiency.

Tables 2-8 indicate variation in desirable and undesirable outputs, and the new value of outputs for oil and gas fields, power plants and transmitter and distributer lines in 10 supply chains.

In Table 2, columns 2 to 5 report the changes in the desirable and undesirable outputs of suppliers 1 and 2, and columns 6 to 9 show the new quantities of the two categories outputs of oil and gas fields in 10 electricity supply chains.

**Table 2** The optimal changes and new values of desirable, undesirable outputs of suppliers under VRS

| DMU | $\Delta g_{1k}^{1}$ | $\Delta g_{1k}^{2}$ | $\Delta b_{1k}^{1}$ | $\Delta b_{1k}^{2}$ | $g_{1k}^{1}+\Delta g_{1k}^{1}$ | $g_{1k}^{2}+\Delta g_{1k}^{2}$ | $b_{1k}^{1}+\Delta b_{1k}^{1}$ | $b_{1k}^{2}+\Delta b_{1k}^{2}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1414.093 | 0 | 0 | 1739.6933 | 2600.309 | 54 | 151.2 |
| 2 | 0 | 2451.172 | 0 | 0 | 40527.9964 | 9654.402 | 1296 | 345.6 |
| 3 | 0 | 49.778 | 0 | 0 | 8895.88282 | 3775.981 | 432 | 183.6 |
| 4 | 0 | 278.390 | 0 | 0 | 26527.1913 | 2208.415 | 972 | 140.4 |
| 5 | 0 | 0 | 0 | 0 | 4552.85776 | 10438.19 | 216 | 367.2 |
| 6 | 0 | 425.316 | 0 | 0 | 23324.3911 | 3775.991 | 756 | 183.6 |
| 7 | 5495.061 | 1030.967 | 0 | 0 | 22575.5321 | 3384..097 | 756 | 172.8 |
| 8 | 4400.007 | 199.298 | 0.002 | 0 | 20272.9206 | 9654.402 | 648.002 | 345.6 |
| 9 | 0 | 588.597 | 0 | 0 | 6062.77171 | 10438.19 | 194.4 | 367.2 |
| 10 | 0 | 0 | 0 | 0 | 25603.3995 | 2208.415 | 1296 | 140.4 |

According to the obtained results of the output-oriented inverse model performance in energy sectors, gas fields of 80% of supply chains have necessary abilities for investment to economic return increment. According to Table 2, the most significant desirable output or gas sold to other companies belong to the gas field of supply chain number 2 while other production factors remain constant. Indeed, the gas field of supply chain number 2 can increment 2451.172 milion cubic meter gas without changing flare gas emissions as other production factors keep constant.

In Tables 3-5, columns 2 to 5 report the changes in the desirable and undesirable outputs of manufacturers 1, 2, and 3. Also, columns 6 to 9 show the new quantities of power plant sectors outputs in 10 electricity supply chains.



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**Table 3** The optimal changes and new values of the undesirable output of manufacturer 1

| DMU | $\Delta g_{1k}^{3}$ | $\Delta b_{1k}^{3}$ | $\Delta b_{2k}^{3}$ | $\Delta b_{3k}^{3}$ | $g_{1k}^{3}+\Delta g_{1k}^{3}$ | $b_{1k}^{3}+\Delta b_{1k}^{3}$ | $b_{2k}^{3}+\Delta b_{2k}^{3}$ | $b_{3k}^{3}+\Delta b_{3k}^{3}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 12.2 | 454610.278 | 23891876.280 | 288025420.100 |
| 2 | 76.399 | -76160 | -1722000 | -48270000 | 88.599 | 226939.8048 | 2485069.806 | 143682930.5 |
| 3 | 0 | 0 | 0 | 0 | 13 | 235104.740 | 195553.061 | 149621794 |
| 4 | 0 | 0 | 0 | 0 | 12.2 | 229464.218 | 12059407.75 | 145380628.200 |
| 5 | 0 | 0 | 0 | 0 | 73.6 | 43498.708 | 38755.471 | 27536231.770 |
| 6 | 0 | 0 | 0 | 0 | 100 | 256638.343 | 217529.667 | 163094448.800 |
| 7 | 0 | 0 | 0 | 0 | 85.5 | 6683.633 | 5954.829 | 9585079.623 |
| 8 | 0 | 0 | 0 | 0 | 85.5 | 15138.687 | 184259.151 | 9585079.623 |
| 9 | 0 | 0 | 0 | 0 | 13 | 92035.892 | 76552.691 | 58572086.910 |
| 10 | 0 | 0 | 0 | 0 | 86.6 | 236364.062 | 196600.528 | 150423232.700 |

**Table 4** The optimal changes and new values of the undesirable output of manufacturer 2

| DMU | $\Delta g_{1k}^{4}$ | $\Delta b_{1k}^{4}$ | $\Delta b_{2k}^{4}$ | $\Delta b_{3k}^{4}$ | $g_{1k}^{4}+\Delta g_{1k}^{4}$ | $b_{1k}^{4}+\Delta b_{1k}^{4}$ | $b_{2k}^{4}+\Delta b_{2k}^{4}$ | $b_{3k}^{4}+\Delta b_{3k}^{4}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 85.5 | 5715.366 | 5092.145 | 3618030.390 |
| 2 | 0 | 0 | 0 | 0 | 12.1 | 283431.105 | 14895617.700 | 179572190 |
| 3 | 0 | 0 | 0 | 0 | 12.2 | 174773.192 | 9070013.802 | 110729096.200 |
| 4 | 0 | 0 | 0 | 0 | 25.2 | 182851.984 | 152090.788 | 116367887.400 |
| 5 | 0 | 0 | 0 | 0 | 12.2 | 49845.037 | 2619587.603 | 3158009.070 |
| 6 | 0 | 0 | 0 | 0 | 85.5 | 27420.014 | 24430.049 | 17357845.530 |
| 7 | 0 | 0 | 0 | 0 | 12.2 | 273496.466 | 14373506.370 | 173277944.500 |
| 8 | 0 | 0 | 0 | 0 | 12.2 | 311634.456 | 21776302.480 | 197440862.200 |
| 9 | 0 | 0 | 0 | 0 | 98.8 | 176752.534 | 147351.908 | 112467128.500 |
| 10 | 0 | 0 | 0 | 0 | 96.6 | 79593.197 | 66419.786 | 50641168.170 |

Table 3 indicates the maximal increase of produced electricity of the first power plant of supply chain number 2 is 76.399 ($10^6$ kWh). In this case, increase of power production and decrease of $NO_X$, $SO_X$ and, $CO_2$ gases emissions maintain power plant efficiency and other production factors without unchanged. Also, the quantities of desirable and undesirable outputs remained constant in the second and third power plant.



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**Table 5** The optimal changes and new values of the undesirable output of manufacturer 3

| DMU | $\Delta g_{1k}^{5}$ | $\Delta b_{1k}^{5}$ | $\Delta b_{2k}^{5}$ | $\Delta b_{3k}^{5}$ | $g_{1k}^{5}+\Delta g_{1k}^{5}$ | $b_{1k}^{5}+\Delta b_{1k}^{5}$ | $b_{2k}^{5}+\Delta b_{2k}^{5}$ | $b_{3k}^{5}+\Delta b_{3k}^{5}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 73.6 | 19603.894 | 17519.680 | 12447945.190 |
| 2 | 0 | 0 | 0 | 0 | 73.6 | 27423877.76 | 24433491.25 | 17360291475 |
| 3 | 0 | 0 | 0 | 0 | 98.8 | 212448.268 | 690393.877 | 135090771.800 |
| 4 | 0 | 0 | 0 | 0 | 13 | 140748.540 | 117070.408 | 89573051.780 |
| 5 | 0 | 0 | 0 | 0 | 87 | 89573051.780 | 9178172.226 | 190308335.200 |
| 6 | 0 | 0 | 0 | 0 | 13 | 77463.980 | 64432.212 | 49298451.340 |
| 7 | 0 | 0 | 0 | 0 | 13 | 471751.939 | 21768344.370 | 299051808 |
| 8 | 0 | 0 | 0 | 0 | 13 | 510495.755 | 21776302.480 | 323709891.900 |
| 9 | 0 | 0 | 0 | 0 | 13 | 94829.614 | 78876.425 | 60350025.180 |
| 10 | 0 | 0 | 0 | 0 | 1.2 | 59895.401 | 3147780.793 | 37947663.670 |

**Table 6** The optimal changes and new values of desirable, undesirable outputs of transmitters under VRS

| DMU | $\Delta g_{1k}^{6}$ | $\Delta g_{1k}^{7}$ | $\Delta b_{1k}^{6}$ | $\Delta b_{1k}^{7}$ | $g_{1k}^{6}+\Delta g_{1k}^{6}$ | $g_{1k}^{7}+\Delta g_{1k}^{7}$ | $b_{1k}^{6}+\Delta b_{1k}^{6}$ | $b_{1k}^{7}+\Delta b_{1k}^{7}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 205.009 | 0 | 219.438 | 990 | 1746.409 | 508.845 | 271.318 |
| 2 | 0 | 0 | 0 | 0 | 1302.3 | 110 | 200.566 | 301.829 |
| 3 | 0 | 0 | 0 | 0 | 1961.5 | 1302.3 | 175.381 | 357.789 |
| 4 | 0 | 0 | 0 | 0 | 1596 | 1302.3 | 328.197 | 117.468 |
| 5 | 0 | 0 | 0 | 0 | 324 | 1961.5 | 67.759 | 263.987 |
| 6 | 0 | 0 | 0 | 0 | 431.3 | 110 | 254.862 | 107.780 |
| 7 | 0 | 0 | 0 | 0 | 1576.2 | 747 | 447.605 | 61.919 |
| 8 | 0 | 1156.606 | 0 | -123.807 | 601.2 | 1542.606 | 373.774 | 78.393 |
| 9 | 0 | 0 | 0 | 0 | 1541.2 | 110 | 273.358 | 84.462 |
| 10 | 0 | 0 | 0 | 0 | 601.2 | 1453.8 | 294.146 | 38.828 |

According to Table 6, the first transmitter line of 10 supply chains have not changes related to desirable and undesirable outputs when two categories of inputs and dual-role factor and efficiency of transmitter line keep constant. In contrary, the new construction of transmission lines of the second transmitter line of supply chain number 1 can be increased to 205.009 (km circuit) as created power loses increment to 219.438 ($10^6$ kWh) while other production factors and efficiency remained constant. However, the notable increase of new transmitter line of supply chain number 8 need to energy loses decrement when production factors keep unchanged.



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**Table 7** The optimal changes and new values of desirable, undesirable outputs of distributers 1, 2

| DMU | $\Delta g_{1k}^{8}$ | $\Delta g_{1k}^{9}$ | $\Delta b_{1k}^{8}$ | $\Delta b_{1k}^{9}$ | $g_{1k}^{8}+\Delta g_{1k}^{8}$ | $g_{1k}^{9}+\Delta g_{1k}^{9}$ | $b_{1k}^{8}+\Delta b_{1k}^{8}$ | $b_{1k}^{9}+\Delta b_{1k}^{9}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 812830 | 146050 | -3.764 | -7.724 | 1389083 | 722303 | 10.446 | 0.306 |
| 2 | 0 | 226460 | 8.370 | -0.481 | 2046151 | 550380 | 15.57 | 2.676 |
| 3 | 0 | 30178 | 0 | 1.280 | 2046151 | 662102 | 15.570 | 10.909 |
| 4 | 0 | 0 | 0 | 0 | 1288350 | 345484 | 15.570 | 10.730 |
| 5 | 0 | 0 | 0 | 0 | 265678 | 662102 | 13.250 | 12.670 |
| 6 | 0 | 0 | 0 | 0 | 2046151 | 513660 | 15.57 | 11.510 |
| 7 | 0 | 0 | 0 | 0 | 497281 | 429044 | 13.600 | 11.050 |
| 8 | 172850 | 265460 | 1.334 | -0.879 | 296307.5 | 634118 | 12.574 | 12.451 |
| 9 | 812830 | 0 | 0.647 | 0 | 1389083 | 513660 | 14.857 | 7.250 |
| 10 | 100950 | 225250 | 1.107 | 0.821 | 570683 | 573018 | 13.647 | 12.051 |

**Table 8** The optimal changes and new values of desirable, undesirable outputs of distributers 3, 4

| DMU | $\Delta g_{1k}^{10}$ | $\Delta g_{1k}^{11}$ | $\Delta b_{1k}^{10}$ | $\Delta b_{1k}^{11}$ | $g_{1k}^{10}+\Delta g_{1k}^{10}$ | $g_{1k}^{11}+\Delta g_{1k}^{11}$ | $b_{1k}^{10}+\Delta b_{1k}^{10}$ | $b_{1k}^{11}+\Delta b_{1k}^{11}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 89231.050 | 0 | -5.641 | 0 | 337310.05 | 327034 | 7.949 | 14.200 |
| 2 | 0 | 74885.213 | 0 | -1.001 | 345484 | 283231.213 | 10.730 | 6.989 |
| 3 | 0 | 0 | 0 | 0 | 429044 | 265678 | 11.050 | 13.250 |
| 4 | 0 | 343630 | 0 | 1.725 | 329071 | 653334 | 7.670 | 13.755 |
| 5 | 0 | 3173.606 | 0 | 2.341 | 429044 | 635097.606 | 11.05 | 13.731 |
| 6 | 60620.867 | 0 | 0.671 | 0 | 268966.867 | 333449 | 8.661 | 7.250 |
| 7 | 0 | 0 | 0 | 0 | 265678 | 2046151 | 13.25 | 15.570 |
| 8 | 0 | 340120 | 0 | 6.148 | 550244 | 1031611 | 8.030 | 14.248 |
| 9 | 128960 | 173400 | -5.641 | 5.923 | 337306 | 805324 | 7.949 | 13.953 |
| 10 | 0 | 340120 | 0 | 6.148 | 550244 | 864891 | 8.030 | 14.248 |

Tabels7 and 8 show the changes and the new quantities of the two categories of the desirable and undesirable outputs of four distribution lines in 10 supply chain. There are supply chains whose distribution lines have capacities of electricity flow increase to power customers as applied resources as such capacity and length of distributor line and specialist workforce remained unchanged. Also, the increase of electricity flow creates increment or decrement of percent of power loses in distributor lines.

For an instance, the first, second, and third of distributor line of supply chain number 1 have increase ability of dispatched electricity flow based on available capacities while they should have nessacery abilities for reduce of distributor line' power loses. It can be easily seen that the most remarkable increment of desirable output occurred in the fourth distribution lines of supply chain number 4 as meter of electricity was increased from 309704 to 653334.



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At the same time, the percent of power loses as 1.725 increased. In addition, the efficiency score remained unchanged.

## 5 Discussion and Conclusion

### 5.1 Assessment of supply chain sustainability

Generally, the results obtained from inverse oriented-output model supply chains are separated into three categories as follows:
(1) Supply chain divisions have capacities adequate for increase of their desirable output to more economic return in industrial activities while this increase causes no changes of undesirable outputs and efficiency score and available resources remained constant. The gas fields were divisions in which the increment of the sold gas to other companies does not create increase or decrease in pollutants emissions. Thus, they have the necessary abilities to confront harmful emissions and energy losses. Indeed, the most considerable desirable output or gas sold to other companies belongs to the gas field of supply chain number 2 while other production factors remain constant.
(2) The divisions of supply chains in which economic boom or desirable output increase create by the decrement of undesirable outputs. Hence, they should have more facilities and improved engineering systems for pollutant emissions reduction and wasted energy control. As a result, they required new technological innovation and enhanced capabilities to confront flares and GHGs. The first power plant number 2 to desirable output increment and new power plants construction and preservation of current efficiency should reduce harmful emissions of greenhouse gases. Also, the notable increase of new transmitter line of supply chain number 8 need to energy loses decrement when production factors remained unchanged.
(3) The divisions of supply chains in which undesirable outputs increase occurred by increment of economic activities. The most remarkable increment of desirable outputs occurred in the fourth distribution line of supply chain number 4 while the percent of power loses increased. The 40% of supply chains' the first distributer line and 50% of supply chains' the fourth distributer line meet power loss increment when increase dispatched electricity to power customers. In this case, they should have the necessary preparation to confront energy wasted.

### 5.2 Conclusions

In recent years, fossil fuel consumption, such as oil, natural gas, and coal, has increased the concentration of $CO_2$ in the Earth's atmosphere and caused climate change. Hence, the management of available resources is essential in the power industry. This study proposed an inverse DEA model for sustainable supply chain improvement in the power industry. An essential feature of the proposed model is that it enables us to identify the optimal value of investment capacities in supply chain divisions by optimal changes of two categories of outputs when other production factors and the divisions' efficiency remained unchanged. Based on the results of the inverse output-oriented DEA model, the gas field of supply chains 80% should enhance their desirable output while current emissions and efficiency levels remain constant. Thus, the gas field should have equipped to improved engineering systems for economic return increment. Also, the oil field and power plant sectors are presented as significantly operational in the power industry as the oil field of supply chains 80% and more than 90% of supply chains' power plants sectors and transmitter lines produced an optimal


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value of energy. In contrast, distributer lines require the specialist workforce to power loses abatement when enhancing power production and economic boom.

## Availability of data and material

https://web.archive.org/web/20210514014836/http://amar.tavanir.org.ir/

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