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Undesirable input in production process: A SBM-DEA model

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Abstract The slack based measure (SBM) of efficiency is one of mostly used model in the data envelopment analysis (DEA) literature. The model acts on inputs and outputs slacks and gave a measure between zero and unity. In the light of global warming and resource scarcity, some units widely consume undesirable (used) inputs for improving their efficiency. Plastic waste or Gray water production are two examples of undesirable input usage in recycling factories. Hence, increasing undesirable inputs raise challenges for handling in production process. This study contributes undesirable inputs through SBM model. Toward this end, the SBM model is developed to cases that consume both desirable and undesirable inputs to produce the final outputs. Addressing the increasing of undesirable input, SBM model is reformulated to identify not only corresponding reduction in desirable inputs but also anticipate a suitable quantity for the undesirable inputs. A real data set is applied to elucidate the model applicability and advantages.

Keyword: Undesirable inputs, Data envelopment analysis (DEA), Decision making unit (DMU), Slack based measure (SBM), Environmental assessment.

1 Introduction

Data Envelopment Analysis (DEA) has pioneered by Charnes et al. [1] and extended by Banker et al. [2] has recently made substantial contribution in efficiency and productivity analysis. Regarding to climate change and global warming, there has been a considerable attention in assessing the production process by taking undesirable outputs into account. Undesirable outputs as the byproducts of economic challenges lead to environmental pollutions and need to reduce. Greenhouse gas emission such as CO2, methane, nitrous oxide are main instance of undesirable outputs. But the key point is the undesirable outputs are produced as good outputs are produced. Hence, the reduction of undesirable outputs leads to proportional reduction in desirable outputs productions. This issue, proportional reduction of both desirable and undesirable outputs (output weak disposability axiom) goes back to Shepard [3]. The author defined a uniform abatement factor for all units in the sample.

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Kuosmanen [4] argued the application of uniform abatement factors and pointed that the firms with lower abatement cost do not satisfy this factor. As another study, Podinovski and Kuosmanen [5] developed other methodologies satisfying relaxed convexity axioms for modeling the simultaneous reduction of desirable and undesirable outputs. Surveying DEA literature reveals that there are various aspects for treating undesirable outputs with different perspectives. For example, Roshdi et.al [6] introduced a new concept of exponential weak disposability assumption for undesired outputs permits various trade-offs between desirable and undesirable outputs. Mehdiloozad and Podinovski [7] noted that the modification of Shepard technology for increasing the undesirable input through a single scaling factor leads to consequences notably in measuring congestion. Mehdiloo et al. [8] argued that the disposability assumption may not be applicable and resulting to meaningless proportions in presence of input or output overlapping or strongly correlated to each other. Pham et al. [9] explored applying the single or multiple scaling factor in different scenarios. Also the authors elucidated the link between various return to scale and weak disposability of desirable and undesirable outputs.

Monzeli et al. [10] addressed the efficiency measurement in presence of undesirable input and output. Their methodology involved three steps. As for the first step, an appropriate production possibility set is defined in accordance to problem assumption. The second phase involves a comprehensive analysis of undesirable effects in DMUs. The final stage evaluates the impact of undesirable performance on efficiency frontier. With reference to output weak disposability, Kao and Hwang [11] applied a same abatement factor for determining the least extent of the undesirable outputs that cannot inherently avoided. Their common-proportional model can indicate the allowable amount of undesirable output. However, Kao and Hwang [12] highlighted two deficiencies of the existing model. The first deficiency involved neglecting inputs and generating undesirable outputs. The second is assuming constant returns to scale while the subsequent technology advocates being variable. By addressing these limitations, the individual-proportion model proposed allowing for individual reduction factors for each DMU. Compared with Kao and Hwang [11], the individual-proportional model offers a precise distinction between the impact of producing undesirable outputs and the inefficiency related to producing desirable outputs in evaluating production unit efficiency. Hosseinzadeh Lotfi et al. [13] discussed the joint production of desirable and undesirable output in the developed network structure based on SBM models. Their model was examined on the real case study of wheat supply chains to represent the models strength and evaluate the sustainability. Recently, Sun and Zhang [14] explored undesirable output generation in environmental evaluation at thirty provincial level in China. The aim of the paper is to reducing environmental consequences and resource consumption in pig production. The output SBM analysis in this paper highlights the role of slack variables. Reverence to unbalanced economic growth and resource heterogeneity, the results in the paper acknowledge the need for various strategies to tackle productivity inefficiency, resource limitation, and environmental degradation within the pig production in mainland of China.

In some real contexts, both desirable and undesirable inputs are produced, hence, recycling procedure seems crucial. Given the scarcity of natural resources and critical need to protect the resources, employing renewable resources has highlighted the concept of undesirable inputs in recent studies. Gray water is one of the main examples of undesirable inputs. Regarding to global drought and lack of water resources, recycling and reutilization the grey water seems vital. As world Commission on Environment and Development (WCED) indicates the pollution are going to raise and the resources scares. Hence, the concept of undesirable input and recycling industry in efficiency management continues to

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discus. However, there has been some contribution for treating undesirable outputs, but the minimum potential is given to undesirable inputs and its role in economic and production process. In this paper, the attention is given to undesirable inputs, then a slack-based measure (SBM) model is developed to incorporate the simultaneous decreasing of desirable inputs and undesirable inputs increments. The main contribution of this paper is developing a slack-based measure for treating both desirable and undesirable inputs.

The reminder of this study is unfolded as follows. The following section reviews some contributions to deal with undesirable inputs. Section 3 discusses the SBM model for treating undesirable inputs. Finally, conclusion will end the paper.

2 Literature Review

These days, modeling undesirable inputs such as, rotten fruits, gray water and plastic wastes has attracted researches attention. This interest arises from the challenge of increasing level of pollutants and reduction of natural resources, posing challenges for cleaner alternatives. To do so, recycling industries plays a main role which effectively lower energy consumptions and environmental pollutants. Also, transforms the waste into materials promoting environmental sustainability. Nevertheless, modeling undesirable inputs has not been investigated in the literature. The seminal work of Fare and Grosskopf [15] was the pioneering research in presenting an approach named as input weak disposability. Jahanshahloo et.al [16] suggested a non-radial DEA-based model for decreasing undesirable outputs and increasing undesirable inputs, simultaneously. Eyni et.al [17] derived a common set of weights in presence of undesirable inputs. The authors transformed undesirable inputs to be desirable. During 2011-2015 Li et al. [18] suggested a model to analyze the waste treatment efficacy for the solid waste. The model merged from the circular economy structure and revealed that efficiency of the pollution and disposal of the solid waste improved during the 11th and 12th five-year plan. Kordrostami et al. [19] explored the conventional definition of weak disposability considering both undesirable inputs and outputs. Their linear formulation supports the simultaneous proportional reduction in desirable and undesirable outputs and proportional augmentation in favorable and unfavorable inputs for the purpose of efficiency analysis. Liu et al. [20] employed a slack based DEA models by treating undesirable inputs and outputs as desirable outputs and inputs assuming the standard strong disposability assumption. Liu and Xu [21] introduced a one stage SBM model with undesirable outputs integrating the weighing preferences and enhanced the super-SBM model. Their contribution of the model not only distinguishes all DMUs but also determines the projections which are strongly efficient. Asanimoghadam et al. [22] reformulated a SBM model as a linear model for evaluating a two stage network structures with undesirable outputs. Hadi et.al [23] defined an expanded space and new frontier. Based on expanded frontier, a unified efficiency score was defined employing SBM methodology. Maghbouli and Pourhabib yekta [24] proposed a non-radial model ensuring a satiable amount of increasing for undesirable inputs. Their model contributes for simultaneous decreasing for desirable inputs. What's more, the proposed model employs the notation of weak input disposability. Yong Ton et al. [25] proposed a modified SBM model for evaluating hotel performance to elucidate the impact of entropy related variable on efficiency score. Assuming there are K DMUs and for DMU_k, data on the vectors of desirable inputs, undesirable inputs and outputs are represented as

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$x_k(x_{1k},...,x_{Nk}) \geq 0$, $v_k(v_{1k},...,v_{Mk}) \geq 0$ and $y_k(y_{1k},...,y_{Jk}) \geq 0$, respectively. Further assume that $x_k \neq 0$, $v_k \neq 0$ and $y_k \neq 0$. The production possibility set can be represented by:

$$P = \{(x,y) \mid x \geq X\lambda, y \leq \lambda Y \ , \sum_{k=1}^{K} \lambda_k = 1, \lambda \geq 0\}$$

Tone [26] developed the slack-based measure, which is based on two assumptions. Firstly, the measurement is constant with respect to input and output measurement. The second axiom is its uniform reduction according to each input and output. The basic SBM model is defined as follows:

$$\begin{aligned}
&Min \quad \rho_o = \frac{1 + \frac{1}{N}\sum_{n=1}^{N} \frac{s_n^-}{x_{no}}}{1 - \frac{1}{J}\sum_{j=1}^{J} \frac{s_j^+}{y_{jo}}} \\
&s.t. \\
&\sum_{k=1}^{K} \lambda^k x_n^k + s_n^- = x_n^o \quad , n = 1,...,N \\
&\sum_{k=1}^{K} \lambda^k y_j^k - s_j^+ = y_j^o, \quad j = 1,...,J \qquad (1) \\
&\lambda_k, s_n^-, s_j^+ \geq 0
\end{aligned}$$

If $x_{no} = 0$, then the expression $\frac{s_n^-}{x_{no}}$ can be omitted from the evaluation. Furthermore, the measure satisfies in the relation $0 \leq \rho_o \leq 1$. What's more, in the case of $y_{jo} \leq 0$, the expression in the denominator $\frac{s_j^+}{y_{jo}}$ is disadvantageous. Yong Tan et.al [27] developed a Slack-based Measure (SBM) as follows:

$$\begin{aligned}
&Min \quad \rho_o = \frac{1 + \sum_{n=1}^{N} \frac{w_n s_n^+}{P_{no}^-}}{1 - \sum_{j=1}^{J} \frac{w'_j s_j^+}{P_{jo}^+}} \\
&s.t. \\
&\sum_{k=1}^{K} \lambda^k x_n^k + s_n^- = x_n^o \quad , n = 1,...,N \\
&\sum_{k=1}^{K} \lambda^k y_j^k + s_j^+ = y_j^o, \quad j = 1,...,J \qquad (2) \\
&\sum_{n=1}^{N} w_n = 1, \\
&\sum_{j=1}^{J} w'_j = 1 \\
&\sum_{k=1}^{K} \lambda_k = 1 \\
&\lambda_k, s_n^-, s_j^+ \geq 0
\end{aligned}$$

The variable $\lambda = (\lambda^1,...,\lambda^k)$ is referred to as intensity variable. The slack variables s_n^- and s_j^+ are denoted as the deviation variables. The coefficient of w_n and w'_j shows the assigned weights for the deviation variables which the summation do not exceed the unity. Furthermore, $P_{no}^- = x_{no} - \min_k \{x_{nk}\} > 0, n = 1,..., N$ and $P_{jo}^+ = y_{jo} - \max_k \{y_{jk}\} > 0, j = 1,..., J$ (Sharp et al., [28]). Let ρ_o^* denotes the optimal value of Model (1). It is proven that $0 \le \rho_o^* \le 1$(Sharp et al., [28]). The model (1) is linear in terms of unknown intensity variables λ . The objective function supports the Russell measure regarding the concept of input and output slacks. Furthermore, if the unit under evaluation has the efficiency score of unity and declares $s_n^{-*} = s_j^{+*} = 0$, it is presented as efficient unit.

3 Undesirable input estimation: A SBM-DEA model

Model (1) discussed in the previous section, solely considers inputs and outputs. What's more, all conventional DEA models tend to increase outputs and decrease inputs. Whilst, the undesirable inputs are desired to increase for incorporating in production process. As an example for undesirable inputs, in garbage recycling, trash can be considered as an undesirable input. Consequently, it seems logical to increase the unfavorable inputs and at the same time decrease the desirable one. To do so, the notation of slack variables can be modified to address the presence of undesirable inputs in SBM model frameworks. From a computational standpoint, it is rational to consider slack variables as deviations to address the augment amount of undesirable inputs, through developing an optimization- slack-based model. To address this issue, again suppose that there are K DMUs and for DMU_k data on the vectors of desirable inputs, undesirable inputs and outputs are represented as $x_k(x_{1k},...,x_{Nk}) \ge 0$, $v_k(v_{1k},...,v_{Mk}) \ge 0$ and $y_k(y_{1k},...,y_{Jk}) \ge 0$, respectively. Further, assume that $x_k \ne 0, v_k \ne 0$ and $y_k \ne 0$. The production possibility set can be set as follows:

$$P(w) = \{(x,v,y) \,\middle|\, x \ge X\lambda, v \le V\lambda, y \le \mathrm{Y}\lambda, \sum_{k=1}^{K} \lambda^k = 1, \lambda \ge 0\}$$

To examine the efficiency of DMUs with the above proposition, the SBM model (1) can be modified accordingly:

$$Min \quad \rho_o = \frac{1+\sum_{n=1}^{N}\frac{w_n s_n^+}{P_{no}^-}}{1-(\sum_{m=}^{M}\frac{w'_m s_m^-}{P_{mo}^+}+\sum_{j=1}^{J}\frac{w''_j s_j^-}{P_{jo}^+})}$$

$$s.t.$$

$$\begin{aligned}
&\sum_{k=1}^{K}\lambda^k x_n^k \le x_n^o - s_n^+ \quad, n=1,...,N \\
&\sum_{k=1}^{K}\lambda^k v_m^k \ge v_m^o - s_m^-, \quad m=1,...,M \\
&\sum_{k=1}^{K}\lambda^k y_j^k = y_j^o - s_j^-, \quad j=1,...,J \qquad (3) \\
&v_m^o - s_m^- \ge 0 \ , \quad m=1,...,M \\
&y_j^o - s_j^- \ge 0 \quad , \quad j=1,...,J \\
&x_n^o - s_n^+ \ge x_n^o \quad , n=1,..., N \\
&\sum_{n=1}^{N} w_n = 1, \\
&\sum_{m=1}^{M} w'_m = 1, \\
&\sum_{j=1}^{J} w''_j = 1, \\
&\sum_{k=1}^{K}\lambda^k = 1, \\
&\lambda^k, s_n^+, s_j^-, s_m^- \ge 0
\end{aligned}$$

Looking closely to the modified model above, λ^k is referred to intensity vector. The vectors w_n , w'_m and w''_j insures the coefficient for deviations not exceed the unity. As the three first constraints in model (3) show, the variables s_n^+, s_m^- and s_j^- are denoted as deviation variables or slack variable for each unit. The first constraint here modified as $\sum_{k=1}^{K}\lambda^k x_n^k \le x_n^o - s^+{}_n \quad , n=1,...,N$ and refers to desirable input reduction. The second and third constraints admit that the outputs and undesirable inputs intended to increase simultaneously. The requirement for dominance constraints $v_m^o - s_m^- \ge 0$, $m=1,...,M$, $y_j^o - s_j^- \ge 0$ and $x_n^o - s_n^+ \ge x_n^o$ intend the simultaneous utilization of inputs to generate the highest quantity of outputs. Although, the least usage of desirable inputs and the highest consideration of undesirable inputs depicted in the model (3). The objective function also defined as $\rho_o = \frac{1+\sum_{n=1}^{N}\frac{w_n s_n^+}{P_{no}^-}}{1-(\sum_{m=}^{M}\frac{w'_m s_m^-}{P_{mo}^+}+\sum_{j=1}^{J}\frac{w''_j s_j^-}{P_{jo}^+})}$ secures the contribution of both desirable and undesirable inputs in the production process. The numerator term indicates the reduction of desirable input as the first priority, where $P_{no}^- = x_{no} - \min_k \{x_{nk}\}, n=1,...,N$. Furthermore, the

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denominator term guarantees the share of undesirable input and output augment, where $P_{mo}^{+} = v_{mo} - \max_k \{v_{nk}\}, m = 1,...,M$ and $P_{jo}^{+} = y_{jo} - \max_k \{y_{jk}\}, j = 1,...,J$. It can be easily demonstrated that model (2) is always feasible.

Proposition_1. Let ρ_0^* be the optimal value of Model (3), then $\rho_0^* \leq 1$.

Proof. The proof is clear and hence omitted. □

In evaluating with model (3), if $\rho_0^* = 1$ then $s_n^{+*} = s_m^{-*} = s_j^{-*} = 0$, which means that the under evaluated unit is called efficient. The efficiency of the inefficient unit is lower than unity. All in all, the model (2) not only does not overestimate the efficiency of the DMUs but also it makes relatively better discrimination on the DMUs compared with existing models.

4 Numerical Example

In order to highlight on the applicability of the data set, a real data set involved thirty units are evaluated. The data set are related to a chemical experiment which the temperature is undesirable input. Two desirable inputs are utilized to generate two favorable outputs. Desirable inputs are categorized as ionic liquid and the metal material and the temperature analyzed as undesirable inputs. That is to say, higher temperature seems more acceptable during experiment. The productions are time (calculated per minute) and the percentage of the material which yielded. Table1 reports the date set.

Table 1 The Data Set

DMU	Undesirable Input	Desirable Input 1	Desirable Input2	Output1	Output2
1	665	437	1438	2015	14667
2	491	884	1061	3296	1162
3	417	1160	9171	2276	1819
4	302	626	10151	1640	555
5	229	374	8416	9564	3287
6	1083	597	3038	5409	1833
7	1053	870	3342	1651	754
8	740	685	9984	4787	1625
9	845	582	8877	3521	1667
10	517	763	2829	2629	1158
11	664	689	6057	6286	1763
12	313	355	1609	20512	9482
13	1206	851	2352	12654	3786
14	377	926	1222	3188	1087
15	792	203	9698	6477	2121
16	524	1109	7141	7613	4565
17	307	861	4391	1539	763
18	1449	249	7856	1205	496
19	1131	652	3173	8957	1819
20	826	364	3314	16195	3515
21	1357	670	5422	1538	363
22	1089	1023	4388	3099	848
23	652	1049	3665	4412	1516

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24	999	1164	8549	2530	985
25	526	1012	5162	5165	1702
26	218	464	10504	3142	1131
27	1339	406	9365	2120	847
28	231	1132	9958	1320	488
29	1431	593	3552	5807	2503
30	965	262	6211	6977	3757

The results of implementing model (3) on the data set of Table (1), are recorded in Table (2).

Table2 Efficiency assessment for thirty chemical data

		Slacks for Desirable Inputs		Slacks for Outputs		Slack for undesirable Input	
DMU	Model (3)	s_1^{+*}	s_2^{+*}	s_1^{-*}	s_2^{-*}	s_1^{-*}	$v_1 - s_1^{-*}$
1	1	0	0	0	0	0	665
2	1	0	0	0	0	0	491
3	0.57	0	0	693	1456	0	417
4	0.76	0	0	69.51	178.10	0	302
5	0.53	0	0	8359	2791	0	229
6	1	0	0	0	0	0	1083
7	1	0	0	0	0	0	1053
8	0.57	0	0	3582	1129	0	740
9	0.57	0	0	2017.01	1276.20	0	845
10	0.83	0	0	0	240.73	0	517
11	0.56	0	0	4703	1400	0	664
12	1	0	0	0	0	0	313
13	0.66	0	0	10501.30	0	379.86	826.14
14	1	0	0	0	0	0	377
15	1	0	0	0	0	0	792
16	0.52	0	0	6030	4202	0	524
17	1	0	0	0	0	0	307
18	1	0	0	0	0	0	1449
19	0.67	0	0	4336.50	0	0	1131
20	1	0	0	0	0	0	826
21	1	0	0	0	0	0	1357
22	0.67	0	0	1546.92	146.70	0	1089
23	0.61	0	0	2795.49	759.23	0	652
24	0.66	0	0	1325	489	0	999
25	0.58	0	0	3700.32	998.41	0	526
26	0.62	0	0	1937	635	0	218
27	0.70	0	0	915	351	0	1339
28	0.93	0	0	92.26	0	0	231
29	0.87	0	0	3045.31	0	534.37	896.63
30	1	0	0	0	0	0	965

As the Table 2 shows, there are twelve efficient units. Columns third and fourth depicts the slacks for desirable inputs are zero. The fifth and sixth columns in Table (2) shows the slack

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variables for outputs. The seventh column of Table (2) reports the slack variables for the undesirable inputs. This value is recorded zero for all units expect unit #13 and #29. The last column titled by $v_1 - s_1^{-*}$, represents the whole usage of the undesirable inputs. There are only two units, #13 and #29 which reports the quantity of 826.14 and 869.63 as the excess of undesirable inputs in this process. That is to say, these units do not consume the whole amount of undesirable inputs, while the rest of units consumed the whole quantity of unwanted inputs to improve their efficiency. In line with the reported slack variables for the desirable inputs, and as the first constraint indicates, the desirable inputs are kept in their current quantity. The undesirable inputs except units#13 and #29 reports the full consumption of the undesirable inputs. The slack variables for the two remaining units are reported as379.68 and 534.37, respectively. What's more, for efficient units, all output slack variables are reported zero. Concurrently, the shortage of final outputs is reported as the output slacks show. In a nutshell, the proposed model is easy to operate and can reflects excesses or shortfalls in undesirable input consumption. From the computational perspective, the model can determine the efficiency scores of all DMUs and simultaneously offer shortages and surpluses of the inputs and outputs. Hence, the model can save much computational time and improve the efficiency of the evaluated units. Additionally, this study may extend the available techniques for handling undesirable inputs in efficiency and productivity analysis. Although, exploiting the concept of slacks may facilitate the development of even more robust methods for addressing the challenges posed by undesirable inputs in efficiency and productivity analysis.

5 Conclusion

In light of global warming and limited availability of natural resources, the concepts of recycling and renewable energy resources have gained researchers' attraction. In many applications, inputs like plastic wastes and rotten fruits are extremely used in the process of recycling and meet an increasing in their usage. DEA is one of the most employed methodologies in the realm of productivity and efficiency analysis to address the undesirable input challenge. One of the favored approaches involves SBM. Despite the numerous studies in the field of undesirable outputs, the debate on undesirable input persists. This study contributes to the ongoing discourse on undesirable inputs. The modified SBM model offers the reduction of desirable input and simultaneously, tends to increase the usage of undesirable input. The quantity of decrement and increment are achieved with reference to slack or deviation variables in the model. Regarding to assigned positive coefficient, the algebraic format of the objective function ensures simultaneous reductions and augments in desirable and undesirable inputs, respectively. The applicability and strength of the proposed example justified through real examples.

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