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A hybrid two-phase approach for the solid transportation problem under certainty conditions based on goal programming and multi-parametric models

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Abstract The solid transportation model is one of the most useful models in linear programming literature. Hence, this study focuses on a development of solid transportation programming with fuzzy cost coefficients and fuzzy-flexible supply and demand constraints and transportation capacity, which aims to minimize costs. Considering the available resources (capacity of supply centers), the capacity of the vehicle is generally considered as the minimum capacity, and the demand is generally considered as the maximum capacity. To adapt to real conditions, a flexible fuzzy hybrid model is studied for the solid transportation model with supply and demand constraints and flexible fuzzy vehicles. Generally, for such models, supply and demand restrictions and vehicles must first be converted into a real form, and then the associated problem is solved in a deterministic way by using the existing real problem techniques. Furthermore, a combined Goal programming and parametric approach is proposed to obtain the best satisfactory solution. Finally, an example is examined to analyze this approach.

Keyword: Fuzzy Linear Programming, Goal Programming, Membership Function, Multi-parametric Fuzzy Flexible Transportation Problem, Solid Transportation Problem.

1 Introduction

Transportation problem represents one of the most extensively examined topics in the field of linear programming. As a critical component of national infrastructure, transportation systems are present in every country. Its activities have an impact on the nation's economic development, but they also experience several qualitative and quantitative shifts during the growth process. A solid transport issue is a specific instance of the traditional transportation problem where the vehicle capacity is considered to minimize the cost of transporting a given commodity from several sources (factories, manufacturing plants) to multiple destinations (warehouses, retail establishments). Additionally, the quantity of units transported from the origin to the destination affects the cost of the service. However, in the real world, transportation problem variables are determined by unpredictable variables, and because of the unregulated

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aspects of the economic system, we encounter uncertainty and complexity. For researchers to address these challenges in non-deterministic settings, they must do so. The challenges of effective transportation in a hazy environment have also been the subject of extensive research. Since most real-world data is inaccurate and the need for a fuzzy model is increasing, numerous academics have studied solid transportation in a fuzzy environment. Zhang et al. [1] attempted to reduce transportation expenses by analyzing the solid transportation issue, in which resources, demands, and transportation capacity are viewed as fixed fees and inaccurate direct expenditures. Their proposed algorithm uses uncertainty theory and the tab search algorithm for model solution, as shown by a comparison of their technique to prior ones. Lastly, they determine the maximum feasible degree and points of rupture using the recommended algorithm. Dos [2] utilized Type 2 fuzzy parameters to explore issues relating to solid transportation with the goal of cutting costs and time. They employed two approaches to address transportation issues. The first model uses fuzzy type 2 for time and expense, while the second model uses it for cost, time, and every other variable. With the weighted method, global standards, and CV-based reduction approach, this method was addressed. With fuzzy goal programming, Rivaz et al. [3] presented a novel model for solving a multi-objective transportation problem. It is possible that altering the weights in the modified model will result in a variety of solutions. A comparison was then made with several existing methods. In an intuitionistic fuzzy environment, Chhibber et al. [4] investigated a fuzzy solid transportation problem. Using linear, hyperbolic, and exponential membership and non-membership functions, they discovered a Pareto-optimal solution to the multi-objective fixed-charge solid transportation problem. The transportation issue, in which supply, demand, and transportation costs are Fermatean Fuzzy Numbers (FFNs), was addressed by Sahoo [5]. He developed an algorithm for solving the transportation problem with Fermatean fuzzy parameters and used arithmetic operations of Fermatean fuzzy numbers to achieve the best solution. Samanta et al. [6] proposed a two-stage solution to the solid transportation problem: first, from the origin(s) to the nearest station(s); second, from the nearest station(s) to the major destination(s). Depending on the extent of transportation, a fuzzy discount policy was implemented in conjunction with a fuzzy fixed charge and fuzzy unit transportation costs. The model was solved using the Genetic Algorithm (GA). Khan et al. [7] proposed the multi-objective pentagonal fuzzy supply and demand after converting it to its precise form using the decomposition approach, and then they solved it nonlinearly using goal programming. Qiuping et al. [8] created a three-dimensional transit model using Triangular Neutrosophic Numbers (TNN) for supply, demand, transportation capacity, and cost. Then, degree of diversity was used to turn the three-dimensional Neutrosophic transport problem into an interval programming problem, and two basic linear programming models were solved to determine the lower and upper bounds of the ideal solution. Nasseri et al. [9] used the goal programming technique to address a linear programming issue involving flexible fuzzy numbers. They used goal programming to determine the best Pareto solution for the simplified multiparametric, multi-objective, linear programming problem that they had created from the original problem through a series of cuts. They then applied this strategy to the many different kinds of flexible linear programming models, depending on their methodology. In continuation of their study, we analyze a Solid Transportation (ST) challenge using adjustable constraints and triangular fuzzy cost coefficients.

We provide an overview of the studies conducted related to the topic of this study (see in Table1).

Table 1 A review of studies conducted in the last 10 years on solid transport problems under certainty conditions based on goal programming and multi-parameter models.

Author(s)	Year	Description
Yu et al. [10]	2015	An interactive approach is developed to solve the multi-objective transportation problem with interval parameters.
Colapinto et al. [11]	2015	This paper provides a state-of-the-art review on the use of goal programming in multi-criteria decision analysis across engineering, management, and social sciences.
Dalman [12]	2016	The paper presents an uncertain programming model for a multi-item solid transportation problem with uncertain parameters.
Das et al.[13]	2017	A profit-maximizing solid transportation model under a rough interval approach is proposed.
Ahmad & Adhami [14]	2018	This paper proposes a neutrosophic programming approach to solve a multi-objective nonlinear transportation problem with fuzzy parameters.
Chong et al. [15]	2019	The paper presents a goal programming optimization model to improve disaster management and distribution of humanitarian aid under uncertainty.
Das.K et al. [16]	2020	This paper proposes a mathematical model for a green solid transportation system with dwell time under carbon tax, cap, and offset policy, using type-2 fuzzy logic to handle supply and demand uncertainties.
Bakhtavar et al. [17]	2020	This paper presents a multi-objective goal programming model to assess renewable energy-based strategies for net-zero energy communities.
Hussain & Kim [18]	2020	This paper develops a goal-programming-based multi-objective optimization model for off-grid microgrids to minimize energy storage degradation and load/renewable curtailment.
Haque et al. [19]	2021	A two-phase planning approach combining centralized and decentralized decision-making processes is proposed for modeling a multi-echelon, multi-period, decentralized supply chain.
Gupta et al. [20]	2021	This paper presents a multi-objective programming model to optimize transportation and inventory costs in a supply chain network under uncertainty.
Mamashli & Javadian [21]	2021	This paper proposes a multi-objective fuzzy robust programming model to design a sustainable municipal solid waste management network under uncertainty.
Jana et al. [22]	2022	This paper presents a bi-criteria optimization approach using fuzzy goal programming to minimize life cycle energy consumption and CO2 emissions in a biofuel supply chain under uncertainty.
Bind et al. [23]	2023	This paper proposes a solution approach for a sustainable multi-objective multi-item 4D solid transportation problem involving triangular intuitionistic fuzzy parameters.
Kaspar & Kaliyaperumal [24]	2024	This paper presents a bi-objective fixed-charge solid transportation problem that minimizes total transportation cost and time under uncertainty using neutrosophic sets.
Vinotha [25]	2025	This paper proposes improved mathematical models for a multi-objective cold fuzzy solid transportation problem with an extra power source to support freezing during vehicle engine shutdown.

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For this purpose, we utilize a multi-parametric approach and goal programming according to the membership function designed to solve the Solid Transportation problem with Flexible Constraints. The best solutions of the goal programming and the suggested model are then contrasted. We also provide for both suggested scenarios, as well as an algorithm, fuzzy flexibility and a parametric approach, we attain the most degree of satisfaction. This paper has been divided into five parts. Section 2 provides some basic ideas as lemmas and theorems as our major instruments to prepare the research to incorporate the models and approaches. Proposed methods and an algorithm are presented in Section 3. We employ an illustrative example of the algorithm in Section 4. Finally, Section 5 is focused on the study's conclusion.

2 Fuzzy Solid transport issue with flexible circumstances

The solid transportation problem is one of the several forms of transportation models and is a generalization of conventional transportation. Basically, in the solid transportation model, it is assumed that each supplier has a supply capacity of s_i, $i=1,2,...,m$, customers have a demand of d_j, $j=1,2,...,n$, and k vehicles have a capacity of e_k, $1 \le k \le K$, and c_{ijk} the cost of shipping a goods unit from supplier i to customer j responds with the decision variable x_{ijk}. Among realistic transportation models, which seek to reduce the transportation cost, are solid Transportation problems with fuzzy restrictions and fuzzy costs. Vehicle $k-th$ moves the item from m suppliers to n clients. Goods from their source of supply to the capacity vehicle transfer items. Demand is now focused online since the COVID-19 pandemic has altered supply. As a result, the terms supply, demand, and transportation no longer have a rigid definition. A suitable framework for the mathematical model in this situation should be created that reflects the actual circumstances of the problem. The subsequent model considers all limitations to be fuzzy flexible data, and uses the symbols $\lesssim$ for "lower than or equal to" and $\gtrsim$ for "upper than or equal to". The mathematical model is displayed below:

<u>*Model I:*</u>

$$Min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{c}_{ijk} x_{ijk} \tag{1}$$

$$\text{s.t.} \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \lesssim s_i , \quad \forall i \tag{2}$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \gtrsim d_j , \quad \forall j \tag{3}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \lesssim e_k , \quad \forall k \tag{4}$$

$$x_{ijk} \ge 0, \quad i=1,2,...,m , \quad j=1,2,...,n , \quad k=1,2,...,K . \tag{5}$$

For Equation (3), the fuzzy membership function is:

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$$\mu_j(x)=\begin{cases}1, & x\le d_j\\ 1-\dfrac{d_j-x}{q_j}, & d_j\le x\le d_j+q_j\ .\\ 0, & x\ge d_j+q_j\end{cases} \tag{6}$$

For equations (2) and (4), Nasseri et al. [26] introduced the fuzzy constraint of membership function as:

$$\mu_i(x)=\begin{cases}1, & x\le s_i\\ 1-\dfrac{x-s_i}{p_i}, & s_i\le x\le p_i+s_i\ .\\ 0, & x\ge p_i+s_i\end{cases} \tag{7}$$

Given the absence of a precise description for conditions (2), (3), and (4), the equivalent model is built to solve this model using a parametric approach.
Let that the tolerance of the $i-th$ constraint of supply is p_i. main of tolerance p_i and its flexibility in its range, we possess $\sum_{j=1}^{n}\sum_{k=1}^{K}x_{ijk}\lesssim s_i$, $i=1,2,...,m$, and $\sum_{j=1}^{n}\sum_{k=1}^{K}x_{ijk}\le s_i+\theta p_i$, which $\theta\in[0,1]$. The tolerance of the $j-th$ constraint of demand is q_j. Based on tolerance q_j and its flexibility in its range, we give:

$$\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}\gtrsim d_j \text{ and } \sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}\ge d_j-\theta q_j,\quad j=1,2,...,n,$$

and the tolerance of the $k-th$ constraint of vehicle is r_k. Main of tolerance r_k and its flexibility in its range, we have:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}\lesssim e_k \text{ , and} \sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}\le e_k+\theta r_k,\quad k=1,2,...,K.$$

The following lemmas can be useful for our discussion.

Lemma 1. The constraint $\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}\gtrsim d_j$ is equivalent to the constraint $\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}\ge d_j-\theta q_j$, for $\theta\in[0,1]$.

Proof. Each feasible solution x_{ijk} which is satisfied in $\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}\gtrsim d_j$ is indeed a fuzzy set with the following membership function:

$$\mu_j\{t_j\}=\begin{cases}1, & t_j\le d_j\\ 1-\dfrac{d_j-t_j}{q_j}, & d_j\le t_j\le d_j+q_j\ ,\\ 0, & t_j\ge d_j+q_j\end{cases} \tag{8}$$

where $t_j = \sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}$, $j = 1,...,n.$

We called this feasible solution as $\beta -$ feasible solution of this constraint.

Now, we will consider the three following cases:

A) If $\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} - d_j \le 0$, then the $j-th$ Constraint is held and equal to 1.

B) If $0 \le \sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} - d_j \le q_j$ then the membership function is monotonically increasing for $j-th$ Constraint. The degree (level) of satisfaction $j-th$ constraint is decreasing.

C) If $\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} - d_j \ge q_j$, the tolerance accepted range is larger than the value which is determined by the decision-maker, Thus, the $j-th$ Constraint has been completely violated, and its membership function is equal to 0.

Hence, because the membership function is continuous, the right-hand sides of the flexible constraint form d_j to $d_j - q_j$ base on the continuous value for θ, from $\theta = 0$ to $\theta = 1$ can be achieved. Therefore, the fuzzy flexible relation can be shown by the following equivalent parametric form, $d_j(\theta) = d_j - \theta q_j$ where $\theta \in [0,1]$.

Lemma 2. Problem I is equivalent to the following multi-parametric linear programming problem:

Model II:

$$Min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} \tilde{c}_{ijk} x_{ijk} \tag{9}$$

$$\text{s.t. } \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le s_i + (1-\alpha_i)p_i , \quad i = 1,2,...,m, \tag{10}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge d_j - (1-\beta_j)q_j , \quad j = 1,2,...,n, \tag{11}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k + (1-\gamma_k)r_k , \quad k = 1,2,...,K , \tag{12}$$

$$x_{ijk} \ge 0, \quad i = 1,2,...,m , \quad j = 1,2,...,n , \quad k = 1,2,...,K , \quad 0 \le \alpha, \beta, \delta \le 1. \tag{13}$$

Proof. To establish the claim, it suffices to show that equations (2), (3), and (4) are respectively equivalent to (10), (11), and (12). Since the procedure is analogous in all three cases, we will focus solely on the second one and leave the remaining cases to the reader. The conclusion follows directly from Lemma 1.

Thus, the main problem can now be reformulated in the following equivalent form:

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Model III:

$$Min\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\tilde{c}_{ijk}x_{ijk} \tag{14}$$

$$\text{s.t. } X \in X_{\alpha} \tag{15}$$

$$x_{ijk} \geq 0,\quad i=1,2,...,m,\quad j=1,2,...,n,\quad k=1,2,...,K,\quad 0\leq \alpha,\beta,\gamma \leq 1. \tag{16}$$

where the set of all feasible solutions of the problem is defined as follows.

Definition 1. Suppose $\alpha=(\alpha_1,...,\alpha_m,\beta_1,...,\beta_n,\gamma_1,...,\gamma_k)\in[0,1]^{m+n+k}$ and

$$X_{\alpha}=\left\{ x_{ijk}\in\mathbb{R} \left| \begin{array}{c} \sum_{j=1}^{n}\sum_{k=1}^{K}x_{ijk}\leq s_i+(1-\alpha_i)p_i, \sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}\geq d_j-(1-\beta_j)q_j,\ \sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}\leq e_k+(1-\gamma_k)r_k \\ x_{ijk}\geq 0,\quad i=1,2,...,m,\quad j=1,2,...,n,\quad k=1,2,...,K,\quad 0\leq\alpha,\beta,\delta\leq 1. \end{array} \right. \right\}$$

So that $X=(x_{ijk})\in X_{\alpha}$, $x_{ijk}\in\mathbb{R}$ is an $\alpha-\beta-\gamma$ feasible solution for Modell III.

3 Transforming model from a flexible fuzzy model into an exact multi-parametric model

Consider the problem with supply constraints in the flexible range of $[s_i, s_i+p_i]$, demand constraints in the flexible range of $[d_j-q_j, d_j]$ and the constraint of vehicle's capacity in the range of $[e_k, e_k+r_k]$. By utilizing the membership function for cost coefficients, we can transform the problem into a Multi-Parametric Solid Transportation (MPST) problem. We assume that the cost coefficients of the objective function are represented as triangular fuzzy numbers. Due to the fuzzy nature of these coefficients, solving the problem directly is not feasible. Therefore, we propose converting it into a crisp objective function using methods like Yager's approach, as referenced in [27]:

Model IV:

$$Min\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\Re(\tilde{c}_{ijk})x_{ijk} \tag{17}$$

$$\text{s.t. } \sum_{j=1}^{n}\sum_{k=1}^{K}x_{ijk}\leq s_i+(1-\alpha_i)p_i,\quad i=1,2,...,m, \tag{18}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}\geq d_j-(1-\beta_j)q_j,\quad j=1,2,...,n, \tag{19}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}\leq e_k+(1-\gamma_k)r_k,\quad k=1,2,...,K, \tag{20}$$

$$x_{ijk}\geq 0,\quad i=1,2,...,m,\quad j=1,2,...,n,\quad k=1,2,...,K,\quad 0\leq\alpha,\beta,\delta\leq 1. \tag{21}$$

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where $\Re(\tilde{c}_{ijk})$ corresponding crisp value of the cost coefficient is determined using a linear ranking function. By solving this problem, we obtain the optimal values for the decision variables as well as the optimal value of the objective function.

3.1 Two-Step of Multi-Parametric Method

In this section, we introduce a new method for solving the flexible fuzzy solid transportation problem by employing a multi-parametric approach. After solving Problem IV, we obtain the optimal solution as $(x^*,\alpha^*,\beta^*,\gamma^*)$ for the decision variables and the corresponding value of the objective function z^*. To further maximize the degree of satisfaction, we proceed to solve the following problem.

The multi-parametric linear programming problem is formulated as Model V:

<u>*Model V:*</u>

$$Max\sum_{i=0}^{m}\alpha_i+\sum_{j=1}^{n}\beta_j+\sum_{k=1}^{K}\gamma_k \qquad (22)$$

$$\text{s.t. } \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}c_{ijk}x_{ijk}\le z^*+(1-\alpha_0)p_0 \qquad (23)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K}x_{ijk}\le s_i+(1-\alpha_i)p_i,\quad i=1,2,...,m, \qquad (24)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}\ge d_j-(1-\beta_j)q_j,\quad j=1,2,...,n, \qquad (25)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}\le e_k+(1-\gamma_k)r_k,\quad k=1,2,...,K, \qquad (26)$$

$$x_{ijk}\ge 0,\quad i=1,2,...,m,\quad j=1,2,...,n,\quad k=1,2,...,K,\quad 0\le\alpha,\beta,\gamma\le 1, \qquad (27)$$

$$\alpha_i^*\le\alpha_i\le 1,\quad \beta_j^*\le\beta_j\le 1,\quad \gamma_k^*\le\gamma_k\le 1. \qquad (28)$$

Solving the second step yields the best solution x^{**}, with an objective function value as z^{**}, and also the degree of efficiency as $(\alpha^{**},\beta^{**},\gamma^{**})$ the second phase produces the highest degree of satisfaction. The following algorithm is given to solve the major transportation problem.

Algorithm (STPFFC Solver):

Assumption: Consider the Solid Transportation Problem with Fuzzy Flexible Constraints (STPFFC), in which the model incorporates a set of parameters: $s_i, d_j, e_k, p_i, q_j, r_k$.

Step 1: A linear ranking function is employed to defuzzify the cost coefficients, thereby obtaining their corresponding crisp values, as demonstrated below:

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$$Min\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\tilde{c}_{ijk}x_{ijk} \text{ which is equivalent to } Min\mathcal{R}\left(z\left(\tilde{c},x\right)\right)=Min\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\mathcal{R}\left(\tilde{c}_{ijk}\right)x_{ijk}.$$

Step 2: The problem is reformulated into a multi-parametric framework. By solving Model IV, one obtains the optimal values of the satisfaction parameters as $\alpha_i^*,\beta_j^*,\gamma_k^*$, the corresponding optimal value of the objective function z^*, and x^* the associated optimal feasible solution.

Step 3: Model V is solved using the corresponding results from Model IV, incorporating the values of the satisfaction parameters denoted by $\alpha_i^*,\beta_j^*,\gamma_k^*$ as determined by an expert decision maker in the first phase, based on their maximum degree of satisfaction. This process yields the optimal solution x^{**} and the corresponding optimal value of the objective function.

In the following section, a numerical example is presented to illustrate the proposed approach. All models are solved using LINGO optimization software.

4 Numerical examples

Consider the parameters of the Solid Transportation (ST) problem as follows:

i : Index of supplies (suppliers; $i=1,2,3$).

j : Index of demands (customers; $j=1,2,3$).

k : Capacity of vehicles ($k=1,2,3$).

c_{ijk} : Unit transportation cost of shipping of product from source i to customer j by using vehicle k ($i=1,2,3$, $j=1,2,3$, $K=1,2,3$).

x_{ijk} : Decision variable representing the number of units transported from source i to customer j by using vehicle k ($i=1,2,3$, $j=1,2,3$, $K=1,2,3$).

z : Total transportation cost, i.e., the objective function to be minimized.

Table 2 Demand and transportation data

	K=1			K=2			K=3			
$i \setminus j$	3	2	1	3	2	1	3	2	1	
1	(2,3,4)	(3,6,9)	(8,9,10)	(6,7,8)	(8,9,10)	(10,12,14)	(5,7,9)	(5,7,9)	(7,9,11)	8
2	(5,6,7)	(7,9,11)	(4,5,6)	(6,8,10)	(8,11,14)	(5,6,7)	(5,6,7)	(1,3,5)	(3,5,7)	9
3	(1,1,1)	(1,2,3)	(1,2,3)	(8,9,10)	(6,7,8)	(1,2,3)	(1,3,5)	(6,7,8)	(1,1,1)	5
		10			5			6		

$$\sum_{i=1}^{3}\sum_{k=1}^{3}x_{i1k}\geq 7,\quad \sum_{i=1}^{3}\sum_{k=1}^{3}x_{i2k}\geq 8,\quad \sum_{i=1}^{3}\sum_{k=1}^{3}x_{i3k}\geq 6.$$

Based on the available fuzzy data, we have the valuable issue of the transportation model with fuzzy costs and flexible constraints. This issue will be solved through the algorithm:

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$$\begin{aligned}
&Min\\
&Z = 9x_{111} + 8x_{121} + 3.5x_{131} + 12x_{112} + 9x_{122} + 7x_{123} + 9x_{113} + 7x_{123} + 7x_{133} + 5x_{211} + 9x_{221} + 6x_{231} + 6x_{212} +\\
&11x_{222} + 8x_{223} + 5x_{213} + 3x_{223} + 6x_{233} + 2x_{311} + 2x_{321} + x_{331} + 2x_{312} + 7x_{322} + 9x_{323} + x_{313} + 7x_{323} + 3x_{333}\\
&s.t. x_{111} + x_{121} + x_{131} + x_{112} + x_{122} + x_{132} + x_{113} + x_{123} + x_{133} \lesssim 8\\
&x_{211} + x_{221} + x_{231} + x_{212} + x_{222} + x_{232} + x_{213} + x_{223} + x_{233} \lesssim 9\\
&x_{311} + x_{321} + x_{331} + x_{312} + x_{322} + x_{332} + x_{313} + x_{323} + x_{333} \lesssim 5\\
&x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} \gtrsim 7\\
&x_{112} + x_{212} + x_{312} + x_{122} + x_{222} + x_{322} + x_{132} + x_{232} + x_{332} \gtrsim 8\\
&x_{113} + x_{213} + x_{313} + x_{123} + x_{223} + x_{323} + x_{133} + x_{233} + x_{333} \gtrsim 6\\
&x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} \lesssim 10\\
&x_{112} + x_{212} + x_{312} + x_{122} + x_{222} + x_{322} + x_{132} + x_{232} + x_{332} \lesssim 5\\
&x_{113} + x_{213} + x_{313} + x_{123} + x_{223} + x_{323} + x_{133} + x_{233} + x_{333} \lesssim 6\\
&x_{ijk} \ge 0, \quad i = 1,2,3, \quad j = 1,2,3, \quad k = 1,2,3, \quad 0 \le \alpha_i \le 1, \quad 0 \le \beta_j \le 1, \quad 0 \le \gamma_k \le 1.
\end{aligned}$$

Step 1: In the process of solving of the above problem, we use the Yager's ranking function to obtain the associate crisp values of fuzzy numbers. Also, we use the suggested process in section 3 to make the following multi-parametric linear programing.

$$\begin{aligned}
&Min\\
&z = 9x_{111} + 8x_{121} + 3.5x_{131} + 12x_{112} + 9x_{122} + 7x_{123} + 9x_{113} + 7x_{123} + 7x_{133} + 5x_{211} + 9x_{221} + 6x_{231} + 6x_{212} +\\
&11x_{222} + 8x_{223} + 5x_{213} + 3x_{223} + 6x_{233} + 2x_{311} + 2x_{321} + x_{331} + 2x_{312} + 7x_{322} + 9x_{323} + x_{313} + 7x_{323} + 3x_{333}\\
&s.t.\\
&x_{111} + x_{121} + x_{131} + x_{112} + x_{122} + x_{132} + x_{113} + x_{123} + x_{133} \le 8 + 3(1-\alpha_1)\\
&x_{211} + x_{221} + x_{231} + x_{212} + x_{222} + x_{232} + x_{213} + x_{223} + x_{233} \le 9 + 4(1-\alpha_2)\\
&x_{311} + x_{321} + x_{331} + x_{312} + x_{322} + x_{332} + x_{313} + x_{323} + x_{333} \le 5 + 2(1-\alpha_3)\\
&x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} \ge 7 - 3(1-\beta_1)\\
&x_{112} + x_{212} + x_{312} + x_{122} + x_{222} + x_{322} + x_{132} + x_{232} + x_{332} \ge 8 - 3(1-\beta_2)\\
&x_{113} + x_{213} + x_{313} + x_{123} + x_{223} + x_{323} + x_{133} + x_{233} + x_{333} \ge 6 - 2(1-\beta_3)\\
&x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} \le 10 + 4(1-\gamma_1)\\
&x_{112} + x_{212} + x_{312} + x_{122} + x_{222} + x_{322} + x_{132} + x_{232} + x_{332} \le 5 + 2(1-\gamma_2)\\
&x_{113} + x_{213} + x_{313} + x_{123} + x_{223} + x_{323} + x_{133} + x_{233} + x_{333} \le 6 + 2(1-\gamma_3)\\
&x_{ijk} \ge 0, \quad i = 1,2,3, \quad j = 1,2,3, \quad k = 1,2,3, \quad \alpha_i^* < \alpha_i \le 1, \quad \beta_j^* < \beta_j \le 1, \quad \gamma_k^* < \gamma_k \le 1.
\end{aligned}$$

Step 2: Use Lingo software to solve the above problem based on the various values of α, β, γ. The following table has the results.

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Table 3 The optimal values of the objective function based on different values of α, β, γ .

α, β, γ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Z	25.5	29.55	33.6	37.65	41.7	45.75	49.8	54.3	59.2	64.1	73

Note that for
$\alpha_1^* = 0.5,\ \ \alpha_2^* = 0.5,\ \ \alpha_3^* = 0.5,\ \ \beta_1^* = 0.5,\ \ \beta_2^* = 0.5,\ \ \beta_3^* = 0.5,\ \ \gamma_1^* = 0.5,\ \ \gamma_2^* = 0.5, \gamma_3^* = 0.5.$

We have the objective function $z_1^* = 45.75$.

Step 3: The goal multiparametric linear programming problem is presented as follows:

$$Min\ D = d_1^- - d_1^+ + d_2^- - d_2^+$$

$s.t.$

$$9x_{111} + 8x_{121} + 3.5x_{131} + 12x_{112} + 9x_{122} + 7x_{123} + 9x_{113} + 7x_{123} + 7x_{133} + 5x_{211} + 9x_{221} + 6x_{231} + 6x_{212} + 11x_{222} + 8x_{223}$$
$$+5x_{213} + 3x_{223} + 6x_{233} + 2x_{311} + 2x_{321} + x_{331} + 2x_{312} + 7x_{322} + 9x_{323} + x_{313} + 7x_{323} + 3x_{333} + d_1^- - d_1^+ = 42.75$$
$$\alpha_1 + \alpha_2 + \alpha_3 + \beta_1 + \beta_2 + \beta_3 + \gamma_1 + \gamma_2 + \gamma_3 + d_2^- - d_2^+ = 9$$
$$x_{111} + x_{121} + x_{131} + x_{112} + x_{122} + x_{132} + x_{113} + x_{123} + x_{133} \le 8 + 3(1-\alpha_1)$$
$$x_{211} + x_{221} + x_{231} + x_{212} + x_{222} + x_{232} + x_{213} + x_{223} + x_{233} \le 9 + 4(1-\alpha_2)$$
$$x_{311} + x_{321} + x_{331} + x_{312} + x_{322} + x_{332} + x_{313} + x_{323} + x_{333} \le 5 + 2(1-\alpha_3)$$
$$x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} \ge 7 - 3(1-\beta_1)$$
$$x_{112} + x_{212} + x_{312} + x_{122} + x_{222} + x_{322} + x_{132} + x_{232} + x_{332} \ge 8 - 3(1-\beta_2)$$
$$x_{113} + x_{213} + x_{313} + x_{123} + x_{223} + x_{323} + x_{133} + x_{233} + x_{333} \ge 6 - 2(1-\beta_3)$$
$$x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} \le 10 + 4(1-\gamma_1)$$
$$x_{112} + x_{212} + x_{312} + x_{122} + x_{222} + x_{322} + x_{132} + x_{232} + x_{332} \le 5 + 2(1-\gamma_2)$$
$$x_{113} + x_{213} + x_{313} + x_{123} + x_{223} + x_{323} + x_{133} + x_{233} + x_{333} \le 6 + 2(1-\gamma_3)$$
$$x_{ijk} \ge 0,\quad i = 1,2,3,\quad j = 1,2,3,\quad k = 1,2,3,\quad \alpha_i^* < \alpha_i \le 1,\quad \beta_j^* < \beta_j \le 1,\quad \gamma_k^* < \gamma_k \le 1$$

The optimal values are those derived from solution of the model:
$d_1^- = 0,\ \ d_1^+ = 182.75,\ \ d_2^- = 3,$ and $d_2^+ = 0$ as deviations.

The optimal solution of the objective function is equal to:
$z^{**} = 228.5 - 182.75 = 45.75$,
with $\alpha_1 = 0.5,\ \ \alpha_2 = 0.5,\ \ \alpha_3 = 1,\ \ \beta_1 = 1,\ \ \beta_2 = 1,\ \ \beta_3 = 0.5,\ \ \gamma_1 = 0.5,\ \ \gamma_2 = 0.5,\ \ \gamma_3 = 0.5$.

According to the results in Table 4.2, raising each parameter from zero to one leads to an increase in the objective function's value, as the function itself decreases. This confirms the accuracy of the tested outputs. These findings underscore the robustness and adaptability of the proposed model in addressing transportation problems characterized by data uncertainty and constraint flexibility. Furthermore, the model's capacity to explicitly capture the sensitivity of the optimal solution to parameter variations enhances its practical relevance, particularly for real-world decision-making scenarios in which ambiguity and tolerance in constraints are inevitable.

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5 Conclusion and Future works

In this study, an advanced extension of the solid transportation model presented by incorporating fuzzy cost parameters and flexible fuzzy constraints related to supply, demand, and vehicle capacity. The approach addresses the uncertainties often encountered in real transportation systems by modeling supply as a flexible lower bound, demand as a flexible upper bound, and transportation capacity as a variable constraint. To transform the fuzzy environment into a solvable form, membership functions were applied, and the model was converted into a deterministic equivalent. A combined method based on goal programming and a multi-parametric strategy was then employed to derive the most satisfactory solution, considering the trade-offs between conflicting goals. The proposed methodology, which integrates goal programming with multi-parametric models, addresses the solid transportation problem under certainty conditions while incorporating fuzzy flexible constraints. The results, as reported in the corresponding tables, reveal that the application of fuzzy flexible constraints ensures that an increase in the degree of desirability does not adversely affect the objective function value. This finding affirms the appropriateness of the proposed method. Furthermore, the outcomes indicate that the developed hybrid model distinguishes itself from existing approaches to the solid transportation problem by incorporating conditions and assumptions that more accurately reflect real-world circumstances, while demonstrating superior performance relative to several established models. Finally, the effectiveness of the proposed model was validated through a numerical example, confirming its suitability for complex and uncertain decision-making scenarios in transportation logistics. In future research, interval constraints can also be used. Moreover, the objective function in Model IV can be replaced by an equivalent multi-objective problem, and then, in the next process, by using a well-known approach for solving the multi-objective problems, a weighted method can be used. Also, extending the model to multi-stage systems and considering stochastic demand scenarios are among the issues that future research could explore.

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