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A novel mixed-integer linear programming model for identifying the most efficient DMU in data envelopment analysis

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Abstract Finding the most efficient decision-making unit (DMU) in Data Envelopment Analysis (DEA) can provide deeper insights into the performance of efficient units. Over the years, several methods have been proposed to improve the ability of DEA models to distinguish between DMUs, often by aiming for stronger ranking capabilities. In this study, we present an enhanced model based on mixed-integer linear programming (MILP) to identify the most efficient DMU. The model is designed so that only one DMU can achieve an efficiency score equal to one, while all others receive scores strictly less than one. This structure enhances the model's ability to fully rank all units, while using fewer constraints compared to traditional full-ranking models. To demonstrate its effectiveness and compare it with two well-known models, the proposed model is applied to two real-world examples from the literature. These findings show that proposed model clearly outperforms of the reviewed models—not just in theory, but in practice too.

Keyword: Data Envelopment Analysis, Most Efficient DMU, Mixed Integer Linear Programming, Ranking.

1 Introduction

Data envelopment analysis (DEA) is a mathematical approach introduced by Charnes et al. [1] to assess the relative efficiency of a homogeneous group of decision-making units (DMUs). DEA successfully divides DMUs into two categories; efficient DMUs and inefficient DMUs. It is not possible to rank efficient units based on their efficiency score, one. Therefore, many models have been examined in the DEA literature to rank these units. Each of these methods ranks efficient units from different perspectives. Among these methods, we can mention cross efficiency ranking methods [2-12], super efficiency ranking methods [11], the common set of weights (CWS) methods[13-21], benchmark ranking methods [22], the linear discriminant analysis [23], discriminant analysis of ratios [24, 25].

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In some cases, the decision-maker must select only one DMU among efficient DMUs which is called the most efficient DMU. Therefore, several studies have been done to find the most efficient unit in DEA. To evaluate the most efficient DMU in advanced manufacturing technology (AMT), Karsak and Ahiska [12] proposed an integrated multi-criteria decision-making (MCDM) DEA model. For overcoming the convergence of the proposed model in [12], Amin et al. [26] modified and improved it. Amin and Toloo [27] proposed a new mixed integer linear programming (MILP) model based upon CSW to find the most efficient unit. For selecting the most BCC-efficient DMU, Toloo and Nalchigar [28] extended this model into variable returns to scale (VRS) situation. Amin et al. [26], Amin [29] introduced a new mixed integer non-linear programming (MINLP) model for overcoming some drawbacks of previous MILP models. Although their models can determine the most efficient unit, they are non-linear and therefore difficult to solve.

Toloo et al. [30] revealed that the problem of finding the most association rule by considering multiple criteria in data mining is an important task and designed an algorithm for prioritizing association rules. This algorithm has some drawbacks that is mentioned and improved by Toloo and Nalchigar [31]. By maximizing the minimum possible distance between a selected unit and the next ranked unit, Foroughi [32] proposed a new MILP model to find the most efficient unit. This approach can also be extended to rank all extreme efficient DMUs. By removing additional constraints in Foroughi's model, Wang and Jiang [33] proposed a new model to identify the most efficient DMU, which is less complex than Foroughi's model. Toloo [34] proposed a new MILP model for selecting the most efficient DMU without explicit input and utilized this model to determine the best efficient professional tennis player.

Toloo [35] excluded the non-Archimedean epsilon and proposed a new model with fewer computations to find the most efficient DMU. Toloo [36] showed that in the supply chain, the selecting and full-ranking of suppliers with imprecise data is a very important issue. Using the CSW method, Toloo [37] introduced a new minimax MILP model for selecting the most efficient DMU. Lam [38] introduced a new MILP model similar to that of the super-efficiency model for directly discovering the most efficient DMU.

Salahi and Toloo [39] illustrated that Lam’s model may be infeasible, and they proposed a modified model to cope with this issue. Toloo [40] proposed a method for finding the most cost-efficient DMU by utilizing the proposed approach in [41] when the prices are fixed and known. Toloo and Salahi [42] developed a new two-step MINP model involving the epsilon which identifies a single efficient DMU whose efficiency score is strictly greater than one. Both non-linear models can be turned into linear models. Based on the proposed model in [42], Özsoy, Örkcü [43] proposed a mixed integer programming model without epsilon with one step for selecting the most efficient unit. This model, with fewer constraints than the model in [42], determines exactly one DMU as the most efficient, with an efficiency score greater than one, while the other DMUs have efficiency scores strictly below one.

Ebrahimi et al. [44] analyzed the two-step method proposed by [38, 39] to identify the most efficient units. They mathematically proved that the first-step model in [39] is sufficient to determine the best DMU, rendering the second step redundant. They improved the first-step model by proposing a modified version, demonstrating that it could identify the best DMU with considerably lower computational effort.

Noori et al. [45] explored the link between the most efficient and extremely efficient units. Their findings showed that an extremely efficient unit can also be considered the most efficient, and the reverse holds true as well. This implies that the defining properties of extremely efficient units are essentially the same as those of the most efficient units.

The major contribution of this study is the development of a single-step MILP model that effectively identifies the most efficient DMU in DEA. Unlike existing models, the proposed approach guarantees that only one DMU attains an efficiency score of one, while all others receive strictly lower scores. This formulation enhances the model's discriminatory power, simplifies its structure, and reduces the number of constraints leading to better computational efficiency. A comparative analysis with two famous models, using benchmark DEA case studies, highlights the superior performance of the proposed method.

The rest of the paper is structured as follows. Section 2 gives a brief overview of two well-known models for identifying the most efficient DMU. In Section 3, we introduce our proposed MILP model and explain how it works. Section 4 presents two numerical examples to illustrate how the model can be applied in practice and to highlight its effectiveness. Finally, Section 5 wraps up the paper with concluding remarks and suggestions for future research.

2 Preliminaries

Suppose there are n DMUs to be evaluated, $DMU_j\,(j=1,2,...,n)$, each using m inputs to produce s outputs. Let $x_{ij}\,(i=1,2,...,m)$ and $y_{rj}\,(r=1,2,...,s)$ represent the input and output values of DMU_j, respectively. Mathematically, the efficiency score of a specific DMU, DMU_p, can be calculated as [46]:

$$e_p=\frac{\sum_{r=1}^{s}u_r y_{rp}}{\sum_{i=1}^{m}v_i x_{ip}},\ j=1,2,...,n.$$

Where $v_i\,(i=1,2,...,m)$ and $u_r\,(r=1,2,...,s)$ be the weights of i *th* input and r *th* output, respectively.

Sueyoshi [22] proposed the following linear programming model for obtaining optimal weights and estimating the best relative efficiency score of DMU_p, under constant returns to scale (CRS):

$$\begin{aligned}
e_p^* = Max\ & \sum_{r=1}^{s} y_{rp} \\
s.t.\ & \sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}\le 0,\ j=1,2,...,n, \\
& \sum_{i=1}^{m}v_i x_{ip}=1, \\
& u_r \ge \frac{1}{(m+s)\max_j\{y_{rj}\}},\ r=1,2,...,s, \\
& v_i \ge \frac{1}{(m+s)\max_j\{x_{ij}\}},\ i=1,2,...,m,
\end{aligned} \tag{1}$$

Let v_i^* and u_r^* be the optimal weights of *ith* input and *rth* output in model (1), respectively. The DMU_p is efficient if and only if ($e_p^* = 1$ and $\sum_{r=1}^{s} u_r^* y_{rp} - \sum_{i=1}^{m} v_i^* x_{ip} = 0$), otherwise it is inefficient.

Definition 1 Let $\left(u_r^*(r=1,2,...,s), v_i^*(i=1,2,...,m)\right) > 0$ be optimal solution of model (1), such that $\sum_{r=1}^{s} u_r^* y_{rp} - \sum_{i=1}^{m} v_i^* x_{ip} = 0$ and moreover $\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} < 0, j \neq p$, then DMU_p is called the most (best) efficient unit [37].

Theorem 1 Any extremely efficient DMU is a candidate for being the most efficient.

Proof see [45].

Remark 1 If the production possibility set (PPS) does not contain any extremely efficient DMUs, then it does not contain any most efficient ones.

2.1 The Wang and Jiang (2012)'s model

Wang and Jiang [33] proposed the following MILP model for finding the most CCR-efficient DMU under CRS.

$$\begin{aligned} Min \ & \sum_{i=1}^{m} v_i \left(\sum_{j=}^{n} x_{ij}\right) - \sum_{i=1}^{m} u_r \left(\sum_{j=}^{n} y_{ij}\right) \\ s.t. \ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le I_j, \ j = 1,2,...,n, \\ & \sum_{j=1}^{n} I_j = 1, \\ & u_r \ge l_r^u, \ r = 1,2,...,s, \\ & v_i \ge l_i^v, \ i = 1,2,...,m, \\ & I_j \in \{0,1\}, \quad j = 1,2,....,n, \end{aligned} \tag{2}$$

Where $l_r^u = ((m+s)\max_j\{y_{rj}\})^{-1}$ and $l_i^v = ((m+s)\max_j\{x_{ij}\})^{-1}$ are lower bounds borrowed from model (1). Model (2) is feasible and its objective is to maximize the overall efficiency of all of the DMUs. In this model, if $I_p^* = 1$ then $\sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} \le 1$, hence, model (2) allows efficiency value of DMU_p to be larger than one and, on the other hand for $I_j^* = 0 (j \neq p)$, the efficiency value of DMU_j is less than or equal to one due to constraint $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0$. So, in model (2), DMU_p is determined as the most efficient DMU if and only if $I_p^* = 1$.

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2.2 The Özsoy, Örkcü [43]'s model

Inspired by the Toloo and Salahi [42]'s model, Özsoy, Örkcü [43] presented a new single-stage MINLP model to find the most efficient DMU as follows:

$$
\begin{aligned}
& h^* = Max \quad h \\
& s.t. \\
& \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le MI_j - h(1-I_j), \quad j = 1,2,...,n, \\
& \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \ge hI_j - M(1-I_j), \quad j = 1,2,...,n, \\
& \quad \sum_{i=1}^{n} I_j = 1, \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (3) \\
& \quad I_j \in \{0,1\}, \ j = 1,2,...,n, \\
& \quad u_r \ge ((m+s)\max_j\{y_{rj}\})^{-1}, \quad r = 1,2,...,s, \\
& \quad v_i \ge ((m+s)\max_j\{x_{ij}\})^{-1}, \quad i = 1,2,...,m,
\end{aligned}
$$

Where M is a large positive number. The minimum possible interval between the first two top-ranking DMUs is [-h^*, h^*], where h^* is strictly positive. Model (3) identifies exactly one DMU ($DMU_p, I_p^* = 1$) as the most efficient, with an efficiency score greater than one, while all other DMUs $(DMU_j, j \ne p)$ have efficiency scores strictly less than one.

Model (3), by using the continuous variable $z_j = hI_j$ and adding the following constraints, is transformed into a MILP model [42, 43].

$$0 \le z_j \le MI_j$$

$$z_j \le h \le z_j + M(1-I_j)$$

3 The proposed model

In this section, we propose the following model for determining the most efficient DMU:

$$
\begin{aligned}
& Max \quad \sum_{j=1}^{n} s_j \\
& s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + s_j = 0, \quad j = 1,2,...,n, \\
& \qquad eps\delta_j \le s_j \le M\delta_j, \ j = 1,2,...,n, \qquad\qquad\qquad (4) \\
& \qquad \sum_{j=1}^{n} \delta_j = n-1, \\
& \qquad \delta_j \in \{0,1\}, \ j = 1,2,...,n, \\
& \qquad u_r \ge l_r^u, \ r = 1,2,...,s, \\
& \qquad v_i \ge l_i^v, \ i = 1,2,...,m,
\end{aligned}
$$

Where M is a large positive number, ***eps*** is a very small positive number and $\delta_j \, (j = 1,2,...,n)$ are binary variables. We use ***eps*** to generate the full ranking and to identify the most efficient DMU according to Definition 1.

Constraints $u_r \geq l_r^u (r=1,2,...,s)$ and $v_i \geq l_i^v (i=1,2,...,m)$ are borrowed from (2) and have been extensively applied in DEA practice.

Let $\left(u_r^*(r=1,2,...,s), v_i^*(i=1,2,...,m), \delta_j^*(j=1,2,...,n)\right)$ be optimal solution of model (4). If $\delta_p^*=0$, then $s_p^*=0$, so $\sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} = 0$. This allows the efficiency of DMU_p to be one. On the other hand, if $\delta_j^*=1 (j=1,2,...,n; j\neq p)$, then $eps \leq s_j^* \leq M$ results in $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} < 0$. This guarantees that the efficiencies of the other DMUs are less than one. There for, DMU_p is the most efficient DMU based on Definition 1.

In the following, we prove some properties of proposed model.

Theorem 2 Model (4) always has a feasible solution.

Proof If PPS includes an extremely efficient DMU, DMU_p, then there exist $(\hat{u}, \hat{v}) > 0$ that satisfies $\left(\hat{u}y_p - \hat{v}x_p = 0, \hat{u}y_j - \hat{v}x_j < 0 (j \in \{1,2,...,n\} - \{p\})\right)$. This completes the proof.

Theorem 3 The optimal objective value of model (4) is bounded.

Proof Let $(\bar{u}, \bar{v}, \bar{I}, \bar{s})$ be any arbitrary feasible solution to model (4). Based on the constraints of this model, it follows that $(n-1)\times eps \leq \sum_{j=1}^{n} \bar{s}_j \leq (n-1)\times M$. This means that the objective function of model (4), for any feasible solution, is bounded both below and above. This concludes the proof.

4 Numerical examples

Example 1 This example is taken from [47] and in it, fourteen banks active in the Czech Republic are evaluated in the light of 5 inputs and 4 outputs .Inputs and outputs are described below and the data set is provided in Table 1:

Inputs: X_1 =number of employees, X_2 =number of branches, X_3 =assets, X_4 =equity, X_5 =expenses

Outputs: Y_1 =deposits, Y_2 = loans, Y_3 = non-interest income, Y_4 = interest income.

Table 1 inputs and outputs of 20 banks

Bank	X_1	X2	X3	X4	X5	Y1	Y2	Y3	Y4
AIR	400	18	33600	2596	745	30696	11135	14	554
CMZRB	217	5	111706	4958	566	86967	16813	634	1700
CS	10760	658	920403	93190	18259	629622	479516	8747	32697
CSOB	7801	322	937174	73930	16087	629622	479516	8747	32697
EQB	296	13	8985	1296	601	7502	5611	19	215
ERB	72	1	33614	464	173	2940	1762	15	131
FIO	59	36	18561	726	347	17174	6465	211	536
GEMB	3346	260	135474	34486	5276	97063	101898	3943	11026
ING	293	10	128425	913	1034	92579	19216	468	5139
JTB	407	3	85087	7233	1333	62085	39330	487	3686
KB	8758	399	786836	100577	13511	579067	451547	8834	35972
LBBW	365	18	31300	2774	1138	20274	2528	128	1046
RB	2927	125	197628	18151	57112	144143	150138	2829	8563
UCB	2004	98	318909	38937	13804	195120	192046	2740	8891

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We apply the models (1), (2), (3), and (4) to the data set given in Table 1, using $M = 10000$ and $eps = 0.0001$. The optimal weights obtained from the proposed model are as follows:

$$v_1^* = 1.0326e\text{-}05, v_2^* = 137.34, \quad v_3^* = 0.18134, v_4^* = 0.40514, v_5^* = 0.39415$$
$$u_1^* = 0.034172, \quad u_2^* = 0.28911, u_3^* = 6.5098, u_4^* = 0.71482,$$
$$\delta_{10}^* = 0, \delta_j^* = 1 (j \neq 10),$$

Table 2 presents the results of models (1), (2), (3), and (4), respectively. The highest efficiency scores achieved by the various models are highlighted in bold. The numbers in parentheses alongside the efficiency scores denote the rankings of the banks. The results show that 12 out of 20 banks are efficient. Model (2) identifies Bank CS as the most efficient, while Bank JTB is selected as the most efficient by models (3) and the proposed model. However, model (2) fails to fully rank all DMUs, whereas models (3) and (4) successfully provide a complete ranking of all banks. Considering the number of constraints, model (4) has a simpler structure than model (3). In all models, ERB Bank is identified as the least efficient.

Table 2 results of models (1), (2), (3), and (4)

DMUs	Bank	CCR Model (1)	Wang and Jiang (2012)-Model (2)	Özsoy et al. (2021)-Model (3)	Proposed model Model (4)
1	AIR	1(1)	0.797188(11)	0.385261(12)	0.47983(12)
2	CMZRB	1(1)	1(2)	0.679386(9)	0.5685(9)
3	CS	1(1)	1.13391(1)	0.990725(2)	0.96691(2)
4	CSOB	1(1)	1(2)	0.974095(4)	0.96007(3)
5	EQB	1(1)	0.618833(12)	0.352771(13)	0.51618(10)
6	ERB	0.473757(14)	0.131031(14)	0.136019(14)	0.12346(14)
7	FIO	1(1)	1(2)	0.540581(10)	0.48195(11)
8	GEMB	1(1)	1(2)	0.92167(6)	0.86898(7)
9	ING	1(1)	0.943912(9)	0.874328(8)	0.60691(8)
10	JTB	1(1)	1(2)	1.165441(1)	1(1)
11	KB	1(1)	1(2)	0.988263(3)	0.95894(4)
12	LBBW	0.824637(13)	0.604593(13)	0.410718(11)	0.3091(13)
13	RB	1(1)	1(2)	0.917785(7)	0.87933(6)
14	UCB	1(1)	0.906461(10)	0.965502(5)	0.93381(5)

We use Spearman's rank correlation to assess the strength of the relationship between the rankings obtained from models (2), (3), and (4). The correlation values are reported in Table 3, with p-values shown in parentheses below each corresponding correlation coefficient. Table 3 indicates a positive correlation between the proposed model and both models (2) and (3).

Table 3 Correlation test of ranking models in Example 1.

		Spearman's rank correlation		
		Proposed model Model (4)	Wang and Jiang (2012)-Model (2)	Özsoy et al. (2021)-Model (3)
Proposed model Model (4)	Correlation	1	0.74399	0.96044
	p-value		(0.002281)	(5.08E-08)
Wang and Jiang (2012)-Model (2)	Correlation		1	0.76746
	p-value			(0.001354)
Özsoy et al. (2021)-Model (3)	Correlation			1
	p-value			

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Table 3 shows a strong correlation between model (4) and models (2) and (3). Specifically, the Spearman's rank correlation coefficient between model (4) and model (3) is 0.96044. These results are statistically significant at the ($\alpha = 0.05$) level. The proposed model, with fewer constraints, successfully ranks all banks in a single step.

Example2 In this example, we use real data from nineteen facility layout designs (FLDs) studied by Ertay, Ruan [48]. Each FLD consumes two inputs, cost (x_1) and adjacency score (x_2) to produce shape ratio (y_1), flexibility (y_2), quality (y_3) and hand-carry utility (y_4) as four outputs. The data appear in columns two through seven of Table 4. In this example, we use $M = 100$ and $eps = 0.001$. The optimal weights obtained from the proposed model are as follows:

$$v_1^* = 0.008697,\ v_2^* = 9.5774\text{e-}06,\ u_1^* = 262.54,\quad u_2^* = 1.947,\ u_3^* = 1.9701,\ u_4^* = 0.0049603,$$
$$\delta_{10}^* = 0, \delta_j^* = 1 (j \neq 10),$$

Since $\delta_{10}^* = 1$, FLD10 is identified as the most efficient FLD by model (4). The columns eight through eleven of Table 4 presents the outcomes of models (1), (2), (3), and (4) for Example 2. The results from model (1) indicate that nine FLDs are efficient. Models (2), (3), and (4) consistently identify FLD10 as the most efficient design. However, model (2) cannot fully distinguish among all DMUs; for instance, FLD3 and FLD12 receive the same rank. In contrast, models (3) and (4) are capable of ranking all DMUs effectively. It is also worth noting that FLD13 is identified as the least efficient DMU across all models.

Table 4 Data set for 19 FLDs and efficiency of FLDs by different models Example 2.

DMUs	Inputs		Outputs				CCR-Model (1)	Wang and Jiang (2,012)-Model (2)	Özsoy et al. (2,021) -Model (3)	Proposed model -Model (4)
	x_1	x_2	y_1	y_2	y_3	y_4				
FLD1	20,309.56	6,405	0.4697	0.0113	0.041	30.89	0.984592 (13)	0.964891 (5)	0.761219 (7)	0.69934 (2)
FLD2	20,411.22	5,393	0.438	0.0337	0.0484	31.34	0.988393 (12)	0.971531 (4)	0.761527 (6)	0.64937 (7)
FLD3	20,280.28	5,294	0.4392	0.0308	0.0653	30.26	0.997428 (11)	1 (2)	0.770702 (3)	0.65547 (6)
FLD4	20,053.20	4,450	0.3776	0.0245	0.0638	28.03	0.949290 (15)	0.894522 (14)	0.673692 (15)	0.57007 (9)
FLD5	19,998.75	4,370	0.3526	0.0856	0.0484	25.43	1 (1)	0.925330 (9)	0.751551 (8)	0.53433 (11)
FLD6	20,193.68	4,393	0.3674	0.0717	0.0361	29.11	0.973342 (14)	0.910794 (13)	0.734339 (10)	0.5511 (10)
FLD7	19,779.73	2,862	0.2854	0.0245	0.0846	25.29	1 (1)	0.790849 (17)	0.552031 (17)	0.43747 (17)
FLD8	19,831	5,473	0.4398	0.0113	0.0125	24.8	0.856831 (17)	0.868210 (15)	0.723427 (13)	0.67025 (3)
FLD9	19,608.43	5,161	0.2868	0.0674	0.0724	24.45	0.889201 (16)	0.834482 (16)	0.630595 (16)	0.44371 (16)
FLD10	20,038.10	6,078	0.6624	0.0856	0.0653	26.45	1 (1)	1.440321 (1)	1.230623 (1)	1(1)
FLD11	20,330.68	4,516	0.3437	0.0856	0.0638	29.46	0.998328 (10)	0.940190 (8)	0.732256 (11)	0.51268 (13)
FLD12	20,155.09	3,702	0.3526	0.0856	0.0846	28.07	1 (1)	1 (2)	0.766601 (5)	0.53069 (12)

FLD13	19,641.86	5,726	0.269	0.0337	0.0361	24.58	0.775852 (19)	0.675683 (19)	0.513299 (19)	0.4148 (19)
FLD14	20,575.67	4,639	0.3441	0.0856	0.0638	32.2	1 (1)	0.941034 (7)	0.723855 (12)	0.50723 (14)
FLD15	20,687.50	5,646	0.4326	0.0337	0.0452	33.21	1 (1)	0.951281 (6)	0.740819 (9)	0.63283 (8)
FLD16	20,779.75	5,507	0.3312	0.0856	0.0653	33.6	1 (1)	0.913958 (11)	0.693781 (14)	0.48355 (15)
FLD17	19,853.38	3,912	0.2847	0.0245	0.0638	31.29	1 (1)	0.769322 (18)	0.534852 (18)	0.43469 (18)
FLD18	19,853.38	5,974	0.4398	0.0337	0.0179	25.12	0.851718 (18)	0.913731 (12)	0.767148 (4)	0.66979 (4)
FLD19	20,355	17,402	0.4421	0.0856	0.0217	30.02	1 (1)	0.923829 (10)	0.790033 (2)	0.65705 (5)

Table 5 Correlation test of ranking models in Example 2.

		Spearman's rank correlation		
		Proposed model Model (4)	Wang and Jiang (2012)-Model (2)	Özsoy et al. (2021)-Model (3)
Proposed model Model (4)	Correlation	1	0.57394	**0.78246**
	p-value		(0.010182)	(7.52E-05)
Wang and Jiang (2012)-Model (2)	Correlation		1	0.81615
	p-value			(2.03E-05)
Özsoy et al. (2021)-Model(3)	Correlation			1
	p-value			

As shown in Table 5, the correlation coefficient between the proposed model and the model by Özsoy, Örkcü [43] is 0.78246 (7.52E-05). This result indicates that the two models are statistically concordant at the significance level ($\alpha = 0.05$). Moreover, the proposed model, with fewer constraints, successfully ranks all FLDs in a single step.

5 Conclusion

This paper presented a straightforward MILP model designed to identify the most efficient DMU using a common set of weights. The model simplifies the evaluation process by reducing the number of constraints while still offering strong discriminatory power. Through testing on well-known case studies, the model proved both effective and practical. Overall, the approach shows promise as a useful tool for efficiency analysis. Looking ahead, future research could explore how different choices for the parameter M affect the results and how the model can be adapted to handle negative data.

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