

# Ranking using TOPSIS method and discriminant analysis model in data envelopment analysis (DEA/DA-TOPSIS)

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**Abstract** Many decisions are so complex that the analyst is different from the person making the final decision. Despite the wide domain of applications of multi-criteria decisions in the real world, this approach also has its limitations and challenges. It is up to the analyst to determine which method to use (to determine the weight or evaluate options) or in what situation to use only part of the method. There exist many various ranking methods for ranking efficient Decision-Making Units (DMUs) in Data Envelopment Analysis (DEA). Nevertheless, since each of these methods considers a certain theory for ranking, they may give different ranks. In practice, select a ranking method, the results of which the Decision Maker (DM) would be able to confident is an important topic. In this paper, we consider some CCR (Charnes, Cooper and Rhodes) efficient DMUs and then rank them by using some ranking methods, each of which is significant. Afterward, by using TOPSIS and Discriminant Analysis (DA) method, we propose the ranks of efficient DMUs.

**Keywords:** Ranking, Efficiency Evaluation, Data Envelopment Analysis, Discriminant Analysis, TOPSIS Technique.

## 1 Introduction

In evaluating organizations and units, one of the most important goals is to rank units based on their performance. Given that current methods of evaluating Decision-Making Units (DMUs) in data envelopment analysis (DEA) may result in several efficient units, always selecting the best unit from all units is one of the main problems in DEA. Numerous methods have been proposed for ranking DMUs, most of which are part of multi-criteria compensatory decision-making methods. Each of these methods is solved by its approach and assumptions. Therefore, in different situations, each will have different answers. Also, one of the basic assumptions is the validity of the weights related to the criteria used in the mentioned techniques. In a general viewpoint, the methods used to determine weight are objective and subjective. A change in the

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results of the weight calculations will affect the performance of the mentioned techniques in achieving the superior option. Therefore, in such a situation, the existence of an experimental or scientific indicator that can measure the validity of the resulting weights and the correctness of the results obtained by implementing these techniques becomes more important than ever. Many researchers have researched in this regard, some of which are mentioned.

Here are some examples of studies that have been done in this area. The DEA technique has long been considered in the evaluation of the efficiency of the decision-making unit and its application is increasing in functional areas. Beazley [1], Breu and Raab [2], Alirezaei and Jahanshahloo [3], Avkiran [4], and Modell [5]. have been among the researchers who have evaluated efficiency using DEA. Methods for selecting the appropriate TOPSIS technique were proposed by individuals such as Hwang and Yoon [6], Hobbes [7], Ozemoy [8]. These methods are often based on the data required by the techniques. Jahanshahloo et al. [9] proposed two methods: The first method uses an ideal line for the sum of common weights for efficiency (DMUs). The second method is first a special line is defined and then all efficient (DMUs) are compared with that line and finally ranked. Hosseinzadeh-Lotfi et al. [10] proposed a two-step method, in the first stage a paired comparative matrix is created for each (DMUs) pair and in the second stage, the ranking is done with the help of goal planning. János et al. [11] presented the method for combining DEA with techniques for eliciting weights from pairwise comparison matrices. The basic idea is to apply a variant of the CCR problem instead of the classic one. The ensuing scores are then utilized to build a nonreciprocal pairwise comparison matrix which serves as the basis for eliciting the ranking values of the DMUs. The main advantage of this method is the wider range of the resulting ranking values which subsequently leads to a better distinction between DMUs. The study conducted by Nouri [12] shows that the entropy method does not have good stability for weighting at all.

Discriminant Analysis (DA) is a decision-making technique for predicting the group membership of a new observation. In DA using a set of observations that the group membership of them are known, we estimate the weights of a discriminant function using different criteria, and then by the estimated discriminant function, we expect that a new observation belongs to which group. Discriminant analysis has many applications in statistics and classification. Using discriminant analysis, it is possible to predict whether a student can enter the university based on his/her educational background or not. According to her medical record, a patient is in the group of patients at low risk of heart attack or in the group of patients at high risk of heart attack and etc. DEA and DA are both mathematical programming decision theories used to evaluate and classify similar units. In both theories, similar factors are measured for observations, and by designing a weighted model and introducing a hyperplane (discriminant function), they seek to distinguish between the success and failure of units.

Retzlaff-Roberts [13] and Sueyoshi [14], given these similarities, made sense to apply the performance approach and principles of DEA to DA issues to take advantage that may have on some issues. The resulting models are called DEA -DA. Sueyoshi and Goto [15] used DEA -DA models to reduce the number of efficient units. They proposed a two-stage approach, DEA -SCSC, and DEA -DA, in which only one efficiency unit adopts one and the other units adopt a modified efficiency value of less than one. Tavassoli et al [16] used the Suyoshi Vegoto [17] model to rank electricity distribution companies in Iran. Lam [18] using the model of Suyoshi and Goto [19] and the model presented in the study by Lam et al. [20] introduces a modified model to solve the problem of multiple solutions in DEA to determine the most efficient DMU. In this paper, we propose a new method for ranking non-extreme efficient units using DA.

But in this paper, we try to provide a prioritization approach by using the DEA-DA model, which is one of the techniques of DEA and TOPSIS.

## 2 Prerequisites

### 2.1 TOPSIS method

**Step. 1** Establishing a performance decision matrix:

$$A_{ij} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \rightarrow \begin{cases} (i=1, \dots, n) \\ (j=1, \dots, m) \end{cases} \quad (1)$$

**Step. 2** Calculate the normalized decision matrix. The normalized values  $n_{ij}$  are calculated as follows and normalized decision matrix  $N_{ij}$  is estimated in the next way:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (2)$$

$$N_{ij} = \begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1n} \\ n_{21} & n_{22} & \cdots & n_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ n_{m1} & n_{m2} & \cdots & n_{mn} \end{bmatrix} \quad (3)$$

**Step. 3** Calculate the weighted normalized decision matrix. In this step, the weight of the indicators is obtained using the Entropy technique (4), and then the weighted normalized value  $v_{ij}$  (3), is calculated as follows:

$$P_{ij} = \frac{a_{ij}}{\sum_{i=1}^m a_{ij}}, \quad E_j = k \sum_{i=1}^m [P_{ij} \cdot \ln P_{ij}]$$

$$d_j = 1 - E_j, \quad w_j = \frac{d_j}{\sum_{j=1}^n d_j} \quad (4)$$

$$V = N \times W = \begin{bmatrix} n_{11} & \cdots & n_{1n} \\ \vdots & \ddots & \vdots \\ n_{m1} & \cdots & n_{mn} \end{bmatrix}_{mn} \times \begin{bmatrix} w_{11} & \cdots & \circ \\ \vdots & \ddots & \vdots \\ \circ & \cdots & w_{nn} \end{bmatrix}_{nn} \quad (5)$$

where  $W_j$  is the weight of the  $j$ th criterion or attribute and  $\sum_{j=1}^n W_j = 1$ .

**Step. 4** Determine the positive ideal ( $A_j^+$ ) and negative ideal ( $A_j^-$ ) solutions.

$$V_j^+ = \{\min V_{i1}, \max V_{i2}, \dots, \max V_{im}\}$$

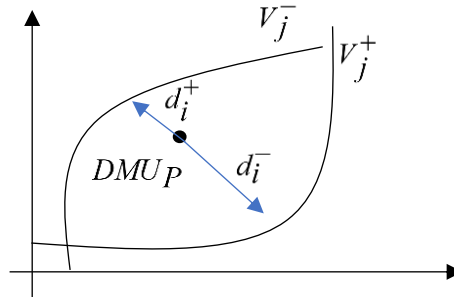
$$V_j^- = \{\max V_{i1}, \min V_{i2}, \dots, \min V_{im}\} \quad (6)$$

Here:

Indicators that have a positive charge the negative ideal is the smallest value of that index.  
Indicators that have a positive charge the positive ideal is the largest value of that index.

Indicators that have a negative charge the negative ideal is the smallest value of that index. Indicators that have a negative charge the negative ideal is the largest value of that index.

**Step. 5** Calculate the dissociation measures using the  $m$ -dimensional Euclidean distance. The dissociation measures of each alternative from (the positive ideal ( $A_j^+$ ) and the negative ideal ( $A_j^-$ )) solution, respectively, are as follows:



**Fig. 1** Distance related to  $DMU_P$  from positive and negative ideals

Figure 1 shows the distance of  $DMU_P$  from positive and negative ideals. Now, the relative proximity of each option to the ideal solution is calculated. The Euclidean distance of each option from the positive and negative ideals will be calculated by the following formula.

$$d_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}, i = 1, 2, \dots, m$$

$$d_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, i = 1, 2, \dots, m \quad (7)$$

**The last step:** Calculate the relative closeness to the ideal solution. To this, we use the following command:

$$cl_i = \frac{d_i^-}{d_i^- + d_i^+} \quad (8)$$

## 2.2 DEA-DA model for ranking

Suppose that there are  $n$   $DMU_s$  and that each  $DMU$  has  $m$  inputs and produces  $s$  outputs. Let  $x_{ij}$  and  $y_{rj}$  represent the  $i$ th input and the  $r$ th output of  $DMU_j$ , respectively, for  $i = \{1, \dots, m\}$  and  $r = \{1, \dots, s\}$  and  $j = \{1, \dots, n\}$ . on the other hand, Kryonushko et al. [21] introduced the number of constraints is equal to  $(2n + 2m + 2s + 3)$  in the DEA-SCSC model. Thus the size of the base matrix is very large. They also showed with two examples that with real medium size data, this base matrix leads to invalid answers for the DEA-SCSC model. Therefore, using the DEA-SCSC model to classify efficient and inefficient units is not economical. It can be used to classify the efficient and inefficient use of the DEA efficiency evaluation model. In addition, in the DEA, if an inefficient unit is declared, it is inefficient, so there is no wrong classification in the category of inefficient units. In addition, in the DEA, if an inefficient unit is declared, it is inefficient, so

there is no wrong classification in the category of inefficient units. In the second stage of the algorithm, Sueyoshi presents the DEA-DA model (9).

$$\begin{aligned}
 \min \quad & M \sum_{j \in E} z_j + \sum_{j \in IE} z_j \\
 s.t. \quad & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + u_0 + Mz_j \geq 0 \quad j \in E \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + u_0 - Mz_j \leq -\varepsilon \quad j \in IE \\
 & \sum_{r=1}^s u_r + \sum_{i=1}^m v_i = 1 \\
 & \sum_{i=1}^m \zeta_i = m, \quad \sum_{r=1}^s \zeta_r = s \\
 & v_i \geq \varepsilon \zeta_i \quad i=1,2,\dots,m, \quad u_r \geq \varepsilon \zeta_r \quad r=1,2,\dots,s \\
 & z_j, \zeta_i, \zeta_r \in \{0,1\}, \forall j, \forall i, \forall r, \quad v_i, u_r \geq 0 \quad \forall i, \forall r, \quad u_0 \text{ free}
 \end{aligned} \tag{9}$$

We add a large  $M$  to the efficient group in the objective of model (9). Therefore, in this classification, the efficient group (E) has more priority than the inefficient group (LE). The criterion of the objective minimum function is the number of incorrect categories and is a MIP model. The discriminant score is expressed by  $-u_0$  and  $-u_0 - \varepsilon$ . The small number  $\varepsilon$  is incorporated into model (9) in order to avoid a case where an observation(s) exists on an estimated discriminant function. variables ( $\zeta_i$  binary  $\forall i$ , and  $\zeta_r$  binary  $\forall r$ ) to count the number of positive weight estimates.

$$H = \left\{ \begin{pmatrix} X \\ Y \end{pmatrix} \middle| U^* Y - V^* X + u_0^* = 0 \right\} \tag{10}$$

Here  $H$  is a supporting hyperplane and the discriminant function. Sueyoshi and Goto [22] show by proving a theorem that by change the location of a supporting hyperplane to separate efficient and inefficient DMUs as a discriminant hyperplane. Now if  $(u_0^*, V^*, U^*)$  is the optimal solution of the model (9),

$$\rho_j = U^* Y_j - V^* X_j + u_0^*, \quad \forall j \tag{11}$$

and

$$\begin{aligned}
 f \min_{1 \leq j \leq n} \{\rho_j\} \geq 0 \Rightarrow & \begin{cases} Rang(A) = \max\{\rho_j\} - \min\{\rho_j\} \\ AE_j = \frac{\rho_j - \min\{\rho_j\}}{Rang(A)} \end{cases} \\
 \text{If } \min_{1 \leq j \leq n} \{\rho_j\} < 0 \Rightarrow & \begin{cases} Rang(B) = \max\{\rho_j\} + |\min\{\rho_j\}| \\ AE_j = \frac{\rho_j + |\min\{\rho_j\}|}{Rang(B)} \end{cases}
 \end{aligned} \tag{12}$$

$AE_j$  (Adjusted Efficiency) is the adjusted Efficiency value of  $j$ , which declares the efficiency of only one unit to be 1 and for the other units to be a modified efficiency value of  $0 \leq AE_j < 1$ . Note that the adjusted efficiency of efficient units may be less than some inefficient units. That is, the rank of an inefficient unit is better than the rank of an efficient unit.

### 3 Suggested method

One of the applications of DEA models is to determine the optimal weights for each unit to achieve the highest efficiency. Therefore, we obtain weights to replace the weights obtained from the entropy method using the DEA model. To do this, we consider the number of options as the number of DMUs, the negative indicators as input, and the positive indicators as output. Then we do the following steps.

**Step 1:** Implement the CCR model (13) to obtain the efficiency score of each decision unit. Units are divided into two categories: efficient and inefficient.

$$\begin{aligned}
 & \text{Min } \theta \\
 & \text{s.t.} \quad \sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ik} \quad , \quad i = 1, \dots, m \\
 & \quad \quad \sum_{j=1}^n \lambda_j y_{rj} \geq y_{rk} \quad , \quad r = 1, \dots, s \\
 & \quad \quad \lambda_j \geq 0 \quad \quad \quad j = 1, \dots, n
 \end{aligned} \tag{13}$$

**Step 2:** We put the two efficient and inefficient categories obtained from the first step in the discriminant analysis model (9) and solve it to produce weights for inputs ( $v_1, \dots, v_m$ ) and outputs ( $u_1, \dots, u_s$ ). We replace the obtained weights with the weights obtained from the entropy method in the third step of TOPSIS, then we perform the next steps of TOPSIS to obtain the rank of each decision unit. In the following, we compare the rank obtained from this method with the rank obtained from the following models : (AP, CHANGING THE REFERENCE SET, MAJ, MODIFIED MAJ, LJK, SBM, SA DEA, L INFINITY, CSW1, CSW2, CROSS EFFICIENCY, ...).

### 4 Practical example

This section presented two examples. Its data from paper Hosseinzadeh-Lotfi et al. (2011) [11].

#### 4.1 Example 1

In this example, the dataset consisting of 6 DMUs each consuming one input to produce one output is given in Table 1.

**Table 1** Raw data for numerical example

| DMUs | I <sub>1</sub> | I <sub>2</sub> | O <sub>1</sub> | O <sub>2</sub> | CCR Efficiency |
|------|----------------|----------------|----------------|----------------|----------------|
| A    | 150            | 0.2            | 14000          | 3500           | 1              |
| B    | 400            | 0.7            | 14000          | 21000          | 1              |
| C    | 320            | 1.2            | 42000          | 10500          | 1              |
| D    | 520            | 2.0            | 28000          | 42000          | 1              |
| E    | 350            | 1.2            | 19000          | 25000          | 0.978          |
| F    | 320            | 0.7            | 14000          | 15000          | 0.868          |

**Table 2** DEA ranking methods which we use

|                 | DEA ranking methods        | References |
|-----------------|----------------------------|------------|
| M <sub>1</sub>  | Ap                         | [1]        |
| M <sub>2</sub>  | CHANGING THE REFERENCE SET | [15]       |
| M <sub>3</sub>  | MAJ                        | [21]       |
| M <sub>4</sub>  | MODIFIED MAJ               | [26]       |
| M <sub>5</sub>  | LJK                        | [29]       |
| M <sub>6</sub>  | SBM                        | [34]       |
| M <sub>7</sub>  | SA DEA                     | [36]       |
| M <sub>8</sub>  | L INFINITY                 | [14]       |
| M <sub>9</sub>  | CSW1                       | [5]        |
| M <sub>10</sub> | CSW2                       | [16]       |
| M <sub>11</sub> | CROSS EFFICIENCY           | [28]       |
| M <sub>12</sub> | THE PROPOSED RANK          | [10]       |
| M <sub>13</sub> | DEA-TOPSIS METHOD          | [37]       |

**Table 3** Ranking related scores assigned to DMUs by ranking methods DEA

| M <sub>1</sub> | M <sub>2</sub> | M <sub>3</sub> | M <sub>4</sub> | M <sub>5</sub> | M <sub>6</sub> | M <sub>7</sub> | M <sub>8</sub> | M <sub>9</sub> | M <sub>10</sub> | M <sub>11</sub> | M <sub>12</sub> | M <sub>13</sub> | New Method | rank |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|------------|------|
| A              | B              | C              | B              | C              | A              | A              | C              | C              | C               | A               | A               | D               | A          | 1    |
| B              | A              | D              | A              | D              | C              | B              | B              | A              | A               | C               | C               | B               | B          | 2    |
| C              | D              | B              | D              | B              | B              | C              | A              | D              | D               | B               | B               | A               | C          | 3    |
| D              | C              | A              | C              | A              | D              | D              | D              | B              | B               | D               | D               | C               | D          | 4    |

As seen, the rank obtained using this method is quite similar to the rank obtained from the M1 and M7 models (See Tables 2 and 3). The fourth column obtained using the model proposed by Yaghoubi [23] has a completely different rank from the rankings of other models, and this shows that the weights obtained from the DEA-DA model perform better than other models.

## 4.2 Example 2

In this example, the dataset consisting of 20 DMUs each consuming three input to produce three outputs is given in Table 4.

Table 4 Raw data for numerical example

| DMUs | O1    | O2    | O3    | I1    | I2    | I3    | CCR   |
|------|-------|-------|-------|-------|-------|-------|-------|
| D 1  | 0.293 | 0.521 | 0.190 | 0.155 | 0.700 | 0.950 | 1.000 |
| D 2  | 0.462 | 0.627 | 0.227 | 1.000 | 0.600 | 0.796 | 0.833 |
| D 3  | 0.261 | 0.970 | 0.228 | 0.513 | 0.750 | 0.798 | 0.991 |
| D 4  | 1.000 | 0.632 | 0.193 | 0.210 | 0.550 | 0.865 | 1.000 |
| D 5  | 0.246 | 0.722 | 0.233 | 0.268 | 0.850 | 0.815 | 0.899 |
| D 6  | 0.569 | 0.603 | 0.207 | 0.500 | 0.650 | 0.842 | 0.748 |
| D 7  | 0.716 | 0.900 | 0.182 | 0.350 | 0.600 | 0.719 | 1.000 |
| D 8  | 0.298 | 0.234 | 0.125 | 0.120 | 0.750 | 0.785 | 0.798 |
| D 9  | 0.244 | 0.364 | 0.080 | 0.135 | 0.600 | 0.476 | 0.789 |
| D10  | 0.049 | 0.184 | 0.082 | 0.510 | 0.550 | 0.678 | 0.289 |
| D11  | 0.403 | 0.318 | 0.212 | 0.305 | 1.000 | 0.711 | 0.604 |
| D12  | 0.628 | 0.923 | 0.123 | 0.255 | 0.650 | 0.811 | 1.000 |
| D13  | 0.261 | 0.645 | 0.176 | 0.340 | 0.850 | 0.659 | 0.817 |
| D14  | 0.243 | 0.514 | 0.144 | 0.540 | 0.800 | 0.976 | 0.470 |
| D15  | 0.098 | 0.262 | 1.000 | 0.450 | 0.950 | 0.685 | 1.000 |
| D16  | 0.464 | 0.402 | 0.115 | 0.525 | 0.900 | 0.613 | 0.639 |
| D17  | 0.161 | 1.000 | 0.090 | 0.205 | 0.600 | 1.000 | 1.000 |
| D18  | 0.068 | 0.349 | 0.059 | 0.235 | 0.650 | 0.634 | 0.473 |
| D19  | 0.111 | 0.190 | 0.039 | 0.238 | 0.700 | 0.372 | 0.408 |
| D20  | 0.764 | 0.615 | 0.110 | 0.500 | 0.550 | 0.583 | 1.000 |

Table 5 Ranking related scores assigned to DMUs by ranking methods

| M1              | M2              | M3              | M4              | M5              | M6              | M7              | M8              | M9              | M10             | M11             | M12             | New Method      | rank |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|------|
| D <sub>15</sub> | D <sub>15</sub> | D <sub>15</sub> | D <sub>15</sub> | D <sub>15</sub> | D <sub>15</sub> | D <sub>15</sub> | D <sub>15</sub> | D <sub>7</sub>  | D <sub>4</sub>  | D <sub>15</sub> | D <sub>15</sub> | D <sub>7</sub>  | 1    |
| D <sub>4</sub>  | D <sub>4</sub>  | D <sub>4</sub>  | D <sub>4</sub>  | D <sub>4</sub>  | D <sub>4</sub>  | D <sub>4</sub>  | D <sub>4</sub>  | D <sub>4</sub>  | D <sub>7</sub>  | D <sub>4</sub>  | D <sub>4</sub>  | D <sub>4</sub>  | 2    |
| D <sub>17</sub> | D <sub>7</sub>  | D <sub>7</sub>  | D <sub>17</sub> | D <sub>7</sub>  | D <sub>17</sub> | D <sub>7</sub>  | D <sub>17</sub> | D <sub>15</sub> | D <sub>20</sub> | D <sub>20</sub> | D <sub>7</sub>  | D <sub>20</sub> | 3    |
| D <sub>20</sub> | D <sub>20</sub> | D <sub>20</sub> | D <sub>7</sub>  | D <sub>17</sub> | D <sub>7</sub>  | D <sub>17</sub> | D <sub>20</sub> | D <sub>12</sub> | D <sub>12</sub> | D <sub>7</sub>  | D <sub>20</sub> | D <sub>17</sub> | 4    |
| D <sub>7</sub>  | D <sub>17</sub> | D <sub>17</sub> | D <sub>20</sub> | D <sub>20</sub> | D <sub>20</sub> | D <sub>12</sub> | D <sub>7</sub>  | D <sub>20</sub> | D <sub>17</sub> | D <sub>12</sub> | D <sub>12</sub> | D <sub>15</sub> | 5    |
| D <sub>12</sub> | D <sub>12</sub> | D <sub>12</sub> | D <sub>12</sub> | D <sub>12</sub> | D <sub>12</sub> | D <sub>20</sub> | D <sub>12</sub> | D <sub>17</sub> | D <sub>15</sub> | D <sub>1</sub>  | D <sub>17</sub> | D <sub>12</sub> | 6    |
| D <sub>1</sub>  | D <sub>1</sub>  | D <sub>1</sub>  | D <sub>1</sub>  | D <sub>1</sub>  | D <sub>1</sub>  | D <sub>1</sub>  | D <sub>1</sub>  | D <sub>1</sub>  | D <sub>1</sub>  | D <sub>17</sub> | D <sub>1</sub>  | D <sub>1</sub>  | 7    |

In this example, as shown in Table 5, the rank obtained using this method is similar to the rank obtained from other models in some cases.

5 Results

Each manager has a special mission to evaluate the performance of the unit under his command, which should try to optimize the use of resources and get closer to the goals so that it can identify the strengths and weaknesses points of the current situation. Performance evaluation is done by various methods, but the capability of mathematical models, including DEA models, in measuring performance is of special importance and application, which can be combined with statistical models to help managers for more accurate evaluation so that they can make the right decision.

## 6 Suggestions for future research

Future studies could extend the proposed DEA–TOPSIS–DA framework in several directions. First, alternative DEA models, such as the BCC, additive, slack-based measure (SBM), network DEA, and dynamic DEA models, could be employed to examine whether the ranking results remain stable under different production assumptions. Second, the effect of different subjective and objective weighting methods, including AHP, BWM, Shannon entropy, and SWARA, could be investigated within the TOPSIS stage. Since real-world performance data are often affected by ambiguity, imprecision, and measurement errors, future research could develop fuzzy, grey, stochastic, or robust versions of the proposed approach. In addition, sensitivity and stability analyses should be conducted to evaluate the impact of changes in input and output values, criterion weights, and the selected ranking methods on the final rankings of efficient DMUs. The proposed framework could also be applied to larger and more diverse datasets in areas such as banking, healthcare, education, energy, transportation, and supply chain management. Finally, future studies may compare the proposed method with machine-learning and classification techniques to improve the discrimination of efficient DMUs and develop more reliable performance-ranking systems for decision makers.

## References

1. Beasley J, (1990). Comparing University Departments, *Omega International Journal*, 18, 171-183.
2. Breu, T, Raab, R, (1994). Efficiency and Perceived Quality of the Nation's Top 25 National Universities and National Liberal arts Colleges: An Application of Data Envelopment Analysis to Higher Education, *Socio-Economic Planning Sciences*, 28, 33-45.
3. Jahanshahloo, G. .Alirezaei, (1994). Technology (CIBTech) 1045 Data envelopment analysis (DEA): case study of the iranian universities, *Indian Journal of Fundamental and Applied Life*. 4 (S1) April-June, 1045-1050
4. Avkiran N, (2000). Investigating Technical and Scale Efficiencies of Australian Universities Through Data Envelopment Analysis, *SocioEconomic Planning Sciences*, 35, 57-80.
5. Modell S, (2000). Goals Versus Institutions: The Development of Performance Measurement in the Swedish University Sector, *Management Accounting Research*, 14, 333-359.
6. Hwang, C, Yoon, K, (1981). Multiple Attribute decision making: A state of the art survey. Springer- Verlag. 58-191.
7. Hobbs, B. F, (1986). What Can We Learn from Experiments in Multi Objective Decision Analysis? *IEEE Transactions on Systems, Man, and Cybernetics*, 16(3), 384-394.
8. onziOzemoy, V., (1987). A framework for choosing the most appropriate discrete alternative MCDM in decision support and expert systems. In: Savaragi, Y, et al. (Eds.), *Toward Interactive and Intelligent Decision Support Systems*. Springer-Verlag, 56-64.
9. Ozemoy, V., (1992). Choosing The 'Best' Multiple Criteria Decision-Making Method. *INFOR* 30,159-171
9. Jahanshahloo G. R, Hosseinzadeh Lotfi F, Khanmohammadi M, Kazemimanesh M, Rezaie V, (2010). Ranking of units by positive ideal DMU with common weights, *Expert Systems with Applications*, 37, 7483–7488.
10. Hosseinzadeh Lotfi1. F, Fallahnejad.R, Navidi. N, (2011). Ranking Efficient Units in DEA by Using TOPSIS Method, *Applied Mathematical Sciences*, 5, 17, 805 – 815.
11. János F, Rita M. S. (2012). Ranking Decision-Making Units Based on Dea-Like Nonreciprocal Pairwise Comparisons, *Acta Polytechnica Hungarica*, 9 (2), 7794.
12. Nouri, Q, (2006). Sensitivity Analysis of Multi-Criteria Decision Problems Compared to The Method Used. *University of Tehran*, 36 (15), 25-38.
13. Retzlaff-Roberts, D.L., (1997). A Data Envelopment Analysis approach to Discriminant Analysis. *Annals of Operations Research*, 73, 299-321.
14. Sueyoshi, T., (1999b). DEA-discriminant analysis in the view of goal programming. *European Journal of Operational Research* 115, 264-582

15. Sueyoshi, T. and Goto, M., (2013). A use of DEA-DA to measure importance of R&D expenditure in Japanese information technology industry, *Decision Support Systems* 54(2), 941-952.
16. Tavassoli, M., Faramarzi G. R., and Saen, R. F., (2015). Ranking electricity distribution units using slacks-based measure, strong complementary slackness condition, and discriminant analysis, *International Journal of Electrical Power & Energy Systems* 64, 1214
17. Sueyoshi, T., and Goto, M., (2012). Efficiency -based rank assessment for electric power industry: a combined use of data envelopment analysis (DEA) and DEA -discriminant analysis (DA). *Energy Economics* 34(3), 634 -644
18. Lam, K. F, (2020). Applying Data Envelopment Analysis and Discriminant Analysis to Determining the Most Efficient Decision-Making Unit. *Journal of Advanced Management Science*, 8(2).
19. Sueyoshi, T. and Goto, M., (2011). A combined use of DEA (data envelopment analysis) with strong complementary slackness condition and DEA-DA (discriminant analysis), *Applied Mathematics Letters* 24(7), 1051-1056 .
20. Lam, K, F, Choo, E, U, Moy, J, W, (1996). Minimizing deviations from the group mean: A new linear programming approach for the two-group classification problem. *European Journal of Operational Research*, 88, 358-367.
21. Krivonozhko, V. E., Førsund, F. R. and Lychev, A. V., (2012). A note on imposing strong complementary slackness conditions in DEA, *European Journal of Operational Research* 220(3), 716-72.
22. , T. and Goto, M., (2013). A use of DEA-DA to measure importance of R&D expenditure in Japanese information technology industry, *Decision Support Systems* 54(2), 941-952.
23. Yaghoubi. M., (2016). A DEA Model to Determine the Relative Weights of Evaluation Criteria and TOPSIS to Rank the Alternatives, *International Journal of Innovation in Science and Mathematics*, 4(4), ISSN (Online): 2347–9051.