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Context-Dependent Data Envelopment Analysis-Measuring Attractiveness and Progress with Interval Data

F. Hosseinzadeh Lotfi, N. Ebrahimkhany Ghazy*, S. Ebrahimkhany Ghazy, M. Ahadzadeh Namin

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Abstract Data envelopment analysis (DEA) is a method for recognizing the efficient frontier of decision making units (DMUs).This paper presents a Context-dependent DEA which uses the interval inputs and outputs. Context-dependent approach with interval inputs and outputs can consider a set of DMUs against the special context. Each context shows an efficient frontier including DMUs in particular levels. The Context-dependent DEA with interval inputs and outputs can measure (i) the attractiveness when DMUs showing weaker performance are selected as an appraisal context, and (ii) the interval progress when DMUs showing better performance are selected as the appraisal context.

Keywords Interval Inputs and Outputs, Context-Dependent Data Envelopment Analysis, Attractiveness, Progress, Value Judgment.

1 Introduction

Data envelopment analysis (DEA), presented by Charnes, Cooper and Rhodes (CCR) [1], is a mathematical programming method used for measuring the relative efficiency of decision making units (DMUs) with several outputs and inputs.

Among DMUs from a given set, DEA recognizes efficient DMUs. All we know is that adding or omitting an inefficient DMU or a set of inefficient DMUs does not change the efficiencies of the DMUs and the efficient frontier. The inefficiency scores alter only if the efficient frontier is changed, i.e., the performance of DMUs is only influenced by the identified efficient frontier. On the other hand, the context often influences the consumer choice, e.g., a circle shows big when small circles surround it, and small when bigger ones surround it. A product also may show attractive against some products with less attractiveness, and unattractive when we compare it with more attractive products [2]. Considering this result of the DEA framework, someone could ask "what is the relative attractiveness of a special DMU when compared to others?" mentioned in [3], someone accepts that existence or non-existence of a third option, say DMU_z (or a group of DMUs), influences the relative attractiveness of DMU_x which are compared to DMU_y. Relative attractiveness depends on the evaluation context made from different choices (or DMUs).

***Corresponding Author.** (✉)
E-mail: nasrinebrahimkhany@yahoo.com (N. Ebrahimkhany Ghazy)

F. Hosseinzadeh Lotfi, N. Ebrahimkhany Ghazy, S. Ebrahimkhany Ghazy
Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.

M. Ahadzadeh Namin
Department of Mathematics, Shahr-e –Qods Branch, Islamic Azad University, Shahr-e –Qods, Iran.

Indeed a set of DMUs can be shared among different levels of efficient frontiers. If we omit DMUs which are efficient in their best and worst conditions, afterwards a new second-level efficient frontier will be made for the remaining (inefficient) DMUs. With omitting this new second-level efficient frontier, a third-level efficient frontier is made, and this process continues, until no DMU is remained. Each efficient frontier provides an assessment context that measures the relative attractiveness, e.g., the second-level efficient frontier is considered as the assessment context for measuring the relative attractiveness of the DMUs located on the first-level (main) efficient frontier. On the other part, the performance of DMUs on the third-level efficient frontier can be measured concerning the first-or second-level efficient frontier. With continuing this procedure, the context-dependent DEA is gained that can measure the relative attractiveness and the relative progress when DMUs having worse performance and better performance are chosen as the assessment context, respectively. The existence or non-existence (or the shape) of the assessment context (efficient frontier) influence the relative attractiveness or progress of DMUs on a different level of efficient frontier. When DMUs in a particular level are observed that have equal performance, the attractiveness value or the progress value lets distinguish the "equal performance" based on the same particular assessment context (or third option). In this paper we have interval inputs and interval outputs so the attractiveness measure and progress measure aren't the same in [4] , context-dependent data envelopment presented by Seiford, et al because they just consider in their crisp condition. For interval inputs and outputs sake also the evaluation contexts change, i.e., we have the best condition and worst condition in efficient frontier so we measured in two states; (i) the attractiveness of DMU considered when DMUs having worse performance are chosen as the evaluation context (ii) the progress of DMU considered when DMUs having better performance are chosen as the evaluation context. Note that different input/output values affect the assessment of a DMU's performance. Thus, the incorporation of value judgment in measuring the relative attractiveness and progress, is very important as well. We use the result of a context-dependent DEA with value judgment presented by Zhu [5] in this paper and incorporate it with a context-dependent DEA. The rest of the paper is as follows: The next section represents the context-dependent DEA with interval inputs and outputs. Then we merge the value judgment into the context-dependent DEA. After that, we use this method in the numerical example in one of the Iran's commercial bank. We provide our conclusions in the last section.

2 Context-dependent DEA

Let DMU_j (j = 1, 2,…, n) are decision making units that produces $y_j = (y_{1j},...,y_{rj})$ by using $x_j = (x_{1j},...,x_{mj})$ ، that $\begin{cases} x_j^L \le x_j \le x_j^U \\ y_j^L \le y_j \le y_j^U \end{cases}$. Assume that $j^l = \{DMU_j; j = 1,...,n\}$ is all n DMUs' set.

Now we define $J^{l+1} = J^l - E_l^{++}$ and $E_l^{++} = \{DMU_j; \phi^{*-} = 1, \phi^{+*} = 1\}$.

E_l^{++} reaches from two models below .

$$\phi^{*-}(l,k) = \underset{\lambda_j,\, \phi^-(l,k)}{\text{Max}} \ \phi^-(l,k) \qquad\qquad \phi^{*+}(l,k) = \underset{\lambda_j,\, \phi^+(l,k)}{\text{Max}} \ \phi^+(l,k)$$

$$\text{s.t.} \ \lambda_k x_k^u + \sum_{\substack{j\in F(J^l)\\ j\neq k}} \lambda_j x_j^L \le x_k^U, \qquad \text{s.t.} \ \lambda_k x_k^L + \sum_{\substack{j\in F(J^l)\\ j\neq k}} \lambda_j x_j^U \le x_k^L,$$

$$\lambda_k y_k^L + \sum_{\substack{j\in F(J^l)\\ j\neq k}} \lambda_j y_j^U \ge \phi^-(l,k) y_k^L, \qquad \lambda_k y_k^U + \sum_{\substack{j\in F(J^l)\\ j\neq k}} \lambda_j y_j^L \ge \phi^+(l,k) y_k^U,$$

$$\lambda_j \ge 0, \ j\in F(J^l). \quad (1) \qquad\qquad \lambda_j \ge 0, \ j\in F(J^l). \quad (2)$$

which (x_k, y_k) is the input and output vector of DMU_k, and $j\in F(J^l)$ means $DMU_j \in J^l$. The below algorithm helps us recognize these efficient frontiers due to models (1) and (2). The efficient frontiers can be easily obtained by using the DEA Excel Solver provided in [6].

Step 1: Set l = 1. Consider the all DMUs in J^l by models (1) and (2) to reach the first-level efficient DMUs, set E_l^{++} (the first-level efficient frontier).

Step 2: Assume that $J^{l+1} = J^l - E_l^{++}$ to omit the efficient DMUs from future DEA runs. If $J^{l+1} = \varnothing$ then stop.

Step 3: assess the new subset, J^{l+1} by models (1) and (2) to obtain a new set of efficient DMUs, E_l^{++} (the new efficient frontier).

Step 4: Let l = l + 1. Go to step 2.

Theorem 1. $\phi^{*+}(l,k) \le \tilde{\phi}^*(l,k) \le \phi^{*-}(l,k)$ that $\phi^{*-}(l,k)$ and $\phi^{*+}(l,k)$ are obtained from (1) and (2) in order and $\tilde{\phi}^*(l,k)$ can be obtained by following model:

$$\tilde{\phi}^*(l,k) = \underset{\lambda_j,\, \phi^-(l,k)}{\text{Max}} \ \tilde{\phi}(l,k)$$
$$\text{s.t.} \ \sum_{j\in F(J^l)} \lambda_j \tilde{x}_j \le \tilde{x}_k,$$
$$\sum_{j\in F(J^l)} \lambda_j \tilde{y}_j \ge \tilde{\phi}(l,k) \tilde{y}_k, \qquad (3)$$
$$\lambda_j \ge 0, \ j\in F(J^l).$$

Proof. First we show that $\tilde{\phi}^*(l,k) \le \phi^{*-}(l,k)$. For this purpose we choose the optimal solution $(\lambda_j^*, \tilde{\phi}^*(l,k))$ of model (3); at the same time with we have:

$$\begin{cases} \sum_{j\in F(J^l)} \lambda_j^* \tilde{x}_j \le \tilde{x}_k \Leftrightarrow \sum_{\substack{j\in F(J^l)\\ j\neq k}} \lambda_j^* x_j^L \le \sum_{\substack{j\in F(J^l)\\ j\neq k}} \lambda_j^* \tilde{x}_j \le \tilde{x}_k \left(1-\lambda_k^*\right) \le x_k{}^U \left(1-\lambda_k^*\right) \\ \sum_{j\in F(J^l)} \lambda_j^* \tilde{y}_j \ge \tilde{\phi}(l,k)\tilde{y}_k \Leftrightarrow \sum_{\substack{j\in F(J^l)\\ j\neq k}} \lambda_j^* y_j^U \ge \sum_{\substack{j\in F(J^l)\\ j\neq k}} \lambda_j^* \tilde{x}_j \ge \tilde{y}_k \left(\tilde{\phi}^*(l,k)-\lambda_k^*\right) \ge y_k{}^L \left(\tilde{\phi}^*(l,k)-\lambda_k^*\right) \end{cases}$$

$$\Rightarrow \begin{cases} \lambda_o{}^* x_{io}{}^U + \sum_{\substack{j\in J^l\\ j\neq o}} \lambda_j{}^* x_{ij}{}^L \le x_{io}{}^U, \qquad i=1,...,m, \\ \lambda_o{}^* y_{ro}{}^L + \sum_{\substack{j\in J^l\\ j\neq o}} \lambda_j{}^* y_{rj}{}^U \ge \tilde{\phi}^*(l,k) y_{ro}{}^L, \ r=1,...,s, \end{cases}$$

So $(\lambda_j^*, \tilde{\phi}^*(l,k))$ is a feasible solution for model (1) and since the objective function of the model (1) has been maximized the theorem is proved, $\tilde{\phi}^*(l,k) \le \phi^{*-}(l,k)$;

In the same way we can prove that; $\phi^{*+}(l,k) \le \tilde{\phi}^*(l,k)$.

Showing that these sets E_l^{++} of DMUs have following properties is easy:

(1) $J^l = \cup E_l^{++}$ and $E_l^{++} \cap E_{l'}^{++}$ for $l \neq l'$.

(2) The DMUs in $E^{l'}$ are Dominated through some of the DMUs in E_l^{++} if $l \le l'$;

(3) DMUs in the set E_l^{++} is efficient with due attention to DMUs in set $l < l'$.

Now context-dependent DEA with E_l^{++} can measure attractiveness of DMUs. Evaluate a specific DMU_q from a specific level E_l^{++} (l=1,..,L). Attractiveness is obtained from models below:

$$\begin{aligned} \Omega_q^{-*}(d) = \max_{\lambda_j, \Omega_q^-(d)} \ & \Omega_q^-(d), \ d=1,...,L-l_o \\ \text{s.t.} \ & \sum_{j\in F(E_{l_o+d}^{++})} \lambda_j x_j^L \le x_q^U, \\ & \sum_{j\in F(E_{l_o+d}^{++})} \lambda_j y_j^U \ge \Omega_q^-(d) y_q^L, \\ & \lambda_j \ge 0, \ j\in F(E_{l_o+d}^{++}). \end{aligned} \tag{4}$$

$$\begin{aligned} \Omega_q^{+*}(d) = \max_{\lambda_j, \Omega_q^+(d)} \ & \Omega_q^+(d), \ d=1,...,L-l_o \\ \text{s.t.} \ & \sum_{j\in F(E_{l_o+d}^{++})} \lambda_j x_j^U \le x_q^L, \\ & \sum_{j\in F(E_{l_o+d}^{++})} \lambda_j y_j^L \ge \Omega_q^-(d) y_q^U, \\ & \lambda_j \ge 0, \ j\in F(E_{l_o+d}^{++}). \end{aligned} \tag{5}$$

$\tilde{\Omega}_q^*(d)$ can be obtained by following model:

$$\tilde{\Omega}_q^*(d) = \underset{\lambda_j,\tilde{\Omega}_q(d)}{\text{Max}}\ \tilde{\Omega}_q(d),\ d = 1,...,L-l_o$$
$$\text{s.t.} \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j \tilde{x}_j \le \tilde{x}_q, \qquad i = 1,...,m,$$
$$\sum_{j\in F(E^{++}_{l_o+d})} \lambda_j \tilde{y}_j \ge \tilde{\Omega}_q(d)\tilde{y}_q, \quad r = 1,...,s, \tag{6}$$
$$\lambda_j \ge 0,\ j \in F(E^{++}_{l_o+d}).$$

where $DMU_q = (x_q, y_q)$ is from a particular level $E^{++}_{l_o}; l_o \in \{1,...,L-1\}$. We have:

$$(1)\begin{cases} A_q^{+^*}(d) > 1 \\ \tilde{A}_q^*(d) > 1 \\ A_q^{-^*}(d) > 1 \end{cases} \text{ for each } d = 1,...,L-l_o .(2)\begin{cases} \Omega_q^{+^*}(d+1) < \Omega_q^{+^*}(d) \\ \tilde{\Omega}_q^*(d+1) < \tilde{\Omega}_q^*(d) \\ \Omega_q^{-^*}(d+1) < \Omega_q^{-^*}(d) \end{cases}.$$

Definition 1.

$A_q^{*+}(d) = \dfrac{1}{\Omega_q^{+^*}(d)}$ is called the (output-oriented) d-degree attractiveness of DMU_q from a specific level $E^{++}_{l_o}$. When DMU_q is in the best condition, and DMUs in the evaluation context $l_o + d$; d=1,…, $L - l_o$ are in their worst condition .

$\tilde{A}_q^*(d) = \dfrac{1}{\tilde{\Omega}_q^*(d)}$ is called the (output-oriented) d-degree attractiveness of DMU_q from a specific level $E^{++}_{l_o}$. When $\begin{cases} x_q^L \le \tilde{x}_q \le x_q^U \\ y_q^L \le \tilde{y}_q \le y_q^U \end{cases}$ and DMUs in the evaluation context $l_o + d$; d=1,…, $L - l_o$ are in $\begin{cases} x_j^L \le \tilde{x}_j \le x_j^U \\ y_j^L \le \tilde{y}_j \le y_j^U \end{cases}$.

$A_q^{*-}(d) = \dfrac{1}{\Omega_q^{-^*}(d)}$ is called the (output-oriented) d-degree attractiveness of DMU_q from a specific level $E^{++}_{l_o}$. When DMU_q is in its own worst condition and DMUs in the evaluation context $l_o + d$; d=1,…, $L - l_o$ are in their own best condition .

Theorem 2. $\Omega_q^{+^*}(d) \le \tilde{\Omega}_q^*(d) \le \Omega_q^{-^*}(d)$ that $\Omega_q^{+^*}$, $\tilde{\Omega}_q^*(d)$ and $\Omega_q^{-^*}$ are obtained from (5), (6) and (4).

Proof. First we show that $\tilde{\Omega}_q^*(d) \le \Omega_q^{-^*}(d)$. For this purpose we choose the optimal solution of the model $(\tilde{2})$, $(\lambda^*, \tilde{\Omega}_q^*(d))$; at the same time with we have:

$$\begin{cases} \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j \tilde{x}_j \le \tilde{x}_q \Leftrightarrow \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j^* x_j^L \le \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j^* \tilde{x}_j \le \tilde{x}_q \le x_q^U \\ \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j \tilde{y}_j \ge \tilde{\Omega}_q^*(d)\tilde{y}_q \Leftrightarrow \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j^* y_j^U \ge \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j \tilde{y}_j \ge \tilde{\Omega}_q^*(d)\tilde{y}_q \ge \tilde{\Omega}_q^*(d) y_q^L \end{cases}$$

$$\begin{cases} \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j^* x_{ij}^L \le x_q^U \\ \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j^* y_j^U \ge \tilde{\Omega}_q^*(d) y_q^L \end{cases}$$

So $(\lambda^*, \tilde{\Omega}_q^*(d))$ is a feasible solution for model (4), and since the objective function of the model (4) has been maximized the theorem is proved, $\tilde{\Omega}_q^*(d) \le \Omega_q^{-*}(d)$.

In the same way we can prove that; $\Omega_q^{+*}(d) \le \tilde{\Omega}_q^*(d)$.

Let each DMU in the first-level efficient frontier represents a choice. We usually compare a specific DMU in $E^{++}_{l_o}$ with other DMUs that are in the same level as well as with relevant choices that be used as evaluation contexts, i.e., the relevant choices are those DMUs in the second-or third-level efficient frontier, etc. with given the alternatives (evaluation contexts), we are enabled by using models (4), (5), (6) to select the best option or the most attractive one. In models (4), (5), (6), each efficient frontier of $E^{++}_{l_o+d}$ is an assessment context for measuring the relative attractiveness of DMUs in $E^{++}_{l_o}$.

Note that $\begin{cases} A_q^{+*}(d) > 1 \\ \tilde{A}_q^*(d) > 1 \\ A_q^{-*}(d) > 1 \end{cases}$. The larger the value of $\begin{cases} A_q^{+*}(d) \\ \tilde{A}_q^*(d) \\ A_q^{-*}(d) \end{cases}$, the more attractive the DMU_q is, because this DMU_q makes itself more distinctive and different from the assessment context $E^{++}_{l_o+d}$. This property enables us to rank the DMUs in $E^{++}_{l_o}$ based upon their attractiveness scores and recognize the best one.

The progress measure for a specific $DMU_q \in E^{++}_{l_o}, l_o \in \{2,...,L\}$ is obtained in the following context-dependent DEA;

$$\begin{aligned} P_q^{*-}(g) = \underset{\lambda_j, P_q^-(g)}{Max} \quad & P_q^-(g), \quad g = 1,...,l_o - 1 \\ s.t. \quad & \sum_{j\in F(E^{++}_{l_o-g})} \lambda_j x_j^L \le x_q^U, \\ & \sum_{j\in F(E^{++}_{l_o-g})} \lambda_j y_j^U \ge P_q^-(g) y_q^L, \\ & \lambda_j \ge 0, \ j \in F(E^{++}_{l_o-g}). \end{aligned} \tag{7}$$

$$\begin{aligned} P_q^{*+}(g) = \underset{\lambda_j, P_q^-(g)}{Max} \quad & P_q^+(g), \quad g = 1,...,l_o - 1 \\ s.t. \quad & \sum_{j\in F(E^{++}_{l_o-g})} \lambda_j x_j^U \le x_q^L, \\ & \sum_{j\in F(E^{++}_{l_o-g})} \lambda_j y_j^L \ge P_q^+(g) y_q^U, \\ & \lambda_j \ge 0, \ j \in F(E^{++}_{l_o-g}). \end{aligned} \tag{8}$$

$\tilde{P}_q^*$ can be obtained by following model:

$$
\begin{aligned}
\tilde{P}_q^*(g) = \underset{\lambda_j, \tilde{P}_q(g)}{\text{Max}} \ \tilde{P}_q(g), \quad g = 1, ..., l_o - 1 \\
\text{s.t.} \sum_{j \in F(E_{l_o-g}^{++})} \lambda_j \tilde{x}_j \le \tilde{x}_q, \\
\sum_{j \in F(E_{l_o-g}^{++})} \lambda_j \tilde{y}_j \ge \tilde{P}_q(g) \tilde{y}_q, \\
\lambda_j \ge 0, \ j \in F(E_{l_o-g}^{++}).
\end{aligned} \tag{9}
$$

Theorem 3. $P_q^{*+}(g) \le \tilde{P}_q^*(g) \le P_q^{*-}(g)$ that $P_q^{*+}(g)$, $\tilde{P}_q^*(g)$ And $P_q^{*-}(g)$ are obtained from (7), (9) and (8).

Proof. First we show that $\tilde{P}_q^*(g) \le P_q^{*-}(g)$. For this purpose, we choose the optimal solution of the of model (9) ; At the same time with we have:

$$
\begin{cases}
\sum_{j \in F(E_{l_o-g}^{++})} \lambda_j^* \tilde{x}_j \le \tilde{x}_q \Leftrightarrow \sum_{j \in F(E_{l_o-g}^{++})} \lambda_j^* x_j^L \le \sum_{j \in F(E_{l_o-g}^{++})} \lambda_j^* \tilde{x}_j \le \tilde{x}_q \le x_q^U \\
\sum_{j \in F(E_{l_o-g}^{++})} \lambda_j^* \tilde{y}_j \ge \tilde{P}_q^*(g) \tilde{y}_q \Leftrightarrow \sum_{j \in F(E_{l_o-g}^{++})} \lambda_j^* y_q^U \ge \sum_{j \in F(E_{l_o-g}^{++})} \lambda_j^* \tilde{y}_j \ge \tilde{P}_q^*(g) \tilde{y}_j \ge \tilde{P}_q^*(g) y_q^L
\end{cases}
$$

$$
\begin{cases}
\sum_{j \in F(E_{l_o-g}^{++})} \lambda_j^* x_j^L \le x_q^U \\
\sum_{j \in F(E_{l_o-g}^{++})} \lambda_j^* y_j^U \ge \tilde{P}_q^*(g) y_q^L
\end{cases}
$$

So $(\lambda^*, \tilde{P}_q^*(g))$ is a feasible solution for model (7) and since the objective function of the model (7) has been maximized the theorem is proved, $\tilde{P}_q^*(g) \le P_q^{*-}(g)$.

In the same way we can prove that; $P_q^{*+}(g) \le \tilde{P}_q^*(g)$.

We have; $\begin{cases} P_q^{+^*}(g+1) > P_q^{+^*}(g) \\ \tilde{P}_q^*(g+1) > \tilde{P}_q^*(g) \\ P_q^{-^*}(g+1) > P_q^{-^*}(g) \end{cases}$.

Definition 2.
$P_q^{+^*}(g)$ is called the (output-oriented) g-degree progress of DMU_q from a specific level $E_{l_o}^{++}$. When DMU_q is in the best condition and DMUs in the evaluation context $l_o - g$; $g = 1, ..., l_o - 1$ are in their worst condition.

$\tilde{P}_q^*(g)$ is called the (output-oriented) g-degree progress of DMU_q from a specific level $E_{l_o}^{++}$.

when $\begin{cases} x_q^L \le \tilde{x}_q \le x_q^U \\ y_q^L \le \tilde{y}_q \le y_q^U \end{cases}$ is and DMU_j ، $j \in F(E_{l_o-g}^{++})$ in the evaluation context $l_o - g \ ; g = 1,...,l_o - 1$ are in $\begin{cases} x_j^L \le \tilde{x}_j \le x_j^U \\ y_j^L \le \tilde{y}_j \le y_j^U \end{cases}$.

$P_q^{-^*}(g)$ is called the (output-oriented) g-degree progress of DMU_q from a specific level $E_{l_o}^{++}$. When DMU_q is in its own worst condition and DMUs in the evaluation context $l_o - g \ ; g = 1,\ldots, l_o - 1$ are in their own worst condition.

Each efficient frontier, $E_{l_o-g}^{++}$, includes a possible purpose for a particular DMU in $E_{l_o}^{++}$ to improve its performance. Here the progress is a level-by-level improvement. A larger $\begin{cases} P_q^{+^*}(g) \\ \tilde{P}_q^*(g) \\ P_q^{-^*}(g) \end{cases}$, Shows more progress for DMU_q. Thus, a smaller value of $\begin{cases} P_q^{+^*}(g) \\ \tilde{P}_q^*(g) \\ P_q^{-^*}(g) \end{cases}$ is preferred.

3 Context-dependent DEA with value judgment

Both attractiveness and progress are measured radially with due attention to different levels of efficient frontiers, in the previous section. The measurement does not need a priori information on the importance of the attributes (input/output) that feature the performance of DMUs. However, different properties play different roles in the evaluation of a DMU's overall performance. Therefore, we present value judgment into the context-dependent DEA.

3.1 Incorporating value judgment into attractiveness measure

In order to incorporate value judgment into our measures of attractiveness and progress, we first specify a set of weights related to the s outputs, u_r (r =1,…, s) such that $\sum_{r=1}^{s} u_r = 1$.

Based upon [5], we develop the following linear programming problem for $DMU_q =$ $(x_q, y_q) = (x_{1q},...,x_{mq}, y_{1q},...,y_{sq})$ in $E_{l_o}^{++}$, $l_o \in \{1,\ldots,L-1\}$:

$$\begin{aligned}
&\Phi_q^{-^*}(d) = \underset{\lambda_j, \Phi_q^{r^-}(d)}{\text{Max}} \sum_{r=1}^{s} u_r \Phi_q^{r^-}(d)\ , \ d = 1,\dots, L - l_o \\
&\text{s.t.} \sum_{j \in F(E_{l_o+d}^{++})} \lambda_j x_{ij}^L \le x_{iq}^U\ , \qquad i = 1,\dots, m, \\
&\qquad \sum_{j \in F(E_{l_o+d}^{++})} \lambda_j y_{rj}^U \ge \Phi_q^{r^-}(d) y_{rq}^L, \ r = 1,\dots, s, \\
&\qquad \Phi_q^{r^-}(d) \le 1, \qquad r = 1,\dots, s, \\
&\qquad \lambda_j \ge 0, \ j \in F(E_{l_o+d}^{++}).
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\Phi_q^{+^*}(d) = \underset{\lambda_j, \Phi_q^{r^+}(d)}{\text{Max}} \sum_{r=1}^{s} u_r \Phi_q^{r^+}(d)\ , \ d = 1,\dots, L - l_o \\
&\text{s.t.} \sum_{j \in F(E_{l_o+d}^{++})} \lambda_j x_{ij}^U \le x_{iq}^L, \qquad i = 1,\dots, m, \\
&\qquad \sum_{j \in F(E_{l_o+d}^{++})} \lambda_j y_{rj}^L \ge \Phi_q^{r^+}(d) y_{rq}^U, \quad r = 1,\dots, s, \\
&\qquad \Phi_q^{r^+}(d) \le 1, \qquad r = 1,\dots, s, \\
&\qquad \lambda_j \ge 0, \ j \in F(E_{l_o+d}^{++}).
\end{aligned} \tag{11}$$

$\tilde{\Phi}_q^*$ can be obtained by following model:

$$\begin{aligned}
&\tilde{\Phi}_q^*(d) = \underset{\lambda_j, \tilde{\Phi}_q^{r}(d)}{\text{Max}} \sum_{r=1}^{s} u_r \tilde{\Phi}_q^{r}(d), \ d = 1,\dots, L - l_o \\
&\text{s.t.} \sum_{j \in F(E_{l_o+d}^{++})} \lambda_j \tilde{x}_{ij} \le \tilde{x}_{iq}, \qquad i = 1,\dots, m, \\
&\qquad \sum_{j \in F(E_{l_o+d}^{++})} \lambda_j \tilde{y}_{rj} \ge \tilde{\Phi}_q^{r}(d) \tilde{y}_{rq}, \ r = 1,\dots, s, \\
&\qquad \tilde{\Phi}_q^{r}(d) \le 1, \qquad r = 1,\dots, s, \\
&\qquad \lambda_j \ge 0, \quad j \in F(E_{l_o+d}^{++}).
\end{aligned} \tag{12}$$

Theorem 4. $\Phi_q^{+^*}(d) \le \tilde{\Phi}_q^*(d) \le \Phi_q^{-^*}(d)$ that $\Phi_q^{-^*}(d)$, $\tilde{\Phi}_q^*(d)$ and $\Phi_q^{+^*}(d)$ are obtained from (10), (12) and(11) in order .

Proof. First we show that $\tilde{\Phi}_q^*(d) \le \Phi_q^{-^*}(d)$. For this purpose we choose the optimal solution of model (12); At the same time with we have:

$$\begin{cases} \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j \tilde{x}_{ij} \le \tilde{x}_{iq} & \Leftrightarrow \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j x^L_{ij} \le \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j \tilde{x}_{ij} \le \tilde{x}_{iq} \le x^U_{iq}, \\ \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j \tilde{y}_{rj} \ge \tilde{\Phi}^{r^*}_q(d)\tilde{y}_{rq} & \Leftrightarrow \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j y^U_{rj} \ge \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j \tilde{y}_{rj} \ge \tilde{\Phi}^{r^*}_q(d)\tilde{y}_{rq} \ge \tilde{\Phi}^{r^*}_q(d) y^L_{rq}, \end{cases}$$

$$\begin{cases} \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j x^L_{ij} \le x^U_{iq}, & i=1,\dots,m, \\ \sum_{j\in F(E^{++}_{l_o+d})} \lambda^*_j y^U_{rj} \ge \tilde{\Phi}^{r^*}_q(d) y^L_{rq}, & r=1,\dots,s, \end{cases}$$

So $(\lambda^*, \tilde{\Phi}^{r^*}_q(d), \tilde{\Phi}^*_q(d))$ is a feasible solution for model (10), and since the objective function of the model (10) has been maximized the theorem is proved, $\tilde{\Phi}^*_q(d) \le \Phi^{-^*}_q(d)$.

In the same way we can prove that; $\Phi^{+^*}_q(d) \le \tilde{\Phi}^*_q(d)$.

Definition 3.

$\bar{\bar{A}}^{+^*}_q(d) \equiv \dfrac{1}{\Phi^{+^*}_q(d)}$ is called the (output-oriented) value judgment (VJ) attractiveness of DMU_q from a specific level $E^{++}_{l_o}$. When DMU_q is in the best condition and DMUs in the evaluation context $l_o + d; d = 1,\dots, L - l_o$ are in their best condition.

$\tilde{\bar{\bar{A}}}^*_q(d) \equiv \dfrac{1}{\tilde{\Phi}^*_q(d)}$ is called the (output-oriented) value judgment (VJ) d-degree attractiveness of DMU_q from a specific level $E^{++}_{l_o}$. When $\begin{cases} x^L_q \le \tilde{x}_q \le x^U_q \\ y^L_q \le \tilde{y}_q \le y^U_q \end{cases}$ is and DMU_j , $j \in F(E^{++}_{l_o+d})$ in the evaluation context $l_o + d; d = 1,\dots, L - l_o$ are in $\begin{cases} x^L_j \le \tilde{x}_j \le x^U_j \\ y^L_j \le \tilde{y}_j \le y^U_j \end{cases}$.

$\bar{\bar{A}}^{-^*}_q(d) \equiv \dfrac{1}{\Phi^{-^*}_q(d)}$ is called the (output-oriented) value judgment (VJ) d-degree attractiveness of DMU_q from a specific level $E^{++}_{l_o}$. When DMU_q is in its own worst condition and DMUs in the evaluation context $l_o + d; d = 1,\dots, L - l_o$ are in their own best condition.

It is obvious that $\begin{cases} \bar{\bar{A}}^{+^*}_q(d) > 1 \\ \tilde{\bar{\bar{A}}}^*_q(d) > 1 \\ \bar{\bar{A}}^{-^*}_q(d) > 1 \end{cases}$. The larger score $\begin{cases} \bar{\bar{A}}^{+^*}_q(d) \\ \tilde{\bar{\bar{A}}}^*_q(d) \\ \bar{\bar{A}}^{-^*}_q(d) \end{cases}$ is, the more attractive the DMU_q shows under the weights u_o (u=1,…,s). Now we can rank DMUs in the same level by their VJ attractiveness scores integrated with the preferences over outputs.

We can prioritize the DMUs with higher values of the r_o th output with increasing the value of the corresponding weight u_{ro}. These user-specified weights appear the relative degree of desirability of the outputs .The constraints of $\begin{cases} \Phi_q^{+^*}(d) < 1 \\ \tilde{\Phi}_q^*(d) < 1 \\ \Phi_q^{-^*}(d) < 1 \end{cases}$ (r = 1,…, s) is for making sure that DMU_q make itself as distinctive as possible, DMU_q is not allowed to decrease some of its outputs to achieve higher levels of other better outputs which is preferred.

Obviously, different weight combinations cause different attractiveness scores. $\begin{cases} \overline{\overline{A}}_q^{+^*}(d) \\ \tilde{\overline{\overline{A}}}_q^*(d) \\ \overline{\overline{A}}_q^{-^*}(d) \end{cases}$ (or $\begin{cases} \Phi_q^{+^*}(d) \\ \tilde{\Phi}_q^*(d) \\ \Phi_q^{-^*}(d) \end{cases}$) is an overall attractiveness of DMU_q in terms of outputs when the inputs don't change their status quo . On the other hand, the attractiveness of DMU_q in terms of each output dimension can be measured through each individual optimal value of $\begin{cases} \dfrac{1}{\Phi_q^{+^*}(d)} \\ \dfrac{1}{\tilde{\Phi}_q^*(d)} \\ \dfrac{1}{\Phi_q^{-^*}(d)} \end{cases}$ (r = 1,…, s).

Note that $\begin{cases} \overline{\overline{A}}_q^{+^*}(d) \\ \tilde{\overline{\overline{A}}}_q^*(d) \\ \overline{\overline{A}}_q^{-^*}(d) \end{cases}$ is not equal to $\begin{cases} \sum_{r=1}^{s} u_r A_q^{r+}(d) \\ \sum_{r=1}^{s} u_r \tilde{A}_q^{r}(d) \\ \sum_{r=1}^{s} u_r A_q^{r-}(d) \end{cases}$ which $\begin{cases} \overline{\overline{A}}_q^{r^{-^*}}(d) = \dfrac{1}{\Phi_q^{+^*}(d)} \\ \tilde{\overline{\overline{A}}}_q^{r^*}(d) = \dfrac{1}{\tilde{\Phi}_q^*(d)} \\ \overline{\overline{A}}_q^{r^{+^*}}(d) = \dfrac{1}{\Phi_q^{-^*}(d)} \end{cases}$.

Definition 4. For $DMU_q \in E_{l_o}^{++}$; $l_o \in \{1,\ldots,L\}$, the optimal value $\begin{cases} \overline{\overline{A}}_q^{r^{-^*}}(d) = \dfrac{1}{\Phi_q^{+^*}(d)} \\ \tilde{\overline{\overline{A}}}_q^{r^*}(d) = \dfrac{1}{\tilde{\Phi}_q^*(d)} \\ \overline{\overline{A}}_q^{r^{+^*}}(d) = \dfrac{1}{\Phi_q^{-^*}(d)} \end{cases}$ is called the (output-oriented) VJ d-degree output-specific attractiveness measure.

Suppose $\begin{cases}\Phi_q^{r^+}(d)y_{rq}^U = y_{rq}^U - s_q^{r^+}(d)\\ \tilde{\Phi}_q^r(d)\tilde{y}_{rq} = \tilde{y}_{rq} - \tilde{s}_q^r(d)\\ \Phi_q^{r^-}(d)y_{rq}^l = y_{rq}^l - s_q^{r^-}(d)\end{cases}$ (r = 1,…, s) in $\begin{cases}(4)^+\\ (\tilde{4})\\ (4)^-\end{cases}$. Since $\begin{cases}\Phi_q^{r^+}(d)\le 1\\ \tilde{\Phi}_q^r(d)\le 1\\ \Phi_q^{r^-}(d)\le 1\end{cases}$, $\begin{cases}s_q^{r^+}(d)\ge 0\\ \tilde{s}_q^r(d)\ge 0\\ s_q^{r^-}(d)\ge 0\end{cases}$, models $\begin{cases}(4)^+\\ (\tilde{4})\\ (4)^-\end{cases}$ is equivalent to the following linear programming problems:

$$\begin{aligned}&\underset{\lambda_j, S_q^{r^-}(d)}{\text{Min}} \sum_{r=1}^{s} \mathrm{D}_r S_q^{r^-}(d),\ d=1,\dots,L-l_o\\ s.t.\ &\sum_{j\in F(E^{++}_{l_o+d})} \lambda_j x_{ij}^L \le x_{iq}^U,\ i=1,\dots,m,\\ &y_{rq}^L - \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j y_{rj}^U = S_q^{r^-}(d), r=1,\dots,s,\\ &S_q^{r^-}(d)\ge 0,\quad r=1,\dots,s,\\ &\lambda_j\ge 0,\quad j\in F(E^{++}_{l_o+d}).\end{aligned}\tag{13}$$

$$\begin{aligned}&\underset{\lambda_j, S_q^{r^+}(d)}{\text{Min}} \sum_{r=1}^{s} \mathrm{D}_r s_q^{r^+}(d),\ d=1,\dots,L-l_o\\ s.t.\ &\sum_{j\in F(E^{++}_{l_o+d})} \lambda_j x_{ij}^U \le x_{iq}^L,\ i=1,\dots,m,\\ &y_{rq}^U - \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j y_{rj}^L = s_q^{r^+}(d), r=1,\dots,s,\\ &s_q^{r^+}(d)\ge 0,\quad r=1,\dots,s,\\ &\lambda_j\ge 0,\quad j\in F(E^{++}_{l_o+d}).\end{aligned}\tag{14}$$

$$\begin{aligned}&\underset{\lambda_j, \tilde{S}_q^{r}(d)}{\text{Min}} \sum_{r=1}^{s} D_r \tilde{S}_q^{r}(d),\ d=1,\dots,L-l_o\\ s.t.\ &\sum_{j\in F(E^{++}_{l_o+d})} \lambda_j \tilde{x}_{ij} \le \tilde{x}_{iq},\ i=1,\dots,m,\\ &\tilde{y}_{rq} - \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j \tilde{y}_{rj} = \tilde{S}_q^{r}(d),\ r=1,\dots,s,\\ &\tilde{S}_q^{r}(d)\ge 0,\quad r=1,\dots,s,\\ &\lambda_j\ge 0,\quad j\in F(E^{++}_{l_o+d}).\end{aligned}\tag{15}$$

Theorem 5. $\underset{\lambda_j, S_q^{r^-}(d)}{Min} \sum_{r=1}^{s} \mathrm{D}_r S_q^{r^-}(d) \ge \underset{\lambda_j, \tilde{S}_q^{r}(d)}{Min} \sum_{r=1}^{s} D_r \tilde{S}_q^{r}(d) \ge \underset{\lambda_j, S_q^{r^-}(d)}{Min} \sum_{r=1}^{s} \mathrm{D}_r S_q^{r^+}(d)$
that

$$\underset{\lambda_j, S_q^{r^-}(d)}{\text{Min}} \sum_{r=1}^{s} \mathrm{D}_r S_q^{r^+}(d),\ \underset{\lambda_j, \tilde{S}_q^{r}(d)}{Min} \sum_{r=1}^{s} D_r \tilde{S}_q^{r}(d) \text{ and } \underset{\lambda_j, S_q^{r^-}(d)}{\text{Min}} \sum_{r=1}^{s} \mathrm{D}_r S_q^{r^-}(d)$$

are obtained from (13), (15) and (14).

Proof. Assume that $(\lambda_j^*, \underset{\lambda_j, S_q^{r^+}(d)}{\text{Min}} \sum_{r=1}^{s} \mathrm{D}_r s_q^{r^+}(d))$ is a optimal solution for model (14). The second constraints of above models can be considered as:

$$\sum_{j\in F(E^{++}_{l_o+d})} \lambda_j y_{rj}^U \ge \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j \tilde{y}_{rj} \ge \sum_{j\in F(E^{++}_{l_o+d})} \lambda_j y_{rj}^L \ge y_{rq}^U \ge \tilde{y}_{rq} \ge y_{rq}^L,$$

So now it is easy to prove.

$\begin{cases} s_q^{r^+}(d) \\ \tilde{s}_q^{r}(d) \\ s_q^{r^-}(d) \end{cases}$ in (13), (15) and (14) can be considered as distance between DMU_q and evaluated contexts (particular efficient frontiers $E_{l_o+d}^{++}$). So, the output-specific attractiveness measure can identify the difference between $DMU_q \in E_{l_o}^{++}$ and $E_{l_o+d}^{++}$ in terms of a particular output. With the output- particular (or input- particular) attractiveness measures. On the other hand, if $\begin{cases} \Phi_q^{r_o^+}(d)=1 \\ \tilde{\Phi}_q^{r_o}(d)=1 \\ \Phi_q^{r_o^+}(d)=1 \end{cases}$, then other DMUs in $E_{l_o+d}^{++}$ or their combinations can also produce the amount of the r_o th output of DMU_q , say, DMU_q does not show better performance with due attention to this specific output dimension. Therefore, DMU_q should improve its performance on the r_o output to identify itself in the future.

3.2 Incorporating value judgment into progress measure

Similar to the development in the previous section, we can define the output-oriented VJ progress measure:

$$\begin{aligned}
\overline{\overline{P}}_q^{-*}(g) = \operatorname*{Max}_{\lambda_j, P_q^{r^-}(g)} \sum_{r=1}^{s} \mathrm{u}_r P_q^{r^-}(g)\ ,\ d=1,\ldots,L-l_o \\
s.t. \quad \sum_{j\in F(E_{l_o-g}^{++})} \lambda_j x_{ij}^{L} \le x_{iq}^{U}\ , i=1,\ldots,m, \\
\sum_{j\in F(E_{l_o-g}^{++})} \lambda_j y_{rj}^{U} \ge P_q^{r^-}(g) y_{rq}^{L}\ ,\quad r=1,\ldots,s, \\
P_q^{r^-}(g) \ge 1, \qquad r=1,\ldots,s, \\
\lambda_j \ge 0, \quad j\in F(E_{l_o-g}^{++}). \qquad (16)
\end{aligned}$$

$$\begin{aligned}
\overline{\overline{P}}_q^{+*}(g) = \operatorname*{Max}_{\lambda_j, P_q^{r^+}(g)} \sum_{r=1}^{s} \mathrm{u}_r P_q^{r^+}(g)\ ,\ g=1,\ldots,l_o-1 \\
s.t. \quad \sum_{j\in F(E_{l_o-g}^{++})} \lambda_j x_{ij}^{U} \le x_{iq}^{L}\ , i=1,\ldots,m, \\
\sum_{j\in F(E_{l_o-g}^{++})} \lambda_j y_{rj}^{L} \ge P_q^{r^+}(g) y_{rq}^{U}\ ,\quad r=1,\ldots,s, \\
P_q^{r^+}(g) \ge 1, \qquad r=1,\ldots,s, \\
\lambda_j \ge 0, \quad j\in F(E_{l_o-g}^{++}). \qquad (17)
\end{aligned}$$

$$\begin{aligned}
\tilde{\overline{\overline{P}}}_q^{*}(g) = \operatorname*{Max}_{\lambda_j, \tilde{P}_q^{r}(g)} \sum_{r=1}^{s} \mathrm{u}_r \tilde{P}_q^{r}(g),\ g=1,\ldots,L-l_o \\
s.t. \quad \sum_{j\in F(E_{l_o-g}^{++})} \lambda_j \tilde{x}_{ij} \le \tilde{x}_{iq}\ ,\quad i=1,\ldots,m, \\
\sum_{j\in F(E_{l_o-g}^{++})} \lambda_j \tilde{y}_{rj} \ge \tilde{P}_q^{r}(g) \tilde{y}_{rq}\ ,\ r=1,\ldots,s, \\
\tilde{P}_q^{r}(g) \le 1, \qquad r=1,\ldots,s, \\
\lambda_j \ge 0, \qquad j\in F(E_{l_o-g}^{++}). \qquad (18)
\end{aligned}$$

Theorem 6. $\overline{\overline{P}}_q^{+^*} \le \tilde{\overline{\overline{P}}}_q^{*} \le \overline{\overline{P}}_q^{-^*}$

that $\overline{\overline{P}}_q^{+^*}$, $\tilde{\overline{\overline{P}}}_q^{*}$ and $\overline{\overline{P}}_q^{-^*}$ are obtained from (16), (18) and (17) in order .

Proof. First we shows that $\tilde{\overline{\overline{P}}}_q^{*} \le \overline{\overline{P}}_q^{-^*}$ for this purpose we choose the optimal solution of model (18); Along with we have:

$$\begin{cases} \sum_{j \in F(E^{++}_{l_o-g})} \lambda^*_{j\,j} \tilde{x}_{ij} \le \tilde{x}_{iq} \Leftrightarrow \sum_{j \in F(E^{++}_{l_o-g})} \lambda^*_j x^L_{ij} \le \sum_{j \in F(E^{++}_{l_o-g})} \lambda^*_j \tilde{x}_{ij} \le \tilde{x}_{iq} \le x^U_{iq}, i=1,\dots,m, \\ \sum_{j \in F(E^{++}_{l_o-g})} \lambda^*_j \tilde{y}_{rj} \ge \tilde{\overline{\overline{P}}}_q^{*}(g)\tilde{y}_{rq} \Leftrightarrow \sum_{j \in F(E^{++}_{l_o-g})} \lambda^*_j y^U_{rj} \ge \sum_{j \in F(E^{++}_{l_o-g})} \lambda^*_j \tilde{y}_{rj} \ge \tilde{\overline{\overline{P}}}_q^{*}(g)\tilde{y}_{rq} \ge \tilde{\overline{\overline{P}}}_q^{*}(g) y^L_{rq}, \end{cases}$$

$$\begin{cases} \sum_{j \in F(E^{++}_{l_o-g})} \lambda^*_j x^L_{ij} \le x^U_{iq}, \quad i=1,\dots,m, \\ \sum_{j \in F(E^{++}_{l_o-g})} \lambda^*_j y^U_{rj} \ge \tilde{\overline{\overline{P}}}_q^{*}(g) y^L_{rq} \quad , r=1,\dots,s, \end{cases}$$

So $(\lambda^*, \tilde{\overline{\overline{P}}}_q^{*})$ is a feasible solution for model (15) and since the objective function of the model (15) has been maximized so that theorem is proved, $\tilde{\overline{\overline{P}}}_q^{*} \le \overline{\overline{P}}_q^{-^*}$.

In the same way we can prove that; $\overline{\overline{P}}_q^{+^*} \le \tilde{\overline{\overline{P}}}_q^{*}$.

Definition 6. The optimal value $\begin{cases} \overline{\overline{P}}_q^{+^*}(g) \\ \tilde{\overline{\overline{P}}}_q^{*}(g) \\ \overline{\overline{P}}_q^{-^*}(g) \end{cases}$ is called the (output-oriented) VJ g-degree progress of DMU_q in a specific level $E^{++}_{l_o}$.

The larger the $\begin{cases} \overline{\overline{P}}_q^{+^*}(g) \\ \tilde{\overline{\overline{P}}}_q^{*}(g) \\ \overline{\overline{P}}_q^{-^*}(g) \end{cases}$ is, the greater progress value is expected for DMU_q . The user-specified weights show the relative degree of desirability of improvement on the individual output levels.

Suppose $\begin{cases} \overline{\overline{P}}_q^{+^*}(g) \\ \tilde{\overline{\overline{P}}}_q^*(g) \\ \overline{\overline{P}}_q^{-^*}(g) \end{cases}$ represent the optimal value of (16), (17), (18) for a specific $g \in \{1,\dots, l_o - 1\}$. By Zhu [4], we know that $\begin{cases} \sum_{j \in F(E^{++}_{l_o-g})} \lambda_j^* y_{rj}^U = P_q^{+^*}(g) y_{rj}^L \\ \sum_{j \in F(E^{++}_{l_o-g})} \lambda_j^* \tilde{y}_{rj} = \tilde{P}_q^*(g) \tilde{y}_{rq} \\ \sum_{j \in F(E^{++}_{l_o-g})} \lambda_j^* y_{rj}^L = P_q^{-^*}(g) y_{rq}^U \end{cases}$ holds in optimal solution for each r = 1,…, s. Take into consideration the following linear programming problem:

$$\begin{aligned} Max \quad & \sum_{r=1}^{m} S_i^{-^-}(g), \ g = 1,\dots,l_o - 1 \\ s.t. \quad & \sum_{j \in F(E^{++}_{l_o-g})} \lambda_j x_{ij}^L + S_i^{-^-}(g) = x_{iq}^U, \ i = 1,\dots,m, \\ & \sum_{j \in F(E^{++}_{l_o-g})} \lambda_j y_{rj}^U = P_q^{r^{-^*}}(g) y_{rq}^L, \quad r = 1,\dots,s, \\ & S_i^{-^-}(g) \ge 0, \qquad i = 1,\dots,m, \\ & \lambda_j \ge 0, \ j \in F(E^{++}_{l_o-g}). \qquad (19) \end{aligned}$$

$$\begin{aligned} Max \quad & \sum_{r=1}^{s} S_i^{-^+}(g), \ g = 1,\dots,l_o - 1 \\ s.t. \quad & \sum_{j \in F(E^{++}_{l_o-g})} \lambda_j x_{ij}^U + S_i^{-^+}(g) = x_{iq}^L, \ i = 1,\dots,m, \\ & \sum_{j \in F(E^{++}_{l_o-g})} \lambda_j y_{rj}^L = P_q^{r^{-^*}}(g) y_{rq}^U, \ r = 1,\dots,s, \\ & s_i^{-^+}(g) \ge 0, \qquad i = 1,\dots,m, \\ & \lambda_j \ge 0, \ j \in F(E^{++}_{l_o-g}). \qquad (20) \end{aligned}$$

$$\begin{aligned} Max \quad & \sum_{r=1}^{s} \tilde{S}_i^-(g), \ g = 1,\dots,l_o - 1 \\ s.t. \quad & \sum_{j \in F(E^{++}_{l_o-g})} \lambda_j \tilde{x}_{ij} + \tilde{S}_i^-(g) = \tilde{x}_{iq}, \ i = 1,\dots,m, \\ & \sum_{j \in F(E^{++}_{l_o-g})} \lambda_j \tilde{y}_{rj} = \tilde{P}_q^{r^*}(g) \tilde{y}_{rq}, \quad r = 1,\dots,s, \\ & \tilde{S}_i^-(g) \ge 0, \qquad r = 1,\dots,s, \\ & \lambda_j \ge 0, \quad j \in F(E^{++}_{l_o-g}). \qquad (21) \end{aligned}$$

Theorem 7. $Max \sum_{i=1}^{m} S_i^{-^+}(g) \ge Max \sum_{r=1}^{s} \tilde{S}_i^-(g) \ge Max \sum_{i=1}^{m} S_i^{-^-}(g)$,

That $Max \sum_{i=1}^{m} S_i^{-^+}(g)$, $Max \sum_{r=1}^{s} \tilde{S}_i^-(g)$ and $Max \sum_{i=1}^{m} S_i^{-^-}(g)$ are obtained from (19), (21) and (20).

Proof. Assume that $(\lambda_j^*, Max \sum_{r=1}^{m} S_i^{-^-}(g))$ is an optimal solution for model (19). Let 's consider the first constraints of above models as;

$\sum_{j\in F(E^{++}_{l_o-g})} \lambda^*_j x^U_{ij} \geq \sum_{j\in F(E^{++}_{l_o-g})} \lambda^*_j \tilde{x}_{ij} \geq \sum_{j\in F(E^{++}_{l_o-g})} \lambda^*_j x^L_{ij} \geq x^U_{iq} \geq \tilde{x}_{iq} \geq x^L_{iq}$, and now it is easy to show that; $Max \sum_{i=1}^{m} S_i^{-^+}(g) \geq Max \sum_{r=1}^{s} \tilde{S}_i^-(g) \geq Max \sum_{i=1}^{m} S_i^{-^-}(g)$.

Definition 7. (Preferred global efficient target and preferred local efficient target) The following points: $\begin{cases} \hat{x}^-_{iq} = x^U_{iq} - S_i^{-^{-*}}(g), i=1,...,m, \\ \hat{y}^-_{rq} = P_q^{r^{-*}}(g) y^L_{rq}, r=1,...,s, \end{cases}$, $\begin{cases} \hat{\tilde{x}}_{iq} = \tilde{x}_{iq} - \tilde{s}_j^{-^*}(g), i=1,...,m, \\ \hat{\tilde{y}}_{rq} = \tilde{P}_q^{r^*}(g) \tilde{y}_{rq}, r=1,...,s, \end{cases}$ and $\begin{cases} \hat{x}^+_{iq} = x^L_{iq} - s_i^{-^{+*}}(g), i=1,...,m, \\ \hat{y}^+_{rq} = P_q^{r^{+*}}(g) y^U_{rq}, r=1,...,s, \end{cases}$.

are preferred global efficient targets for $DMU_q \in E^{++}_{l_o}$, $l_o \in \{2,\ldots,L\}$,

if g = $l_q - 1$; otherwise, if g $< l_q - 1$, they represents preferred local efficient targets, where $\begin{cases} P_q^{*+}(g) \\ \tilde{P}_q^*(g) \\ P_q^{*-}(g) \end{cases}$ are the optimal value in (16), (17) and (18) , and $\begin{cases} s_i^{-^{+*}}(g) \\ \tilde{s}_i^{-*}(g) \\ s_i^{-^{-*}}(g) \end{cases}$ present the optimal values in (19), (21) and (20).

4 Numerical example

Now we will consider the branches of one of the Iran's commercial banks with 3inputs and 5 outputs as our DMUs. The inputs are payable interest, personnel and non-performing loans and the outputs are the total sum of four main deposits, other deposits, loans granted, received interest and fee. These data were collected in 2005.

E_1^{++}={17,11,10,9.,8,7,6,4,1,19}

E_2^{++}={18,16,15,14,5,3,2}

E_3^{++}={20,13,12}

Table 1 Inputs and outputs

Inputs	Outputs
Payable interest	The total sum of four main deposits
Personnel	Other deposits
Non-performing loans	Loans granted
	Received interest
	Fee

Table 2 Input − data for the 20 bank branches

DMU_j	x_{1j}^L	x_{1j}^U	x_{2j}^L	x_{2j}^U	x_{3j}^L	x_{3j}^U
1	5007.37	9613.37	36.29	36.86	87243	87243
2	2926.81	5961.55	18.8	2016	9945	12120
3	8732.7	17752.5	25.74	27.17	47575	50013
4	945.93	1966.39	20.81	22.54	19292	19753
5	8487.07	17521.66	14.16	14.8	3428	3911
6	13759.35	27359.36	19.46	19.46	13929	15657
7	587.69	1205.47	27.29	27.48	27827	29005
8	4646.39	9559.61	24.52	25.07	9070	9983
9	1554.29	3427.89	20.47	21.59	412036	413902
10	17528.31	36297.54	14.84	15.05	8638	10229
11	2444.34	4955.78	20.42	20.54	500	937
12	7303.27	14178.11	22.87	23.19	16148	21353
13	9852.15	19742.89	18.47	21.83	17163	17290
14	4540.75	9312.24	22.83	23.96	17918	17964
15	3039.58	6304.01	39.32	39.86	51582	55136
16	6585.81	13453.58	25.57	26.52	20975	23992
17	4209.18	8603.79	27.59	27.95	41960	43103
18	1015.52	2037.82	13.63	13.93	18641	19354
19	5800.38	11875.39	27.12	27.26	19500	19569
20	1445.68	2922.15	28.96	28.96	31700	32061

Table 3 Output − data for the 20 bank branches

$\mathrm{DMU_j}$	y_{1j}^L	y_{1j}^U	y_{2j}^L	y_{2j}^U	y_{3j}^L	y_{3j}^U	y_{4j}^L	y_{4j}^U	y_{5j}^L	y_{5j}^U
1	2696995	3126798	263643	382545	1675519	1853365	108634.76	125740.28	965.97	6957.33
2	340377	440355	95978	117659	377309	390203	32396.65	37836.56	304.67	749.4
3	1027546	1061260	37911	503089	1233548	1822028	96842.33	108080.01	2285.03	3174
4	1145235	1213541	229646	268460	468520	542101	32362.8	39273.37	207.98	510.93
5	390902	395241	4924	12136	129751	142873	12662.71	14165.44	63.32	92.3
6	988115	1087392	74133	111324	507502	574355	53591.3	72257.28	480.16	869.52
7	144906	165818	180530	180617	288513	323721	40507.97	45847.48	176.58	370.81
8	408163	416416	405396	486431	1044221	1071812	56260.09	73948.09	4654.71	5882.53
9	335070	410427	337971	449336	1584722	1802942	176436.81	189006.12	560.26	2506.67
10	700842	768593	14378	15192	2290745	2573512	662725.21	791463.08	58.89	86.86
11	641680	696338	114183	241081	1579961	2285079	17527.58	20773.91	1070.81	2283.08
12	453170	481943	27196	29553	245726	275717	35757.83	42790.14	375.07	559.85
13	553167	574989	21298	23043	425886	431815	45652.24	50255.75	438.43	836.82
14	309670	342598	20168	26172	124188	126930	8143.79	11948.04	936.62	1468.45
15	286149	317186	149183	270708	787959	810088	106798.63	111962.3	1203.79	4335.24
16	321435	347848	66169	80453	360880	379488	89971.47	165524.22	200.36	399.8
17	618105	835839	244250	404579	9136507	9136507	33036.79	41826.51	2781.24	4555.42
18	248125	320974	3063	6330	26687	29173	9525.6	10877.78	240.04	274.7
19	640890	679916	490508	684372	2946797	3985900	66097.16	95329.87	961.56	1914.25
20	119948	120208	14943	17495	297674	308012	21991.53	27934.19	282.73	471.22

Table 4 The attractiveness scores in E^1 when E^2 and E^3 is chosen as the evaluation context

DMUs	Level	$A_q^{+*}(1)$	$A_q^{-*}(1)$	$A_q^{+*}(2)$	$A_q^{-*}(2)$
1	1	8.474	2.032	18.867	6.25
4	1	17.857	3.021	55.555	16.129
6	1	3.267	1.557	6.289	2.949
7	1	12.987	1.680	58.823	12.345
8	1	22.222	5.714	50	22.222
9	1	12.195	2.298	55.555	17.857
10	1	24.390	9.433	34.482	22.222
11	1	166.666	40	333.333	17.857
17	1	17.241	5.524	30.030	18.867
19	1	13.333	3.861	45.454	13.888

Table 5 The attractiveness scores in E^2 when E^1 and E^3 is chosen as the evaluation context

DMUs	Level	$A_q^{+*}(1)$	$A_q^{-*}(1)$	$P_q^{+*}(1)$	$P_q^{-*}(1)$
2	2	15.625	4.329	1.706	13.510
3	2	23.809	1.941	0.735	4.446
5	2	3.610	2.985	1.300	1.493
14	2	7.194	1.153	1.373	71.062
15	2	17.543	4.255	0.313	6.027
16	2	7.194	1.930	0.768	11.686
18	2	7.692	1.745	1.047	3.610

Table 6 Output-specific attractiveness scores in E^1 (in the best condition) when E^2 is chosen as the evaluation context

<table>
<tr><th>DMUs</th><th>The total sum of four main deposits</th><th>Other deposits</th><th>Loans granted</th><th>Received interest</th><th>Fee</th></tr>
<tr><td rowspan="2">1</td><td>13.698</td><td>3.225</td><td>2.958</td><td>1.481</td><td>7.299</td></tr>
<tr><td colspan="5">DMU 15</td></tr>
<tr><td rowspan="2">4</td><td>28.571</td><td>12.048</td><td>4.578</td><td>2.451</td><td>2.898</td></tr>
<tr><td colspan="5">DMU 15</td></tr>
<tr><td rowspan="2">6</td><td>2.386</td><td>9.433</td><td>1.587</td><td>2.427</td><td>1.517</td></tr>
<tr><td colspan="5">DMU2, DMU3, DMU5</td></tr>
<tr><td rowspan="2">7</td><td>6.211</td><td>1.298</td><td>4.405</td><td>4.608</td><td>3.300</td></tr>
<tr><td colspan="5">DMU 15</td></tr>
<tr><td rowspan="2">8</td><td>1.941</td><td>62.5</td><td>4.716</td><td>4.081</td><td>14.705</td></tr>
<tr><td colspan="5">DMU2, DMU3, DMU 5</td></tr>
<tr><td rowspan="2">9</td><td>5.813</td><td>12.195</td><td>9.259</td><td>7.194</td><td>8.474</td></tr>
<tr><td colspan="5">DMU 15</td></tr>
<tr><td rowspan="2">10</td><td>2.202</td><td>1</td><td>12.987</td><td>29.411</td><td>1</td></tr>
<tr><td colspan="5">DMU3, DMU5, DMU 16</td></tr>
<tr><td rowspan="2">11</td><td>13.888</td><td>333.333</td><td>142.857</td><td>12.820</td><td>250</td></tr>
<tr><td colspan="5">DMU 5</td></tr>
<tr><td rowspan="2">17</td><td>2.247</td><td>8.771</td><td>35.714</td><td>1</td><td>7.246</td></tr>
<tr><td colspan="5">DMU3, DMU15, DMU18</td></tr>
<tr><td rowspan="2">19</td><td>2.066</td><td>27.777</td><td>9.174</td><td>2.518</td><td>2.463</td></tr>
<tr><td colspan="5">DMU2, DMU3, DMU5</td></tr>
</table>

Table 7 Output-speci6c attractiveness scores in E^1 (in the worst condition) when E^2 is chosen as the evaluation context

DMUs	The total sum of four main deposits	Other deposits	Loans granted	Received interest	Fee
1	2.023	1	1	1	1
	DMU3, DMU18				
4	5.076	1.663	1.027	1	1
	DMU3, DMU15				
6	1.706	1	1	1	1
	DMU3, DMU5, DMU 16				
7	1	1.808	1	1	1
	DMU15, DMU18				
8	1	5.555	3.460	1	10.416
	DMU2, DMU5, DMU 16				
9	1	5,007	2.028	2.5	1
	DMU3, DMU15				
10	1.477	1	5.714	23.255	1
	DMU3, DMU5				
11	41.666	32.258	90.909	2.369	58.823
	DMU16				
17	1	1	5.524	1	1
	DMU 2, DMU3, DMU5				
19	1	2.247	3.397	1	1
	DMU2, DMU3, DMU5				

5 Conclusions

To measure the attractiveness and progress of DMUs with due attention to a given evaluation context, context-dependent DEA is developed.

Different levels of efficient frontiers are used as evaluation contexts instead of the traditional first-level efficient frontier. In the basic form of DEA, adding or omitting inefficient DMUs does not change the efficiencies of the existing DMUs and the efficient frontier, but the context-dependent DEA, adding or omitting inefficient DMUs changes the performance of both efficient and inefficient DMUs, i.e., the context-dependent DEA performance depends on not only the efficient frontier, but also the inefficient DMUs. DEA is made by this change more powerful and allowed to locally and globally recognize better choices. Value judgment is integrated into the context-dependent DEA through a particular set of weights showing the preferences over different output (or input) measures. Specially, the attractiveness measure can be used to (i) recognize DMUs that have better performance and (ii) can rank DEA efficient DMUs.

In our application, we are able to differentiate the performance of choices. We have not considered the section 3.3, in [5], that has wrongly obtained the weights.

The input-oriented context-dependent DEA is considered as same as the output-oriented context dependent DEA, though in this case, what we obtain from the attractiveness model is the attractiveness value, and what we obtain from the progress model is the inverted form of the progress value.

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