

# A Probabilistic Model of Learning Fields in Islamic Economics and Finance

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**Abstract** In this paper an epistemological model of learning fields of probabilistic events is formalized. It is used to explain resource allocation governed by pervasive complementarities as the sign of unity of knowledge. Such an episteme is induced epistemologically into interacting, integrating and evolutionary variables representing the problem at hand. The end result is the formalization of a probabilistic model of learning that has no optimal points; only simulating the objective criterion. In such an objective function, variables and their representations are simply learned continuously, and this generates evolutionary general equilibrium points. Such points can be computed by computational general equilibriums under the impact of learning in unity of knowledge. Of all approaches to such a study only the doctrine of the paired universe of the Qur'an stands out as universal and unique. In this regard the Qur'an says (36:36), "Glory to God, Who created in pairs all things that the earth produces, as well as their own kind and things of which they have no knowledge." Upon the episteme of unity is constructed the probabilistic model of learning fields. In it we refer to the specific issue of money-real economy linkages as a resource mobilization problem involving an asset valuation approach. Some empirical evidence is placed in respect of the consequences expected from the study of probabilistic models of learning fields applied to economic and financial problems.

**Keywords** Epistemology, Mathematical Social Science, Simulation, Computational General Equilibrium, Islamic Worldview.

## 1 Introduction

### 1.1 The objectives

The objective of this paper is to substantiate a process-oriented learning that takes place in fields of events governed by pervasive complementarities between representative variables. Such complementarities arise from a foundational knowledge induction of the variables. This phenomenon then triggers a chain of circular causation between the knowledge-induced variables. An epistemological question is invoked being premised on unity of knowledge. The emergent problem to articulate mathematically invokes a probability model of events defined by variables that are continuously learned and complemented between them. The resulting mathematical treatment involves a simulation model that can only be solved according to a probabilistic computational evolutionary general equilibrium approach having no optimum either in the local or global sense – except in the instantaneous sense, which though is of no interest to the learning fields of relations. Only sequences of learned simulacra of approximate

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evaluation exist [1]. A resource allocation problem is explained using such an epistemological process-oriented probabilistic model of computational evolutionary model.

The epistemological premise of this paper is premised on the only known foundation of the worldview of unity of knowledge and the world-system. This is the oneness of the divine law and its application to 'everything'. Such a worldview is referred to as *Tawhid* in the *Qur'an*. Upon this epistemic premise is constructed the probabilistic model of learning fields with especial reference to resource allocation in the case of money-real economy unifying relations in Islamic economics and finance.

## 2 The process-oriented model of learning in unity of knowledge: The Islamic worldview

In none of the received socio-scientific theories is the divine law invoked to establish a substantive worldview of unity of knowledge. We have to turn to the Islamic worldview as the singular case in this respect to explain how the divine law dwells on problems of the world-system in the light of unity of divine knowledge. The claim of this paper is that substantively, unity of knowledge is not possible in any other methodological way.

We explain the methodology of unity of divine knowledge in the Islamic worldview in the following way. Refer to Fig. 1.

Mapping (1) in Fig. 1 denotes derivation of fundamental rules from  $(\Omega, S)$ . This all-pervasive episteme of divine oneness, from which worldly rules and practically guided instruments are derived, is the *Qur'anic* law of monotheism (*Tawhid*). It is explained further by the Guidance of the Prophet Muhammad as the principal teacher of *Qur'anic* monotheism to all the worlds. Subsequently, elements of  $(\Omega, S)$  are taken up for social discourse and understanding in respect of specific issues and problems under investigation.

Mapping (2) in Fig. 1 represents policies, mechanisms and identification of ways and means for enacting the fundamental *Tawhidi* episteme of unity of divine knowledge into actions in the world-systems for specific issues and problems under investigation. This kind of discursive and interactive medium involves the *Shura* once it is induced by the consciousness of the worshipping world (*Tasbih*). A discursive process that does not invoke *Tasbih* cannot recognize the endogenous power of the moral and ethical values and their knowledge-induced processes and worldly determinations. Only the *Tasbih-Shura* initiation of knowledge in specific issues and problems under investigation, and their interconnections with widening domains of the world-systems endogenizes the individual and social preferences, productive menus, institutions and management, policies and programs in the Islamic worldly affairs (*Muamalat*).

Mapping (3) in Fig. 1 is the formation of the paired variables that span the issues and problems under investigation in the *Tawhidi* world-system. The paired variables signify unity of knowledge generated from  $(\Omega, S)$  by organic learning, both intra-system and inter-systems in the sense of a generalized knowledge  $(\theta)$ -induced vectors and matrices of variables  $\mathbf{x}(\theta)$  of investigated world-system spanned by  $\{(\theta, \mathbf{x}(\theta), t)\}$  over time 't'.

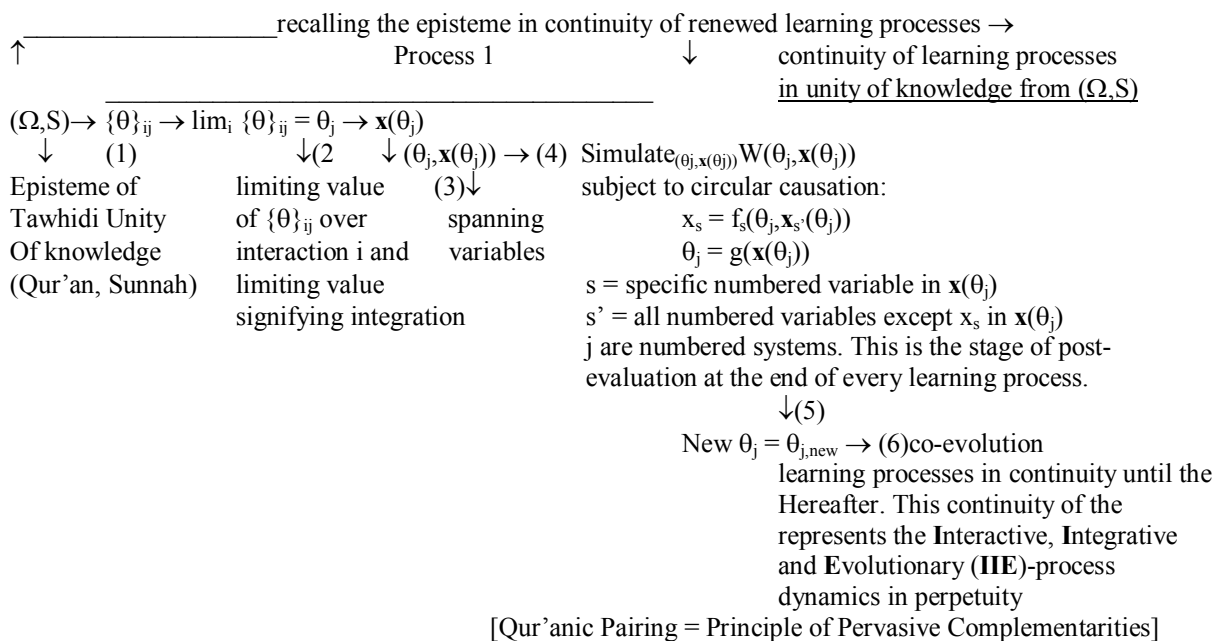
Mapping (4) in Fig. 1 denotes the post-evaluation stage that estimates the degree to which unity of knowledge has been attained intra-system and inter-system by the preceding learning processes in unity of knowledge. The simulation exercise points out the continuous learning processes of interaction, integration and evolution (IIE) that takes place by continuously recalling the episteme  $(\Omega, S)$  at every stage of systemic learning.

In the simulative post-evaluation stage, circular causation shows the estimated values of the interconnected variables under the learning rules, policies and programs arising from the formation of  $(\theta, \mathbf{x}(\theta))$  in stage (3). There is no assertion of a completely unified world-system in the light of  $(\Omega, S)$ . Only dynamic improvement and reconstruction leading to higher levels of systemic unity enabled by organic learning are relevant.

Mapping (5) of Fig. 1 denotes continuity of the Interactive (stage 1), Integrative (stage 1 and stage 2) and Evolutionary (stage 4) dynamics (IIE) of the complete one-process of the learning phase. Thus, the *Tasbih-Shura* learning dynamics is carried into matter and mind complements by the IIE-process methodology as it is derived from and recalled by  $(\Omega, S)$ .

It is important to note that the estimated  $\theta$ -value in any learning process is averaged from the  $\theta$ -weights linked with the performance of the corresponding  $\mathbf{x}(\theta)$ -values. But the  $\theta_{\text{new}}$ -value is derived from fresh discourse by recalling  $(\Omega, S)$  along the evolutionary learning experience in unity of knowledge. See Choudhury, Zaman and Nassar [2] for some details on the method used in respect of  $\theta$ -values. Algorithmic methods of assigning  $\theta$ -values to degrees of complementarities or departure from there from can be developed.

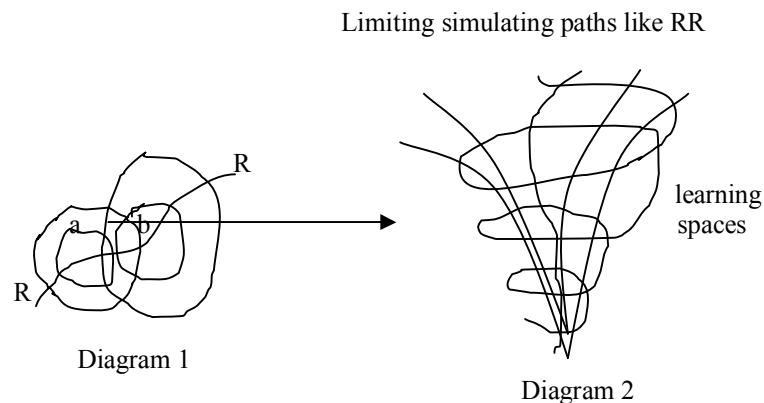
The entire IIE-process dynamics is characterized by learning in unity of knowledge premised on  $(\Omega, S)$ . Besides, in every process, post-evaluation by simulation of  $W(\cdot)$  subject to circular causation between the variables defining  $W(\cdot)$  measures the attained and reformulated strategies for evolution into higher levels of unity of knowledge and the unified world-system. In this way, our terminology of the Principle of Pervasive Complementarities is carried through by the IIE-process methodology in view of the *Qur'anic* paired universe [3]. Pervasive complementarities are carried across intra- and inter- world-systems, given the specific issues and problems to investigate.



**Fig. 1** Characterization of the *Tawhidi* Learning Processes in Unity of Knowledge and the World-System

### 3 An example of resource allocation using the pervasive complementary principle of unity of knowledge

The process-oriented learning evoked by Fig. 1 is entrenched in the learning field shown by the region around 'a' in Fig. 2. The region expands in continuums around 'a', and hence all along the resource allocation path RR in Fig. 2. But since every path of RR experiences the same synergetic dynamics, therefore in inter-systemic sense the system embedded in 'a' interacts, integrates and evolves in complementary connection with 'b', etc. RR therefore represents the continuum of simulated trajectory with learning in unity of knowledge by circular causation between the variables defining events in the regions like 'a', 'b' etc in Fig. 2. Such circular causations and their expansion by the IIE-dynamics are inherent in the phenomenological dynamics of Fig. 1 and now shown in Fig. 2.



**Fig. 2** Resource allocation in learning fields

As an example of resource allocation in Fig. 2 we take  $M$  as the monetary side of the economy signifying monetary policy transmission using the quantity of money. Let  $F$  denote the fiscal side signifying productive spending in the good things of life. When the rate of return ' $r$ ' and not the rate of interest influences  $M$ , then ' $r$ ' is found to influence  $M$  and  $F$  as complements. A complementary system of feedback is generated. This is referred to here as circular causation between complementary variables. This contradicts the marginalist substitution between  $M$  and  $F$  as in the case of received macroeconomic theory.

The complementarities between ' $r$ ', savings = investment, money and interest-free financing instruments are embedded in a learning field of the type shown around 'a' in Fig. 2. 'a'-region makes up an organic synergistic field. The same property is generalized all along the trajectory  $RR$ , wherein pervasive complementarities between the good things of life exist. They can be expanded further by the ethical and social forces of replacing interest by the rate of return everywhere. The learning trajectory  $RR$  complementing  $(M, F)$  in the money-finance and real economy organic interrelations (positive circular causation) under the impact of  $\theta$ -value is the path of socioeconomic sustainability. In the case of the money-real economy relationship for effective monetary transmission the European Central Bank for example, recommends transparent discourse for establishing healthy money-market interrelations and thereby establishing price stability [4].

### 3.1 Perturbations in Fig. 2

Define  $\{.\}$  as the perturbation set of possibilities like in the region bounded by 'a' in Fig. 2. Now  $\{dM/dF\} > 0$ , because as  $\theta$ -value increases so does the complementarities between M and F, and conversely. Also  $\{d^2M/dF^2\}$  can take all possible signs. Thus, no directions toward optimality and steady-state equilibrium can be made. Yet the learning equilibriums of region 'a' are stable in nature. That is because each gravitating point in this field is subject to the continuously evolutionary yet limiting  $\theta$ -values. Such as monetary policy transmission synergizes with the market order along the stable path of sustainability.

### 3.2 The dynamics within the 'a'-region of Fig. 2

Circular causation between the money-finance-real economy variables under the endogenous influence of  $\theta$ -value simulates the wellbeing criterion,  $W(.)$ :

$$\text{Simulate}_{\theta} \quad W=W(M, F, Y, r) [\theta]; \quad (1)$$

$$\text{s.t.} \quad x = f(\theta, x'(\theta)) \quad (2)$$

$$\theta = g(x(\theta)) \quad (3)$$

each of the variables in  $x = (M, F, Y, r)$  is influenced by  $\theta$ . Y is the real output with which M and F complement in the money-real economy circular causation relations.

$\theta$  is assigned ordinal values in respect of the ranking of x-variables.  $g(x(\theta))$  is a monotone of W. Hence estimation of  $\theta$ -function is sufficient for a monotonic estimation of  $W(x(\theta))$ .

New  $\theta$ -value arises by a polity-market restructuring of  $x(\theta)$  at the end of specific learning process in unity of knowledge between the interacting entities.

In the above expressions, each element in  $x = (M, F, Y, r)$  depends on the rest through the common embedding of these variables by  $\theta$ .  $x'$  is the x-vector excluding the selected dependent variable.

The relational system comprising (1) or (3) and (2) forms a stochastic form of the computational general equilibrium model with probabilities that are unlike the frequency probability measure. That is because the 'a'-region contains events that are determined by a combination of historical data and new information arising out of institutional discourse and ethical valuation of future actions. Restructuring is done along these grounds. Thus, the probability measure used in forming the stochastic values of the variables like  $(\theta, x(\theta))$  are measure-theoretic and are governed by conditional probability functions for learning fields. It is understandable that learning fields by their nature of incompleteness (non-optimality) will be intrinsically probabilistic in nature.

#### 4 Probability functions for learning fields

The probability analytics can be summarized as follows [5]:

$$\text{Prob}(x_1(\theta_1) | \theta_1) = \text{Prob}(\theta_1, x_1(\theta_1)) / \text{Prob}(\theta_1); \quad (4)$$

$$\text{i.e. } \text{Prob}(\theta_1, x_1(\theta_1)) = \text{Prob}(x_1(\theta_1) | \theta_1) \cdot \text{Prob}(\theta_1). \quad (5)$$

Likewise,

$$\text{Prob}(\theta_2, x_2(\theta_2)) = \text{Prob}(x_2(\theta_2) | \theta_2) \cdot \text{Prob}(\theta_2),$$

etc.

$$\text{Prob}(\theta_n, x_n(\theta_n)) = \text{Prob}(x_n(\theta_n) | \theta_n) \cdot \text{Prob}(\theta_n) \quad (6)$$

$$\text{Prob}(\theta_1, \theta_2, \dots, \theta_n, x_1(\theta_1), x_2(\theta_2), \dots, x_n(\theta_n)) = \prod_{i=1}^n \text{Prob}(x_i(\theta_i) | \theta_i) \prod_{i=1}^n \text{Prob}(\theta_i). \quad (7)$$

Expression (7) on compound conditional probabilities can be re-written in its equivalent probability density functions (f) and cumulative distribution function (F), respectively as,

$$f(\theta_1, \theta_2, \dots, \theta_n, x_1(\theta_1), x_2(\theta_2), \dots, x_n(\theta_n)) = \prod_{i=1}^n f_i(x_i(\theta_i) | \theta_i) \prod_{i=1}^n f_i(\theta_i) \quad (8)$$

$$F(\theta_1, \theta_2, \dots, \theta_n, x_1(\theta_1), x_2(\theta_2), \dots, x_n(\theta_n)) = \int_{-\infty}^{x_1} \dots \int_{-\infty}^{x_n} \int_{-\infty}^{\theta_1} \dots \int_{-\infty}^{\theta_n} \prod_{i=1}^n f_i(x_i(\theta_i) | \theta_i) \cdot f_i(\theta_i) dx_i \cdot d\theta_i \quad (9)$$

Expression (9) conveys the fact that each event in the set  $\{(\theta_1, \theta_2, \dots, \theta_n, x_1(\theta_1), x_2(\theta_2), \dots, x_n(\theta_n))\}$  is rewritten in the form with  $\text{Event}_i = \{(\theta_i, x_i(\theta_i))\}$  as  $\{E_1, E_2, \dots, E_n\}$ , subject to learning probabilities conveyed by  $\text{Prob}(\theta_i)$  for each  $E_i$ , respectively. These are compounded by the probabilities of occurrence of the state variables  $(x_1(\theta_1), \dots, x_n(\theta_n))$  within themselves and by the occurrence of  $(\theta_1, \theta_2, \dots, \theta_n)$  within their specified system (intra-system). In other words, these events mark the interactively developed integration in the total set  $\{E_1, E_2, \dots, E_n\}$ . This implies the interactive joint occurrence of the two activities, namely market forces and endogenized ethical value guidance, programs and policies. The latter ones are generated by discursive dynamics [6]. The probability of occurrence of such an event increases as learning indicated by a limiting value of  $\theta$  in the interactive, integrative and evolutionary (IIE) set of knowledge variables denoted by  $(\theta_1, \theta_2, \dots, \theta_n)$  increases. We note here that each  $E_i$ , with  $i = 1, 2, \dots, n$  comprises a process in Fig. 1 that leads to subsequent process by continuous learning.

Thus a further simplification is possible:

$$\text{Let } \text{plim}_L \{\theta_1, \theta_2, \dots, \theta_n\} = \theta + \xi_n(\theta); \quad (10)$$

$\xi_n(\theta) > 0$  is a monotonic increasing  $\theta$ -induced impetus to learning probabilities in 'a'-region

Here let  $L$  denote a finite range of interaction leading to the state of consensus (integration) in a derived  $\theta$ -value.  $L$  applies to intra-system, as in Mapping 1 of Fig. 1. Equally it applies to inter-systems, as in evolutionary Diagram 2 of Fig. 2. Besides, since the limiting values emerge over learning sets, there is also evolutionary learning behind the derivation process of  $\theta$ -value.

Likewise, with similar arguments as given above, and the fact that compound probability measures exist also in the socioeconomic set  $\{x_1(\theta_1), \dots, x_n(\theta_n)\}$ , and that there are conditional probabilities for these state values in the set,  $\{(\theta_1, \theta_2, \dots, \theta_n, x_1(\theta_1), x_2(\theta_2), \dots, x_n(\theta_n))\}$ , therefore, the following relations are derived:

$$\text{plim}_L \{x_1(\theta_1), \dots, x_n(\theta_n)\} = x(\theta) + \lambda_n(x(\theta)); \lambda_n(x(\theta)) > 0 \text{ as for } \xi_n(\theta); \quad (11)$$

$$\text{and } \text{plim}_L \{(\theta_1, \theta_2, \dots, \theta_n, x_1(\theta_1), x_2(\theta_2), \dots, x_n(\theta_n))\} = (\theta, x(\theta)) \quad (12)$$

$\mathbf{x}$  denotes the vector of  $x$ -variables.

Expressions (10)-(12) in the sense of  $\text{plim}_L$  now reduce to,

$$\text{Prob}(\theta, x(\theta)) = \text{Prob}(x(\theta) | \theta) \cdot \text{Prob}(\theta) + (\xi_n(\theta), \lambda_n(x(\theta))) \quad (13)$$

$$f(\theta, x(\theta)) = f(x(\theta) | \theta) \cdot f(\theta) + \alpha(\xi_n(\theta), \lambda_n(x(\theta))). \quad (14)$$

$$\text{Let, } \alpha(\theta, x(\theta)) = \alpha((\xi_n(\theta), \lambda_n(x(\theta))))$$

$\alpha(\theta, x(\theta))$  denotes a monotonic positive transformation around the functionals of  $(\theta, x(\theta))$ . Such variations reflect the RR-paths of IIE-processes in learning spaces of  $L$ , as indicated in Diagram 2 of Fig. 2.

$$F(\theta, x(\theta)) = \text{Convolution} \int_{-\infty}^x \int_{-\infty}^0 f(x(\theta) | \theta) \cdot f(\theta) dx \cdot d\theta + \int_{-\infty}^x \int_{-\infty}^0 \alpha(\theta, x(\theta)) dx \cdot d\theta \quad (15)$$

Because of the discoursed learning variables and their induced state variables the data of the probability distributions come from a combination of historical sources discursively or algorithmically generated information in the system of pervasively participatory endogenous relations. They have in them the essence of social interaction leading to integration (consensus) over evolutionary learning experience [7]. Yet such probabilities are computable to convey measures of progressively unraveled socioeconomic states. That is because such probability measures are obtained by progressive experience in learning fields. They are therefore different from those based on the frequency notion of probability measures. They may be compared better with Quantum probabilities [8]. Learning and ethics of relations in unity of knowledge thus brings consciousness into socio-scientific reasoning and analytics, whereas these are absent in the present state of socio-scientific inquiry.

## 5 Application 1: Multivariate Taylor Series expansion in probabilistic learning fields

Consider the nature of multivariate form of Taylor's expansion of the probability distribution function  $f(\theta, x(\theta)) = f(x(\theta) | \theta) \cdot f(\theta) + \alpha(\theta, x(\theta))$  in the neighborhood of 'a', 'b', etc. in the IIE-regions shown in Fig. 2 corresponding to Fig. 1. See Dadkhah [9] for mathematical details. Let the  $\text{plim}\{(\theta, x(\theta))\}$  point be denoted by  $(\theta^*, x^*(\theta^*)) + \alpha^*(\theta^*, x^*(\theta^*))$ .

$$\begin{aligned} f(\theta, x(\theta)) &= f(\theta^*, x^*(\theta^*)) + (\theta - \theta^*) \cdot (\partial/\partial\theta)(f(\theta^*, x^*(\theta^*))) + \\ &(\mathbf{x}(\theta) - \mathbf{x}^*(\theta^*)) \cdot (\partial/\partial\mathbf{x}(\theta))(f(\theta^*, x^*(\theta^*))) + \text{higher terms.} \\ &= f(\mathbf{x}(\theta) | \theta) \cdot f(\theta) + \alpha(\theta, x(\theta)) \end{aligned} \quad (16)$$

Thus,

$$\begin{aligned} f(\mathbf{x}(\theta) | \theta) \cdot f(\theta) &= [f(\theta^*, x^*(\theta^*)) - \alpha(\theta, x(\theta))] + (\theta - \theta^*) \cdot (\partial/\partial\theta)(f(\theta^*, x^*(\theta^*))) + \\ &(\mathbf{x}(\theta) - \mathbf{x}^*(\theta^*)) \cdot (\partial/\partial\mathbf{x}(\theta))(f(\theta^*, x^*(\theta^*))) + \text{higher terms} > 0 \end{aligned} \quad (17)$$

The Taylor expansion is taken in the vector multiplication sense in the terms. Clearly then,

$$[\alpha(\theta, \mathbf{x}(\theta)) - f(\theta^*, \mathbf{x}^*(\theta^*))] > (\theta - \theta^*) \cdot (\partial/\partial\theta)(f(\theta^*, \mathbf{x}^*(\theta^*))) + (\mathbf{x}(\theta) - \mathbf{x}^*(\theta^*)) \cdot (\partial/\partial\mathbf{x}(\theta))(f(\theta^*, \mathbf{x}^*(\theta^*))) + \text{higher terms} \quad (18)$$

All multiplications of variables and terms of expressions are to be taken in the sense of vector-multiplication.

We note that  $(\partial/\partial\theta)(f(\theta^*, \mathbf{x}^*(\theta^*))) > 0$  and  $(\partial/\partial\mathbf{x}(\theta))(f(\theta^*, \mathbf{x}^*(\theta^*))) > 0$ , because of the circular causation relationships between events and their variables under the principle of pervasive complementarities.

Besides,  $\theta > \theta^*$  implies  $\mathbf{x}(\theta) > \mathbf{x}^*(\theta^*)$ . Hence  $[\alpha(\theta, \mathbf{x}(\theta)) - f(\theta^*, \mathbf{x}^*(\theta^*))] > 0$ . The converse is also true around the learning regions of variations along the RR-paths in Fig. 2. We have thus introduced ethical consciousness by learning process in unity of knowledge into socio-scientific problems. Sen [10] refers to this kind of phenomenon as deontological consequentialism.

One such problem we have alluded to in this paper is the money-real economy complementary linkage problem. In this case, the results of expression (18) imply that everyone of the variables in  $\mathbf{x} = (M, F, Y, r)[\theta]$  must be complementary to the rest under the primal impact of  $\theta$ -induction. This is the principal epistemological problem of measured consciousness in socio-scientific systems under the principle of unity of knowledge.

Taylor expansion is applied to the study of random fields of security prices in analytical finance [11]. But this paper has suggested how Taylor expansion of functions in neighborhoods of learning fields is modified by the learning parameter. Thereby, The effect of  $\theta$ -variable is used to stabilize financial security prices by establishing sustainability through the money (M)-real economy (F, Y) and financial (r) complementary linkages [12].

## 6 Application 2: Asset valuation in probabilistic learning fields

Another application of the probabilistic model of learning fields is in respect of the risk and return analysis of complementary processes. The asset valuation model in such a case does not assume rational choice behavior of mainstream finance. That is because the resource scarcity axiom of mainstream economics and finance is abandoned. It is replaced by a continuous reproduction of resources through the medium of continuous learning. The implication is this: With an increase in learning by participation between money, real economy, markets, and agents (investors) a consensus is arrived at as to the nature of economic evolution after considering various different contingencies of the state of nature [13]. Asset evaluation is then done at the proximity of a forward-looking perspective of states of nature and knowledge building. This is referred to as the forward-looking overlapping generation model of asset valuation [14].\*

\*The overlapping generation model for the Islamic case is formalized briefly as follows: There are knowledge-induced cash-flows denoted by  $\{A_t(\theta_t, \mathbf{x}_t(\theta_t))\}$  over time  $(t_0, t_1, \dots, t_m)$ , which are generated by circular causation relations between  $(\theta, \mathbf{x}(\theta))$ -variables in the sense of knowledge-induced simulations of the wellbeing criterion. The circular causation relations implicative of complementarities between the  $\mathbf{x}(\theta)$ -variables are expressed as follows:

$$x_t^*(\theta_t) = f(\theta_{t-1}, \mathbf{x}_t(\theta_{t-1})).$$

Here  $\mathbf{x}_t(\theta_t)$  denotes the vector of variables except the one that becomes the dependent variable sequentially,  $x_t^*(\theta_t)$ , at every point of time in the simulation system of recursive *interrelations*.



Accordingly, evolutionary trends in  $\theta$  are causally interrelated as follows:

$$\theta \uparrow \Leftrightarrow [\theta \rightarrow \theta^*], [x(\theta) \rightarrow x^*(\theta^*)] \quad (19)$$

$$\Leftrightarrow E[f(\theta, x(\theta))] = f(\theta^*, x^*(\theta^*)) + \text{higher positive terms.} \quad (20)$$

$$\text{i.e. } E[f(\theta, x(\theta))] \uparrow$$

$$\text{Besides, } \theta \uparrow \Leftrightarrow \text{Var}[f(\theta, x(\theta))] \downarrow \text{ but non-vanishing.} \quad (21)$$

$$\text{Therefore, } \{dE[f(\theta, x(\theta))]/d\theta\} / \{d\text{Var}[f(\theta, x(\theta))]/d\theta\} < 0 \quad (22)$$

This result is contrary to the one obtained on risk-return analysis by optimization of the expected utility function. The contrary result in our case is caused by the appearance of  $\theta$ -value in the expected utility function, which renders the utility function to be non-optimizable. Only simulation over  $\theta$ -values and the consequential  $E[x(\theta)]$  persist.  $\text{Var}[x(\theta)]$  is possible under the condition of declining risk with increasing expected value caused by the risk- and product- diversifications occurring conterminously.\*

Finally, we reflect back on the probabilistic field surrounding the emergence of  $(\theta, E(x(\theta)), \text{Var}(x(\theta)))$  in the overlapping generation model of asset valuation. Mathematically, the intricacies of the conditional probabilities signify the interactive, integrative and evolutionary learning between market realities, ethical learning by institutional guidance, preference change and unity of knowledge. But in the practical sense of asset valuation for the practitioner, the probability measures are assigned by discourse along the proximity of dated flows of  $z = (\theta, E(x(\theta)), \text{Var}(x(\theta)))$  in asset valuation along the forward movement of the overlapping generation model. Such probabilities never attain the value 1. But they are monotonically related to  $\theta$ -values in the limiting sense of expression (10). Consequently, higher asset valuation is obtained under the following condition:

- $\theta \rightarrow$  monotonically discursively assigned probability values
- $\rightarrow$  monotonically  $\{z = (\theta, E(x(\theta)), \text{Var}(x(\theta)))\}$
- $\rightarrow$  monotonically asset valuation in the overlapping generation model
- $\rightarrow$  monotonically social wellbeing function connected with the overlapping generation model

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Furthermore, for the simulated value of  $\theta$ -values we note:

$$\begin{aligned} \theta'_t &= g(\theta_{t-1}, x_t(\theta_t)), \\ \theta_t &= \text{plim}_{j=1 \rightarrow m} \{\theta_{tj}\}, \end{aligned}$$

The above expressions are true for asset evaluation taking place 'nearest' to every point of time  $t$ .  $j = 1, 2, \dots, m$  are learning processes *within* each time period, for  $t = 1, 2, \dots, n$ .

The overlapping generation valuation model based on the simulated  $(\theta_t, x_t(\theta_t))$ -values with social wellbeing function is,

$$\begin{aligned} \text{Simulate}_{\{\theta_t\}} W(\theta) &= \sum_{t=1}^n [A_t(\theta_t, x_t(\theta_t))] - I_0, \\ &\text{with } \theta \text{ denoting the sequence of consensual } \theta_t\text{-values over time,} \\ &\text{subject to the knowledge-flows and time-dependent circular causationrelations.} \end{aligned}$$

\* The wellbeing objective function, as in expression (1) now is  $U = U(\theta, E(x(\theta)), \text{Var}(x(\theta)))$ . Hence,  $dU/d\theta = (\partial U/\partial E).dE/d\theta + (\partial U/\partial \text{Var}).d\text{Var}/d\theta + \partial U/\partial \theta > 0$ , since each of the terms is non-negative with increasing  $\theta$ -values along the evolutionary learning processes, and with  $(\partial U/\partial \text{Var}) < 0$  and  $d\text{Var}/d\theta < 0$ , as product and risk diversifications are deepened with increasing  $\theta$ -values. This is a non-optimizing condition; it can only be simulated over  $(\theta, E(x(\theta)), \text{Var}(x(\theta)))$

## 7 Evidence\* : non-optimal nature of risk and return relation qua rational behavior

The number of rich households and their wealth in the GCC region (CAGR for liquidity was 7% between 2003 and 2007; number of wealthy households increased by 6 per cent CAGR between 2003 and 2007) increased even as GDP and privatization picked up (CAGR of nominal GDP was 19 per cent between 2002 and 2006). Within the wealth formation there was a 40 per cent share of the capital market that preferred Islamic over conventional funds. There was preference for high returns over stable return among this class of investors. Additionally, 20 per cent of the market share in the GCC region was made up of investors that preferred pure Islamic funds. These investors preferred *Shari'ah*-compliant investments over conventional ones, no matter what was the performance of Islamic funds. The rest of the 40 per cent share of the capital market was of investors in the GCC region preferring conventional funds.

The composition of the Islamic capital market was made up of portfolios of equity (*Sukuk*, CAGR 232 percent between 2002 and 2006; private equities CAGR 36 per cent between 2000 and 2006), real estate (CAGR 36 per cent between 2002 and 2006), and Islamic insurance (*Takaful*, CAGR 13 per cent between 2002 and 2005). In general, Islamic funds increased by CAGR 22 per cent between 2000 and 2006, but with a focus on medium to small enterprises.

The Islamic investment picture for the MENA region is similar. Forty per cent of MENA Islamic investments were in real estate. This is followed by 17 per cent in financial services; 14 per cent in travel and tourism; and 29 per cent in private equities. In all, Islamic funds in MENA experienced a CAGR of 130 per cent between 2002 and 2006.

These trends bring to light the fact that moderate but stable (approximately 7 per cent in Islamic bank-share deposits) expected returns satisfy Islamic investors. This preference over conventional investment funds (60 per cent of all investors in the GCC region) intensifies with increasing Islamic orientation (the  $\theta$ -value). Such investors face risk levels that are lower than the conventional ones. This fact is actualized by a phenomenal increase in *Sukuk* (Islamic bond) funds representing Islamic equities and real estate investments that by their very nature are low risk and with good returns for rich investors. On the other hand, conventional investment funds in the GCC area experienced a sharp market correction, declining in returns by 38 per cent between 2005 and 2006. This decline is followed by very weak recovery in 2007.

Our implications of money-finance-real economy circular causation relations are reflected in the choice of Islamic diversified portfolio contrary to the independence of the financial sector as a competing one with the real sector. Thus, our results of expressions (19) – (22) are supported by the real evidence of the GCC area. The same is found true of MENA Islamic investments and South-East Asia (Bangladesh, Malaysia, and Indonesia).

## 8 Conclusion

It is high time to introduce ethical and scientific consciousness into economics wellbeing after the same kinds of thinking have been introduced in hard core science [15]. Development of consciousness and learning in unity of knowledge produce synonymous results of

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\* Mega Middle East Global Advisors, *The Islamic Funds and Investments Report 2006*, UAE.

sustainability and stability. Ethics and morality are endogenized in such broader fields of choices involving embedded systems. When it is so formalized, the emergent methodology departs away from that of mainstream economics.

The dynamics of pervasive complementarities arising from learning by unity of knowledge, and considerations of ethics and morality remain exogenous factors in economic theory. This paper has formalized ethical endogeneity in models of decision-making with probabilistic learning fields that remain embedded in complementary economic, social and ethical values.

The results obtained by such substantive changes in methodology and reasoning are both epistemological and novel in nature. They contradict many of the existing reasoning and methods underlying mainstream economics and finance. The new reasoning and methodology arise from Islamic epistemological reasoning in socio-scientific issues and systems. They have significant practical evidence for the kind of probabilistic model of asset valuation referred to in this paper. The emergent concepts result in a phenomenological model of unity of knowledge between knowledge, morality and issues of the world systems' intra- and inter-systems over the dimensions of knowledge, space and time. The formalism evokes serious socio-scientific attention, especially for the development of Islamic economics and finance as a distinct worldview in worldly matters (*Muamalat*).

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