International Journal of Applied Operational Research
Vol. 11, No. 3, pp. 21-30, Summer 2023

Journal homepage: ijorlu.liau.ac.ir

Fuzzy free replicability model with restricted variation

M. Jahani Sayyad Noveiri*, B. Ahmadi

Received: 3 January 2023 ; **Accepted:** 18 May 2023

Abstract In conventional data envelopment analysis (DEA) models, inputs, and outputs are usually considered as precise and continuous factors. Furthermore, inputs and outputs of inefficient decision-making units (DMUs) change arbitrarily for meeting the efficient frontier. Nevertheless, there are situations in the real world where the performance of DMUs with fuzzy and integer-valued measures must be evaluated while input and output variables are restricted by the decision-maker. Therefore, the current paper proposes a DEA-based method for assessing the relative efficiency of DMUs with imprecise and integer-valued factors when restricted variations are observed. To illustrate, the free replicability (FR) model is extended for incorporating fuzzy numbers and some visible limitations like restrictions on resources. Furthermore, the method is developed for situations where flexible measures are presented. A numerical example is used to illustrate the approach.

Keyword: DEA, Fuzzy, Integer Value, Flexible Measure, Restricted Variation.

1 Introduction

Data envelopment analysis (DEA), introduced by Charnes et al. [1], is a mathematical technique to evaluate the relative efficiency of decision making units (DMUs) with multiple inputs and outputs. In traditional DEA models, performance measures are deemed as crisp and continuous values. Moreover, the inputs and outputs of inefficient DMUs alter arbitrarily in order to reach the efficient frontier. However, there are occasions in real-world applications that DMUs with vague and integer-valued factors need to be evaluated. Additionally, there are instances where decision-makers limit the variations of inputs and outputs. Indeed, DMUs confront constraints in terms of resources and their abilities. For this reason, the current paper proposes a DEA-based method for measuring the performance of DMUs in the presence of fuzzy and integer values in which variations are restricted. Kordrostami et al. [2] proposed a method for evaluating the comparative efficiency of DMUs with undesirable factors when restricted variations are presented. Also, Kordrostami et al. [3] provided radial and non-radial approaches to assess the relative efficiency of DMUs with restricted input and output variables.

* **Corresponding Author.** (✉)
E-mail: Monirehjahani@yahoo.com (M. Jahani Sayyad Noveiri)

M. Jahani Sayyad Noveiri
Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran

B. Ahmadi
Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran

On the other hand, Kordrostami and Jahani Sayyad Noveiri [4, 5] suggested approaches for assessing the performance of DMUs in the existence of integer and fuzzy data. Nevertheless, the restricted variations of performance measures were not included. Consequently, this study expands upon the free replicability (FR) model [6] to accommodate scenarios involving fuzzy data and constraints on variability.

Moreover, there are occasions in the real world that input and/or output status of some factors is unknown. In the DEA literature, these factors are called "flexible measures" [7-13]. Kordrostami et al. [14] classified performance measures when integer factors are presented. Performance analysis in the presence of bounded, discrete and flexible indicators has also been addressed by Kordrostami and Jahani Sayyad Noveiri [15]. Accordingly, the suggested technique in this study is extended for situations that there are flexible measures. Numerical examples are applied to clarify the proposed approaches. In general, the contribution of this study can be summarized as follows:

i. Measuring the performance of entities with fuzzy and integer-valued factors, and also restricted variations,
ii. Assessing the relative efficiency of processes in the presence of the restricted fuzzy integer-valued flexible measures,
iii. Proposing fuzzy free replicability approaches with restricted variations.

The subsequent sections of this paper are organized as follows. In Section 2, relevant and related reviews are discussed. The proposed approaches are provided in Section 3. A set of data is used to illustrate the approaches in Section 4. Finally, conclusions are drawn in Section 5.

2 Preliminaries

DEA is one of the most popular techniques for evaluating the relative efficiency of DMUs with multiple inputs and multiple outputs. For describing the relative efficiency, take n DMUs, DMU_j $(j = 1, \ldots, n)$, with m precise and continuous inputs x_{ij} $(i = 1, \ldots, m)$ and s precise and continuous outputs y_{rj} $(r = 1, \ldots, s)$. The relative efficiency of each DMU, DMU_o, can be assessed by the following model (1):

$$\begin{aligned} & Min\ \theta \\ & s.t.\ \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io}, i = 1, 2, \ldots, m, \\ & \quad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, r = 1, 2, \ldots, s, \\ & \quad \lambda_j \ge 0, \forall j. \end{aligned} \tag{1}$$

DMUs are classified as efficient and inefficient units after computing model (1) n times. θ indicates the efficiency score. $\lambda_j (j = 1, \ldots, n)$ is intensity variables. Nevertheless, sometimes only integer inputs and outputs exist. Therefore, Tulknes [6] proposed the next model for measuring the efficiency of DMUs with integer-valued factors:

$$\begin{aligned} & Min\ \theta \\ & s.t.\ \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta x_{io}, i=1,2,\dots,m, \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro}, r=1,2,\dots,s, \\ & \lambda_j \ge 0, \forall j, \text{integer}. \end{aligned} \tag{2}$$

The above model is called "free replicability" (FR) model.

Also, there are situations in real-world applications that some factors can play either input or output roles. Amirteimoori and Emrouznejad [8] proposed the following model for incorporating these factors (i.e. flexible measures) and evaluating the efficiency of DMUs:

$$\begin{aligned} & Min\ \ \theta \\ & s.t.\ \ \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta x_{io}, i=1,2,\dots,m, \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro}, r=1,2,\dots,s, \\ & \sum_{j=1}^{n}\lambda_j z_{kj} \le \theta z_{ko} + Md_k, k=1,2,\dots,K, \\ & \sum_{j=1}^{n}\lambda_j z_{kj} \ge z_{ko} - M\left(1\text{-}d_k\right), k=1,2,\dots,K, \\ & \lambda_j \ge 0, \forall j, d_k \in \{0,1\}, \forall k. \end{aligned} \tag{3}$$

that $z_{kj}(k=1,\dots,K)$ denotes flexible measures. M is a large positive number. d_k is a binary variable. If $d_k=0$, z_k is an input and if $d_k=1$, z_k is an output.

In the next section, at first, the FR model is extended for situations where integer and fuzzy measures are presented, while restricted variations are imposed by decision-makers. Then, a model is suggested for evaluating the comparative efficiency of DMUs in the presence of fuzzy, integer and flexible measures while the variation levels of measures are pre-defined by the management.

3 Fuzzy free replicability model (FFR) with restricted variation

Consider n DMU, DMU_j $(j=1,\dots,n)$, that each DMU consumes m integer-valued inputs x_{ij} $(i=1,\dots,m)$ and produces s integer-valued outputs y_{rj} $(r=1,\dots,s)$. Integer-valued inputs and outputs are considered as triangular fuzzy numbers, i.e. $\tilde{x}_{ij}=(x_{ij1},x_{ij2},x_{ij3})$, $\tilde{y}_{rj}=(y_{rj1},y_{rj2},y_{rj3})$. Suppose the i-th input of DMU_o is limited to decrease to $\tilde{x}_{io}-\tilde{\alpha}_{io}\ge 0$. Similarly, the r-th output of DMU_o is limited to increase to $\tilde{y}_{ro}+\tilde{\beta}_{ro}\ge 0$. In other words,

$$\begin{aligned} & \tilde{x}_{io} \to \tilde{x}_{io}-\tilde{\alpha}_{io},\ i=1,2,\dots,m, \\ & \tilde{y}_{ro} \to \tilde{y}_{ro}+\tilde{\beta}_{ro},\ r=1,2,\dots,s, \end{aligned} \tag{4}$$

in which $\tilde{\alpha}_{io} = (\alpha_{io1}, \alpha_{io2}, \alpha_{io3})$ and $\tilde{\beta}_{ro} = (\beta_{ro1}, \beta_{ro2}, \beta_{ro3})$.

Herein we propose the following fuzzy free reliability model in the presence of restricted variability. The third and fourth restrictions indicate the limitations related to variations.

$$
\begin{aligned}
& Min \quad \theta \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \theta \tilde{x}_{io}, i = 1, 2, ..., m, \\
& \qquad \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{io}, r = 1, 2, ..., s \\
& \qquad \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \ge \tilde{x}_{io} - \tilde{\alpha}_{io},\ i = 1, 2, ..., m, \\
& \qquad \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \le \tilde{y}_{ro} + \tilde{\beta}_{ro},\ r = 1, 2, ..., s, \\
& \lambda_j \ge 0, \ \text{integer.}
\end{aligned}
\tag{5}
$$

Therefore, the above model is proposed for evaluating the performance of DMUs with fuzzy and integer factors where restricted variations exist. Two approaches are used for transforming the above fuzzy mixed integer linear programming problem into a mixed integer linear programming problem.

At first, it is assumed that all weights and variables are crisp and precise. Therefore, model (5) can be rewritten as follows:

$$
\begin{aligned}
& Min \quad \theta \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij1} \le \theta x_{io1}, i = 1, 2, ..., m, \\
& \qquad \sum_{j=1}^{n} \lambda_j x_{ij2} \le \theta x_{io2}, i = 1, 2, ..., m, \\
& \qquad \sum_{j=1}^{n} \lambda_j x_{ij3} \le \theta x_{io3}, i = 1, 2, ..., m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj1} \ge y_{ro1}, r = 1, 2, ..., s, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj2} \ge y_{ro2}, r = 1, 2, ..., s,
\end{aligned}
$$

$$\sum_{j=1}^{n}\lambda_j y_{rj3} \geq y_{ro3}, r=1,2,...,s,$$
$$\sum_{j=1}^{n}\lambda_j x_{ij1} \geq x_{io1}-\alpha_{io1},\ i=1,2,...,m,$$
$$\sum_{j=1}^{n}\lambda_j y_{rj1} \leq y_{ro1}+\beta_{ro1},\ r=1,2,...,s$$
$$\sum_{j=1}^{n}\lambda_j x_{ij2} \geq x_{io2}-\alpha_{io2},\ i=1,2,...,m,$$
$$\sum_{j=1}^{n}\lambda_j y_{rj2} \leq y_{ro2}+\beta_{ro2},\ r=1,2,...,s,$$
$$\sum_{j=1}^{n}\lambda_j x_{ij3} \geq x_{io3}-\alpha_{io3},\ i=1,2,...,m,$$
$$\sum_{j=1}^{n}\lambda_j y_{rj3} \leq y_{ro3}+\beta_{ro3},\ r=1,2,...,s$$
$$\lambda_j \geq 0,\ \text{integer.} \tag{6}$$

Alternatively, the fuzzy expected value approach [16] can be used. Thus, model (5) can be substituted with the following model:

$$\begin{aligned}
&Min\quad \theta \\
&s.t.\quad \sum_{j=1}^{n}\lambda_j E(\tilde{x}_{ij}) \leq \theta E(\tilde{x}_{io}), i=1,2,...,m, \\
&\qquad \sum_{j=1}^{n}\lambda_j E(\tilde{y}_{rj}) \geq E(\tilde{y}_{io}), r=1,2,...,s, \\
&\qquad \sum_{j=1}^{n}\lambda_j E(\tilde{x}_{ij}) \geq E(\tilde{x}_{io})-E(\tilde{\alpha}_{io}),\ i=1,2,...,m, \\
&\qquad \sum_{j=1}^{n}\lambda_j E(\tilde{y}_{rj}) \leq E(\tilde{y}_{ro})+E(\tilde{\beta}_{ro}),\ r=1,2,...,s, \\
&\qquad \lambda_j \geq 0,\ \text{integer.}
\end{aligned} \tag{7}$$

Notice that if $\tilde{w}=(a,b,c)$ is considered as a triangular fuzzy number, then $E(\tilde{w})=(1/4)(a+2b+c)$ in model (7).

Definition 1. DMU_o is said efficient in models (6) and (7) if and only if $\theta^*=1$. Otherwise, it is called inefficient.

In the next stage, we incorporate flexible measures in model (5). Assume there are K flexible measures z_{kj} $(k=1,\dots,K)$. Moreover, suppose that the k-th flexible measure of DMU_o is limited to decrease to $\tilde{z}_{ko}-\tilde{\delta}_{ko}\geq 0$ and to increase to $\tilde{z}_{ko}+\tilde{\gamma}_{ko}\geq 0$. Model (5) is rewritten as follows:

$$
\begin{aligned}
& Min \quad \theta \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \theta \tilde{x}_{io}, i = 1, 2, ..., m, \\
& \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{io}, r = 1, 2, ..., s, \\
& \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \ge \tilde{x}_{io} - \tilde{\alpha}_{io}, \ i = 1, 2, ..., m, \\
& \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \le \tilde{y}_{ro} + \tilde{\beta}_{ro}, \ r = 1, 2, ..., s, \\
& \sum_{j=1}^{n} \lambda_j \tilde{z}_{kj} \le \theta \tilde{z}_{ko} + Md_k, k = 1, 2, ..., K, \\
& \sum_{j=1}^{n} \lambda_j \tilde{z}_{kj} \ge \tilde{z}_{ko} - M\left(1\text{-}d_k\right), k = 1, 2, ..., K, \\
& \sum_{j=1}^{n} \lambda_j \tilde{z}_{kj} \ge \tilde{z}_{ko} - \tilde{\delta}_{ko}, k = 1, 2, ..., K, \\
& \sum_{j=1}^{n} \lambda_j \tilde{z}_{kj} \le \tilde{z}_{ko} + \tilde{\gamma}_{ko}, k = 1, 2, ..., K, \\
& \lambda_j \ge 0, \ \text{integer}, d_k \in \{0,1\}, \forall k,
\end{aligned}
\tag{8}
$$

that $\tilde{\delta}_{ko} = (\alpha_{ko1}, \delta_{ko2}, \delta_{ko3})$ and $\tilde{\gamma}_{ko} = (\gamma_{ko1}, \gamma_{ko2}, \gamma_{ko3})$.
For calculating model (8), we use the fuzzy expected value approach similar to model (7). Therefore, model (8) can be rewritten as follows:

$$
\begin{aligned}
& Min \quad \theta \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j E(\tilde{x}_{ij}) \le \theta E(\tilde{x}_{io}), i = 1, 2, ..., m, \\
& \sum_{j=1}^{n} \lambda_j E(\tilde{y}_{rj}) \ge E(\tilde{y}_{io}), r = 1, 2, ..., s, \\
& \sum_{j=1}^{n} \lambda_j E(\tilde{x}_{ij}) \ge E(\tilde{x}_{io}) - E(\tilde{\alpha}_{io}), \ i = 1, 2, ..., m, \\
& \sum_{j=1}^{n} \lambda_j E(\tilde{y}_{rj}) \le E(\tilde{y}_{ro}) + E(\tilde{\beta}_{ro}), \ r = 1, 2, ..., s, \\
& \sum_{j=1}^{n} \lambda_j E(\tilde{z}_{kj}) \le \theta E(\tilde{z}_{ko}) + Md_k, k = 1, 2, ..., K, \\
& \sum_{j=1}^{n} \lambda_j E(\tilde{z}_{kj}) \ge E(\tilde{z}_{ko}) - M\left(1\text{-}d_k\right), k = 1, 2, ..., K, \\
& \sum_{j=1}^{n} \lambda_j E(\tilde{z}_{kj}) \ge E(\tilde{z}_{ko}) - E(\tilde{\delta}_{ko}), k = 1, 2, ..., K, \\
& \sum_{j=1}^{n} \lambda_j E(\tilde{z}_{kj}) \le E(\tilde{z}_{ko}) + E(\tilde{\gamma}_{ko}), k = 1, 2, ..., K, \\
& \lambda_j \ge 0, \ \text{integer}, d_k \in \{0,1\}, \forall k.
\end{aligned}
\tag{9}
$$

M is a large positive number. d_k is a binary variable. If $d_k = 0$, z_k is an input and if $d_k = 1$, z_k is an output. Moreover, the majority rule can be applied to classify flexible measures.
Notice that we have not used the first approach (i.e. similar to model (6)) for transforming model (8) to a mixed integer linear programming problem because of the large number of restrictions and variables.
Also, in the presence of trapezoidal fuzzy data, i.e. $\widetilde{w} = (a, b, c, d)$, $E(\widetilde{w})$ can be assessed as follows: $E(\widetilde{w}) = (1/4)(a + b + c + d)$.
In the next section, a dataset is applied to illustrate the models provided.

4 Example

Assume there are 6 DMUs with one input (I1) and one output (O1). Inputs, outputs, and variation levels are shown in Table 1. As can be seen all data are indicated as triangular fuzzy numbers. Models (6) and (7) are calculated. The results are given in Table 2

Table 1 Fuzzy data and variation levels

DMU	$I1$	$O1$	α_1	β_1
1	(9, 14, 16)	(10, 10, 15)	(4, 4, 6)	(8, 9, 9)
2	(10, 10, 10)	(15, 16, 18)	(3, 3, 3)	(2, 4, 8)
3	(12, 14, 16)	(12, 13, 14)	(3, 4, 5)	(3, 3, 5)
4	(9, 12, 15)	(16, 18, 20)	(4, 5, 6)	(8, 10, 15)
5	(4, 6, 8)	(21, 22, 23)	(1, 2, 3)	(8, 8, 8)
6	(8, 9, 10)	(10, 15, 20)	(4, 4, 5)	(9, 10, 11)

Table 2 Results

DMU	Efficiency		
	Model (6)	Model (7)	Model (7) with non-negative weights
1	1	0.68	0.66
2	1	1	0.70
3	1	0.71	0.71
4	1	1	0.58
5	1	1	1
6	1	0.67	0.53

Notice that with computing model (6), all DMUs are determined as efficient while three DMUs, 1,3 and 6, are efficient via model (7). Also, DMU 6 has obtained the least efficiency score by calculating model (7). The comparison of the results achieved from two models (6) and (7) shows that model (7) is more able for discriminating.

For more analysis, we compute model (7) with considering $\lambda_j \geq 0$ similar to model (1). The findings appear in Column 4 of Table 2. DMU 5 is examined as efficient. Furthermore, DMU 6 is ascertained as the most inefficient unit in both model (7) and model (7) with non-negative weights. As can be found, the scores obtained from models (6) and (7) are not less than model (7) with non-negative weights. These results are in line with the

description presented in [17]. It is clear that the integer-valued weights play an important role in determining the efficiency values obtained.
Now we consider one flexible measure (F1) with variation levels that are shown in Table 3.

Table 3 Flexible measure, variation levels and results

DMU	$F1$	δ_1	γ_1	Efficiency of model (9)	d_k	Efficiency of model (9) with non-negative weights	d_k
1	(5, 6, 7)	(2, 2, 3)	(5, 6, 6)	0.75	0	0.66	0
2	(4, 4, 4)	(2, 2, 2)	(3, 4, 5)	1	0 or1	0.70	1
3	(6, 7, 7)	(4, 5, 5)	(4, 6, 7)	0.71	0	0.71	0
4	(5, 7, 9)	(5, 6, 6)	(3, 4, 4)	1	0 or 1	0.58	0
5	(3, 4, 6)	(2, 2, 4)	(6, 6, 6)	1	0 or 1	1	0 or 1
6	(3, 5, 5)	(2, 4, 4)	(7, 8, 9)	0.94	0	0.67	0

Model (9) is calculated. The results are shown in Columns 5 and 6 of Table 3. Column 5 shows the efficiency scores. The input or output status of the flexible measure is indicated in column 6. In this case, DMUs 2, 4 and 5 are obtained as efficient. Also, DMU 3 is the most inefficient unit with the score 0.71. As can be seen in Column 6, the status of flexible measure is considered as an input in 3 DMUs. Thus, according to the majority rule, the role of flexible measure is specified as an input.

To compare the findings, we solve model (9) with non-negative weights, i.e. $\lambda_j \geq 0$. Outcomes are provided in Columns 7 and 8 of Table 3. As shown, only DMU 5 is efficient and DMU 4 is obtained as the most inefficient entity. Also, the role of the flexible measure is identified as the input based on the majority rule. Also, the efficiency values achieved from model (9) are not less than those resulted from model (9) with non-negative weights.
The achievements indicate that the integer-valued weights influence the efficiency values gained.

4.1 Discussion

Performance analysis of entities is a crucial task in various fields such as operations research, finance, and economics. It involves evaluating and comparing the efficiency and effectiveness of different DMUs. Traditionally, performance analysis has been conducted using crisp data and measures. However, the use of fuzzy and integer measures, along with the restricted variations of performance measures, is significant.

One important aspect of using fuzzy measures in performance analysis is that they allow for the quantification of uncertainty and imprecision in data. This is particularly useful when dealing with qualitative or partially-known information, which is common in decision-making processes.

Moreover, the restricted variations of performance measures provide more comprehensive insights into the measured entity's performance by limiting the range of variations allowed in the measures. By imposing restrictions on certain factors, the analysis becomes more accurate and realistic. These restrictions enhance the reliability of the obtained efficiency scores as they enable a more accurate representation of the decision-making context.

Moreover, the extension of the approach in the presence of flexible measures allows for a more holistic evaluation of DMUs. Flexible measures consider the possibility of adjusting the inputs and outputs to optimize the performance of the DMU.

The outcomes achieved from analyzing the dataset reveal that the weights that are only whole numbers and the limitations on variations are significant factors in determining the obtained values of performance.

In summary, the importance of performance analysis using the FFR model with restricted variations lies in its ability to capture uncertainties, imprecision, and restricted variations in the integer-valued performance data. Additionally, the extension of such approaches to incorporate flexible measures enables a more agile and versatile evaluation of DMUs' performance. These advancements contribute to more accurate and realistic performance assessments, aiding decision-makers in identifying areas for improvement and making informed decisions. The algorithm of the methodology can be seen in Figure 1.

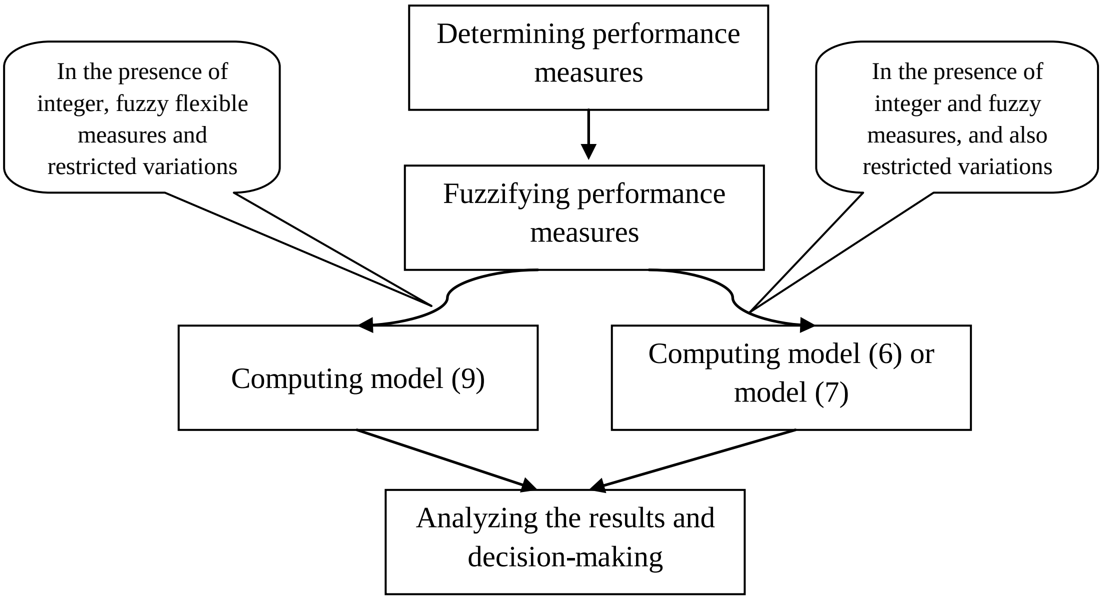

Fig. 1 The algorithm of the methodology

5 Conclusions

There are instances in practical scenarios where it is necessary to assess the efficacy of DMUs that have factors which are imprecise and take on integer values. Additionally, decision-makers may impose restrictions on the variability of performance measures. It is true that DMUs face constraints in terms of resources and their abilities. Accordingly, in the current paper, a model has been proposed for determining the efficiency of entities with fuzzy and integer-valued measures where restricted variations are presented. Two methods have been introduced for calculating the proposed fuzzy mixed integer linear programming and transforming it into the mixed integer linear programming. Moreover, the suggested model has been extended for occasions that flexible measures are present. A data set has been used to clarify the approach. The results obtained from examining the dataset demonstrate that the presence of integer weights and the constraints on variations are notable elements for determining the performance values obtained. Moreover, the incorporation of flexible measures impacts the findings. In this research, the process has been considered as a black-box and a special period of time.

Therefore, the introduced technique can be extended for analyzing the performance of multi-period and multi-division processes. Ranking and discriminating solutions can further be addressed. Also, the development of the advanced method for situations that there are undesirable factors is an interesting topic for future examination.

References

1. Charnes, A., Cooper, W. W., Rhodes, E. (1978). Measuring the efficiency of decision making units. European journal of operational research, 2(6), 429-444.
2. Kordrostami, S., Amirteimoori, A., Jahani Sayyad Noveiri, M. (2015). Restricted variation in data envelopment analysis with undesirable factors in nature. International Journal of Biomathematics, 8(3), 1550034.
3. Kordrostami, S., Amirteimoori, A., Jahani Sayyad Noveiri, M. (2016). Flexibility of variations in radial and non-radial data envelopment analysis models. International Journal of Industrial Mathematics, 8(3), 269-278.
4. Kordrostami, S., Jahani Sayyad Noveiri, M. (2015, September). Data envelopment analysis with integer-valued factors in a fuzzy environment. In 2015 4th Iranian Joint Congress on Fuzzy and Intelligent Systems (CFIS) (pp. 1-3). IEEE.
5. Kordrostami, S., Amirteimoori, A., Jahani Sayyad Noveiri, M. (2018). Fuzzy integer-valued data envelopment analysis. RAIRO-operations research, 52(4-5), 1429-1444.
6. Tulkens, H. (1993). On FDH efficiency analysis: some methodological issues and applications to retail banking, courts, and urban transit. Journal of productivity analysis, 4(1-2), 183-210.
7. Cook, W. D., Zhu, J. (2007). Classifying inputs and outputs in data envelopment analysis. European journal of operational research, 180(2), 692-699.
8. Amirteimoori, A., Emrouznejad, A. (2011). Flexible measures in production process: a DEA-based approach. RAIRO-operations research, 45(1), 63-74.
9. Toloo, M. (2012). Alternative solutions for classifying inputs and outputs in data envelopment analysis. Computers & mathematics with applications, 63(6), 1104-1110.
10. Amirteimoori, A., Emrouznejad, A., Khoshandam, L. (2013). Classifying flexible measures in data envelopment analysis: A slack-based measure. Measurement, 46(10), 4100-4107.
11. Kordrostami, S., Jahani Sayyad Noveiri, M. (2014). Evaluating the performance and classifying the interval data in data envelopment analysis. International journal of management science and engineering management, 9(4), 243-248.
12. Kordrostami, S., Jahani Sayyad Noveiri, M. (2012). Evaluating the efficiency of decision making units in the presence of flexible and negative data. Indian journal of science and technology, 5(12), 3776-3782.
13. Jahani Sayyad Noveiri, M., Kordrostami, S., & Amirteimoori, A. (2023). Multi-period efficiency analysis with flexible measures: Oriented and non-oriented approaches. Iranian Journal of Management Studies, 16(3), 627-645.
14. Kordrostami, S., Amirteimoori, A., Jahani Sayyad Noveiri, M. (2019). Inputs and outputs classification in integer-valued data envelopment analysis. Measurement, 139, 317-325.
15. Kordrostami, S., Jahani Sayyad Noveiri, M. (2021). Performance analysis in the presence of bounded, discrete and flexible measures. Operations Research and Decisions, 31(3).
16. Wang, Y. M., Chin, K. S. (2011). Fuzzy data envelopment analysis: A fuzzy expected value approach. Expert systems with applications, 38(9), 11678-11685.
17. Bogetoft, P., Otto, L. (2010). Benchmarking with DEA, SFA, and R (Vol. 157). Springer Science & Business Media.