

Sequential benchmarking to achieve the closest cross-sectional targets in DEA

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Abstract The models that set the closest targets have made an important contribution to data envelopment analysis (DEA) as tool for best-practice benchmarking of decision making units (DMUs). These models may help define plans for improving the require less effort from the DMUs. One of the important issues in the process of benchmarking and target setting, is to set realistic and achievable targets for inefficient units. Because in practice and in the real world, we often face units that perform poorly and the targets for them are not available in one step, to solve this problem, in this study, an algorithm is presented that takes advantage of the onion layering method has three main advantages over other step-by-step benchmarking methods: firstly, in each step, it offers a better and closer target and benchmark to the manager sequentially. Secondly, by adjusting the number of jumps in the layers according to the conditions, it provides the possibility of more adjustments and flexibility in targets for the manager. Thirdly, by classifying the decision-making units based on the level of efficiency and performance, it specifies a benchmark and a realistic achievable target for the inefficient units at each stage. The proposed method has been implemented on the data of 24 Portuguese bank branches and And targets are specified sequentially for each ineffective unit.

Keyword: Data Envelopment Analysis, Sequential Benchmarking, Closest Targets.

1 Introduction

Nowadays, benchmarking is one of the common and popular methods in many organizations. Benchmarking is a process that allows organizations to take action from the best companies and institutions to improve themselves. In evaluating performance based on efficiency, decision-making units are divided into two categories: efficient and inefficient units. Inefficient units to improve their performance need a benchmark that they can follow to achieve efficiency. Inefficient units will be able to identify and apply the changes they need for logical benchmarking by studying the efficient unit in different dimensions (technology, procedures, processes, management, etc.). It is a powerful tool for introducing useful

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benchmarks and targets for inefficient units. Various benchmarking approaches have been presented by many authors. But one of the problems of the DEA method, which is always mentioned in most researches, is the process of determining a reference or benchmark for inefficient DMUs and improving the efficiency of these DMUs to reach the efficiency frontier, which sometimes reaches the efficiency frontier in only one step for An inefficient DMU is completely impossible, especially when the inefficient DMU is far from the efficiency frontier.

In this regard, some authors believe in the improvement program based on target setting in several steps. For example, Zhu [1] believes that in a special case, an inefficient DMU may not be able to immediately improve its performance at the first level of the best performance frontier, this may happen due to limitations such as management expertise, lack of resources, etc. Intermediate may be desirable for an inefficient DMU. Barr et al. [2] have discussed the onion layering of DEA. Lim et al. [3] by layering DMU in nested efficiency boundaries based on the approach of Seiford and Zhu [4] selected benchmarks based on three criteria of attractiveness, progress and feasibility. Lim et al. [3] claimed that it may be impractical and far from realistic for an inefficient DMU to reach an efficient goal in one step, in the sense that if the inefficient DMU is too far from the efficiency boundary, it can be said It will be impossible to reach the efficiency limit with one displacement step, so the more suitable alternative is to make a gradual and step-by-step progress to reach the final goal. Another similar approach in DEA for sampling is the method of Lozano and Villa [5-6]. They believe that the goal may be too far for an inefficient unit and reaching that goal requires a large decrease in input or a large increase in output. Or in other words, if the input and output changes are large, it is very difficult to do them simultaneously, so they have proposed a gradual improvement strategy with consecutive intermediate goals. Fang [7] applied a similar idea to centralize benchmarks. In the article of Kwon et al. [8], we see that the authors presented a better performance benchmarking approach instead of the best performance benchmarking approach. Despite the usefulness of these resources in sampling, DEA benchmarks mostly to determine goals and benchmarks that are often far from the DMU under evaluation and are difficult to achieve even if it is step by step. In this regard, close determination benchmarks The most important goals for minimizing the distance of DMUs from the efficiency frontier have made significant progress. These benchmarks have created ways for inefficient DMUs to achieve efficiency with less effort. Fukuyama et al. [9-10], Aparacio et al. [11], Aparacio and Pasteur [12], Aparacio et al. [13], Ruiz et al. [14], Ruiz and Sirvent [15], Roman et al. [16] and Cook et al. [17] And Ruiz and Servant [18] had researched in this field. Rostamzadeh et al. [19] have reviewed modeling methods in various articles. Ruiz and Sirvent [20] have presented a new method of identifying the appropriate benchmark in the path of achieving targets with using DEA. An et al. [21], they presented a real benchmarking path with a target setting approach with limited change. This method has been used to determine goals and modeling for inefficient units in the discussion of energy in the transportation sector. However, often, the closest targets are not available for some inefficient DMUs. This paper focuses on evaluation and benchmarking for these units. In this paper, a sequential multi-stage benchmarking is proposed by applying Zhu's layering approach and based on determining the closest targets in each stage. The special feature of this article compared to other step-by-step benchmarking approaches, is that in this article we will use models that minimize the distance from the efficiency frontiers at each stage and by adjusting the number of mutations in the layers, it involves the manager's opinion in the benchmarking process. This makes the process of benchmarking and target setting not only technically accessible, but also favorable and accessible in terms of previous knowledge and

expert opinion, and targets and benchmarks are more realistic and attainable and more flexible determined at each stage. This allows the manager to make different plans to improve the performance of inefficient units. The article is organized in this way. In the second part, a review of the subject literature and preliminary concepts is presented. In the third part, the proposed algorithm for determining the multi-stage goal is presented, and in the fourth part, to explain the subject, the proposed algorithm is implemented on the data of 24 Portuguese bank branches and the objectives are specified sequentially for each inefficient unit.

2 Determining the closest targets

Because classical DEA benchmarking methods usually achieve targets that are far from the unit under evaluation, this reduces the incentive to improve the inefficient units because in practice, whatever the target size and benchmark for inefficient units is far, its incentive for improvement will be less so the distance to the image should be minimized so that the resulting targets are as similar as possible to the inputs and outputs of the units under evaluation and it is easier to eliminate inefficiency, the idea behind it is that the closer targets propose improvement paths for inefficient units, which may lead to efficiency with less effort. However, it should be noted that determining the closest targets is not computationally easy. This problem is due to the complexity of determining the minimum distance from the production technology frontier from an inefficient point (inefficient DMU). This is actually minimizing the distance from the non-convex complement of a multidimensional set that is computationally difficult. So there are two main approaches in the DEA to determine the closest targets: the first approach is based on defining all the efficient aspects of the efficiency frontier and then obtaining the minimum distance from the frontier as the minimum distance of the aspects in a multi-step process. The second approach proposed by Aparicio et al. [11] to overcome the shortcomings of the first approach is based on mixed integer linear programming without explicitly determining all the efficient aspects. Therefore, because it avoids explicitly determining all aspects of efficiency, it makes it much easier to set these targets. Ruiz [20] has also modeled by setting the closest targets and identifying peer groups with the most similar performance in the same way. In all of these articles, explicit definition of different aspects of the production frontier has been avoided. Although this method is now mature in the DEA literature, it is not yet complete and there is possible for expansion. One of these areas is the selection of models and more realistic target setting and match the different conditions for inefficient units, which is addressed in this article.

2.1 Layered evaluation of DMUs in DEA

In DEA benchmarks, it is assumed that all DMUs are in perfect competition, but when there are several specific DMUs in the evaluation benchmark, this condition disappears. In fact, the frontier generated by several specific units to evaluate the performance of all units are imposed on them. To solve this problem, a method called layer evaluation is proposed in DEA.

In this method, instead of using only one efficient frontier for all units, multiple frontiers are used in the review of DMUs, so that the first efficient frontier is identified with DEA benchmarks and we run the benchmark again by removing the frontier-constructing unit or units and specify the second efficiency frontier, and by repeating this process in a finite number of iterations, we achieve the initial classification of DMUs. By layering DMUs,

improvement strategies for these units in each layer are done in such a way that each DMU obtains its improvement solution according to the location and based on expert opinions from the higher layer or layers. In this paper, we will use the Seiford and Zhu [4] approach for layering units.

2.2 DMU classification in DEA

To classify the DMU, we will use the method of Charnes et al. [22]. They briefly divide the units into six categories E, E', F, NE, NE', NF , as shown in Figure 1.

Category E: The extreme efficient units T_C is a unit that cannot be represented as a non-negative linear combination of $n-1$ remaining units.

Category E' : Pareto efficient unit that is not in the category E.

Category F: It is a weak efficient unit.

Category NE, NE', NF : Inefficient units whose radial image is made on the unit (strong and weak efficient) on the unit E, E', F, \dots

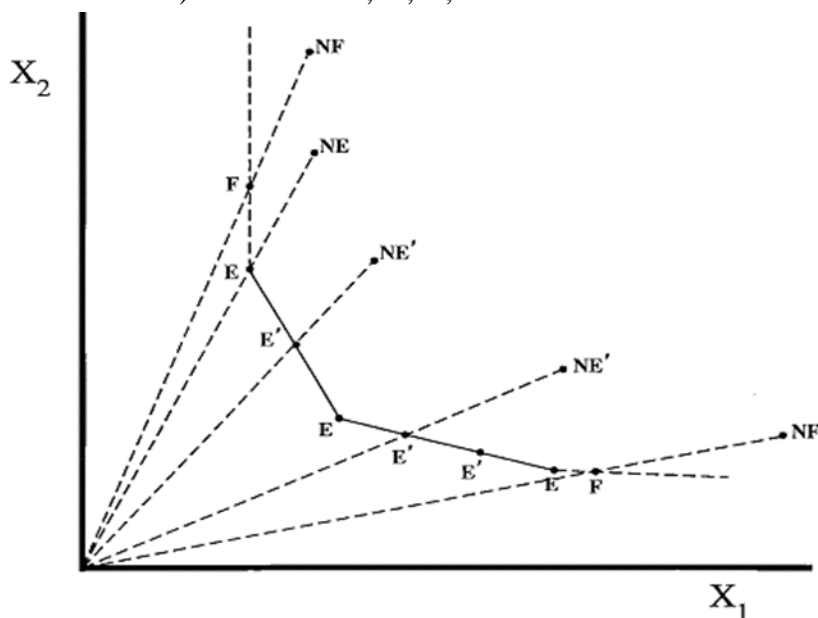


Fig. 1 DMU classification

Given that we will use the category E (extreme efficient units) in this paper, so the method of finding them with the method presented by Charnes et al. [22] in T_C will be as follows:

$$\begin{aligned}
 & \text{Min } \lambda_o \\
 & \text{s.t.} \\
 & \sum_{j=1}^n \lambda_j x_{ij} \leq x_{io}, \quad i = 1, \dots, m, \\
 & \sum_{j=1}^n \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \dots, s, \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n.
 \end{aligned} \tag{1}$$

Assume that λ_o^* is the optimal value of the above benchmark, if $\lambda_o^* = 1$, then $\lambda_o^* = 1$ is an extreme efficient unit.

3 Sequential benchmarking and target setting

Suppose we have n DMUs that use m inputs to generate s outputs, which we represent as $(X_j, Y_j), j = 1, \dots, n$. The relative efficiency of each DMU is determined by referring to a set called the production possibility set, which can be considered with a non-parametric structure of observations and with certain assumptions and principles. To determine the targets and benchmarking in this paper, the production possibility set $T = \{(X, Y) / X \text{ can produce } Y\}$ under constant returns to scale (CRS) is considered as follows:

$$T_{CRS} = \left\{ (x, y) \in R_+^m \times R_+^s : x \geq \sum_{j=1}^n \lambda_j x_j, y \leq \sum_{j=1}^n \lambda_j y_j, \lambda_j \geq 0, j = 1, \dots, n \right\}$$

Model (2), which minimizes the distance from the frontier, can usually be used to determine the closest target for each DMU_o .

$$\begin{aligned} &Min \quad \|(x, y) - (x_o, y_o)\| \\ &s.t \quad (x, y) \in \partial(T) \\ &\quad \quad (x, y) \text{ dominates } (x_o, y_o) \end{aligned} \tag{2}$$

Taking into account the characterization of in terms of a set of linear constraints (see, for instance, Aparicio et al. [10]), we have the following operative formulation of model (2), which is expressed in terms of the usual slacks. We determine the closest targets with $\|\cdot\|_1$ and variable returns to scale as follows: [22]

$$\min z = \sum_{i=1}^m s_{io}^- + \sum_{r=1}^s s_{ro}^+ \quad (3.1)$$

$$s.t \sum_{j \in E} \lambda_j x_{ij} = x_{io} - s_i^-, \quad i = 1, \dots, m \quad (3.1)$$

$$\sum_{j \in E} \lambda_j y_{rj} = y_{ro} + s_r^+, \quad r = 1, \dots, s \quad (3.2)$$

$$\sum_{j \in E} \lambda_j = 1 \quad (3.3)$$

$$-\sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r y_{rj} + u_0 + d_j = 0 \quad j \in E \quad (3.4)$$

$$v_i \geq 1, \quad i = 1, \dots, m \quad (3.5)$$

$$u_r \geq 1, \quad r = 1, \dots, s \quad (3.6) \quad (3)$$

$$s_i^- \geq 0, \quad i = 1, \dots, m \quad (3.7)$$

$$s_r^+ \geq 0, \quad r = 1, \dots, s \quad (3.8)$$

$$\lambda_j \geq 0 \quad j \in E \quad (3.9)$$

$$d_j \leq M b_j \quad j \in E \quad (3.10)$$

$$\lambda_j \leq M(1 - b_j) \quad j \in E \quad (3.11)$$

$$d_j \geq 0 \quad j \in E \quad (3.12)$$

$$b_j \in \{0, 1\} \quad j \in E \quad (3.13)$$

$$u_0 \text{ free}$$

Where M is a large positive number and by using the optimal answer of model (3) the targets (x^*, y^*) can be achieved as follows:

$$x^* = x_o - s^{*-} = \sum_{j \in E} \lambda_j^* X_j$$

$$y^* = y_o + s^{*+} = \sum_{j \in E} \lambda_j^* Y_j$$

Now, in this article, using the approach of Aparicio et al., with layering DMUs in different layers, we will determine the closest targets in each stage for each of the units under evaluation with $\|\cdot\|_\infty$ in a consecutive and sequential manner. The advantage of the presented method is that firstly, in each step, it offers a better and closer target and benchmark to the manager sequentially. Secondly, by adjusting the number of jumps in the layers according to the conditions, it provides the possibility of more adjustments and flexibility in targets for the manager and according to this, long-term or medium-term and even short-term targets are set for inefficient units. Thirdly, by classifying the decision-making units based on the level of efficiency and performance, it specifies a benchmark and a realistic achievable target for the inefficient units at each stage.

To determine the target in different layers with constant returns to scale and $\|\cdot\|_\infty$ we will use model (4) as follows:

$$z = \text{Min}_{x,y} \max_{i,r} \{|x_i - x_{io}|, |y_r - y_{ro}|\}$$

$$\text{s.t.} \quad (x, y) \in \partial(T)$$

If we consider $(x, y) = (x_o - s^-, y_o + s^+)$, then we will have:

$$z = \text{Min}_{s^-,s^+} \max_{i,r} \{|s_i^-|, |s_r^+|\}$$

$$\text{s.t.} \quad (x_o - s^-, y_o + s^+) \in \partial(T)$$

If we assume $z = \max_{i,r} \{|s_i^-|, |s_r^+|\}$ in this case, because (x, y) overcomes (x_o, y_o) , then we will have:

$$\begin{aligned} & \text{Min} \quad z_{Jt} \\ & \text{s.t} \quad \sum_{j \in E_{k-p}} \lambda_j x_{ij} = x_{ij}^{t-1} - s_i^- \quad i = 1, \dots, m \\ & \quad \sum_{j \in E_{k-p}} \lambda_j y_{rj} = y_{rj}^{t-1} + s_r^+ \quad r = 1, \dots, s \\ & \quad - \sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s \mu_r y_{rj} + d_j = 0 \quad j \in E_{k-p} \\ & \quad v_i \geq 1 \quad i = 1, \dots, m \\ & \quad \mu_r \geq 1 \quad r = 1, \dots, s \\ & \quad d_j \leq Mb_j \quad j \in E_{k-p} \\ & \quad \lambda_j \leq M(1 - b_j) \quad j \in E_{k-p} \\ & \quad b_j \in \{0, 1\}, d_j, \lambda_j \geq 0 \quad j \in E_{k-p} \\ & \quad s_i^- \geq 0 \quad s_r^+ \geq 0 \quad \forall i, \forall r \\ & \quad s_i^- \leq z_{Jt} \quad s_r^+ \leq z_{Jt} \quad \forall i, \forall r \end{aligned} \tag{4}$$

x_{ij}	The input value i consumed by unit j .
y_{rj}	The output value r produced by unit j .
J	Operating unit index (unit under evaluation)
t	Target Sequence Index
x_{iJ}^t	Intermediate target, according to input i from unit J in step t $x_{iJ}^0 = x_{iJ}$
y_{rJ}^t	Intermediate target, according to the output r from unit J in step t $y_{rJ}^0 = y_{rJ}$
K	Layer number index
E_k	Extreme efficient DMUs located in the k -th layer
P	Number of mutation steps in layers
M	A large positive number

3.1 Algorithm for benchmarking and determining the sequential target

The step-by-step algorithm for determining close and realistic target can be used as follows.

- I) First layer the DMUs in different layers and specify the number of layers.
- II) Obtain the set of efficient units in each layer.
- III) Specify the unit under evaluation J .
- IV) Determine the value of K (layer number on which the unit under evaluation is located).
- V) Solve benchmark (4) and obtain the closest goals in each stage for
- VI) End condition $K=1$

Note 1: To calculate the amount of improvement between the two goals t and $t-1$, you can easily use the following measure:

$$\eta_{jt} = \sum_i \frac{s_{it}^-}{x_{iJ}} + \sum_r \frac{s_{rt}^+}{y_{rJ}}$$

4 Empirical illustration

In this section, we implement the proposed approach on the data of [23], the data is related to 24 Portuguese bank branches, which includes two inputs (employee costs and other costs) and three outputs (current accounts - credit - profit stocks). We implement the proposed approach in CRS mode on these data. Note that in this case, 6 of the DMUs, which are B10, B11, B16, B26, B29, B50, are functional and are placed in the first layer in the layering, continuing the layering of B5, B17 units. , B20, B51, B53 are placed in the second layer - by continuing this process, the next layer will be formed by units B13, B21, B27, B56, B58, B59, and units B3, B9, B19, B52 will be placed in the fourth layer and the rest of the units, which are B15, B22, B57, are placed in the last layer. Using the method of Charnes *et al.*, the efficient units of the apex in each layer are determined, and all the above units are except the efficient units of the apex in each layer. By implementing the algorithm to determine the goals and close benchmarks step by step for different units of Table 1 will be obtained.

Table 1 Target set for bank branches

step	x_1	x_2	y_1	y_2	y_3	η_t^* (%)
DMU b15						
0	11.717	29.314	4070.630	6418.995	40.328	-
1	11.717	22.06	4070.630	7863.33	41.11	50
2	10.79	19.72	4070.630	8516.45	41.11	27
3	9.95	19.72	4070.630	8516.45	47.60	24
4	8.37	16.65	4073.70	8519.52	47.60	16
DMU b22						
0	16.166	26.062	3946.813	7358.401	46.214	-
1	12.94	20.39	4018.58	7557.75	46.214	79
2	11.60	20.39	4018.58	9435.73	46.214	35
3	10.04	19.96	4018.58	9435.73	48.45	20
4	8.40	16.90	4021.64	9438.79	48.45	31
DMU b57						
0	9.747	13.004	2107.062	5012.420	24.202	-
1	7.15	12.04	2109.65	5012.42	24.96	47
2	6.2	11.48	2110.6	5013.37	25.70	21
3	5.31	10.42	2111.66	5014.43	25.70	23
4	4.49	8.83	2113.25	5016.02	25.70	30
DMU b3						
0	16.819	24.471	4892.629	10238.76	52.234	-
1	13.36	24.471	4892.629	10634.1	52.234	25
2	11.99	24.24	4892.629	10634.1	57.20	21
4	9.87	20.48	4896.39	10637.86	57.20	34
DMU b9						
0	18.441	35.509	6450.385	12479.115	64.644	-
1	17.03	31.1	6450.385	13427.21	64.644	28
2	15.77	31.1	6450.385	13427.21	75.55	24
3	13.34	26.24	6455.24	13432.07	75.55	31
DMU b19						
0	12.211	24.411	3663.067	10103.516	49.062	-
1	11.37	22.68	3664.802	10105.25	49.37	15
2	11.37	17.45	3786.68	10227.13	49.37	27
3	9.19	15.27	3788.86	10229.31	49.37	31
DMU b52						
0	14.146	22.291	4391.541	8259.170	50.503	-
1	12.67	22.291	4391.541	10308.13	50.503	35
2	11.22	20.37	4391.541	10308.13	55.36	30
3	10.18	17.28	4394.64	10311.22	55.36	24
DMU b13						
0	12.979	23.658	4991.984	10194.377	48.583	-
1	12.22	23.658	4991.984	10194.377	58.72	27
2	10.52	19.96	4995.69	10198.08	58.72	30
DMU b21						
0	12.689	25.489	4797.797	10281.063	48.822	-
1	11.71	23.76	4797.798	10281.063	55.78	68
2	9.63	20.05	4801.51	10284.77	55.80	
DMU b 27						
0	10.021	16.780	3394.509	8269.236	39.565	-
1	8.71	16.02	3394.509	8269.236	43	27
2	7.82	13.60	3396.93	8271.66	43	25
DMU b56						
0	9.073	19.259	2888.434	8694.691	39.974	-
1	8.28	15.85	3173.20	8694.691	40.86	39

2	7.18	13.46	3175.59	8694.691	40.94	28
DMU b58						
0	10.639	22.566	3344.774	10293.887	43.311	-
1	9.81	18.76	3756.84	10293.887	48.37	49
2	8.50	15.93	3759.67	10293.887	48.48	31
DMU b59						
0	13.338	24.820	4354.301	10889.840	57.033	-
1	13.06	19.590	4430.860	10966.400	57.033	26
2	10.57	17.44	4433.35	10968.89	57.033	30
DMU b5						
0	11.243	23.558	4777.107	8756.227	52.449	-
1	9.98	18.56	4782.07	8761.19	54.90	37
DMU b17						
0	16.505	31.574	6322.393	17323.595	81.404	-
1	14.31	26.81	6327.16	17323.595	81.57	28
DMU b20						
0	11.981	17.857	3899.831	10658.024	51.052	-
1	9.47	15.85	3902.34	10660.54	51.052	33
DMU b51						
0	15.178	21.418	5758.861	6007.936	64.210	-
1	13.02	18.91	5758.861	6813.03	64.43	39
DMU b53						
0	12.959	20.117	5372.053	7323.490	64.076	-
1	12.79	18.91	5618.72	7570.16	64.076	15

For example, assuming $P = 1$ that the target for the unit B_{15} is determined in four steps, step 0 represents the initial position $DMU_{B_{15}}$ after solving model(4), the first intermediate target for this DMU, in step 1 as: (11.717, 22.06, 4070.630, 7863.33, 41.11). The relative improvement in this step is 50%. In the next step, the second intermediate target for this unit is (10.79, 19.72, 4070.630, 8516.45 41.11) that the relative improvement in this step is 24%. And in the third step, the intermediate target is (9.95, 19.72, 4070.630, 8516.45, 47.60) . The relative improvement in this step is 16%. In the fourth step, the target is as follows: (8.37, 16.65, 4073.708519.52, 47.60) Because at this stage we have reached the initial layer (k=1), then the target obtained will be the final target for using the above method. You should note that the units on the first layer do not need any changes or improvements because they are on the border of efficiency and the first level of performance. It should also be noted that the number of steps to reach the best performance boundary (first level) is different for each DMU and the farther the unit under evaluation is from the boundary, the number of steps and the number of intermediate target set for that unit will be higher. It is noted that the advantage of this method compared to other methods of setting goals is that, firstly, it provides the closest target in each stage for the unit in question, and secondly, it provides a sequence of goals for the unit under evaluation. It is for the manager to decide on his target, according to the conditions.

5 Discussion and conclusion

DEA is a useful tool for benchmarking, whose main assumption is the homogeneity of DMUs whose performance is under evaluation. Therefore, inefficient DMUs according to this assumption should be close and agree with the goals set by DEA models. because the DMUs are all comparable, however, in many conditions, some DMUs perform very poorly and reaching the targets set by DEA models are not possible for these units, at least in the short

term, therefore, in order to find benchmarks and targetsetting for such DMUs, we presented a multi-stage benchmarking method dependent on DEA benchmarks, which can be used as a map. The way forward is in a long-term perspective, while allowing for more realistic and achievable targets at an intermediate boundary of better performance, established by existing DMUs at a similar performance level. This approach is in the framework of developed benchmarks that minimize the distance to the efficiency limit, and the usefulness of this method is evident in the example that was presented, in the sense that, firstly, it creates motivation in highly inefficient units for performance is improved, and secondly, it suggests different options to improve performance to the management so that they can choose options that suit their conditions, taking into account the conditions, resources, and facilities, to improve performance, secondly, by categorizing units Decision-making based on the level of efficiency and performance determines a realistic achievable benchmark and target for inefficient units at each stage.

6 Suggestions for future researchers

The results obtained from the implementation of the algorithm on bank data show that the sequential patterning method can be a very attractive method for inefficient units because it determines the goals for the inefficient units in each stage in a stepwise manner compared to the goals of the previous stage. The main efficiency is closer and it goes step by step towards the main goal. One of the very important benefits of this method is that it motivates inefficient units to improve themselves with the least effort, because in this layered modeling method, we are the first to use models to determine the closest goals. As a suggested future work, the method can be extended to radial models.

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