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A fully fuzzy centralized resource allocation model using lexicographic multi-objective DEA

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Abstract Effective resource allocation by a central authority is often hampered by data imprecision and uncertainty. This study develops a novel Fully Fuzzy Centralized Data Envelopment Analysis (FFCDEA) framework, designed to empower a Central Decision-Making Unit (CDM) to evaluate and simultaneously optimize the performance of all Decision-Making Units (DMUs). The proposed model extends the non-radial Enhanced Russell Measure (ERM) into a fully fuzzy environment and is structured as a lexicographic multi-objective linear program (LMOLP) to effectively handle the inherent uncertainty in input and output data, which are modeled as triangular fuzzy numbers. The inherent fractional nature of the model is linearized using the Charnes-Cooper (C-C)Transformation, rendering it readily solvable. A real-world application on 14 bank branches validates the model's practical utility, demonstrating its capability to derive specific, actionable targets for both individual branches and the system as a whole. By systematically handling data ambiguity and optimizing performance from a holistic, system-wide perspective, our FFCDEA model provides a more robust and realistic tool for strategic planning compared to traditional centralized or fuzzy DEA approaches.

Keyword: Data Envelopment Analysis, Lexicographic Multi-Objective Linear Programming, Centralized Resource Allocation, Fuzzy Mathematical Programming.

1 Introduction
1.1 From independent evaluation to centralized resource allocation

Efficiency assessment has long been a central topic in performance evaluation. Following Farrell's [1] foundational concepts of productive efficiency, Charnes et al. [2] developed the well-known CCR model, which revolutionized the evaluation of decision-making units (DMUs) with multiple inputs and outputs. Traditional DEA models inherently assume that DMUs operate independently and aim to maximize their own individual efficiency. However, in centralized systems—where a central authority oversees all units and allocates shared resources—this assumption is unrealistic and often leads to sub-optimal system-wide performance.

To address this, researchers shifted their focus toward Centralized DEA (CDEA) models. Golany et al. [3] and Athanassopoulos [4] were among the first to propose frameworks for

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system-wide efficiency and central grant allocation, though early models faced computational and feasibility limitations. A significant breakthrough was made by Lozano and Villa [5, 6], who introduced radial and non-radial CDEA models under variable returns to scale. Their models aimed to optimize total resource consumption and output production simultaneously from a macroeconomic perspective. This foundation sparked a series of methodological advancements in CDEA. Subsequent studies expanded centralized frameworks to include BCC models [7], enhanced Russell measures [8], and simplified linear structures [9]. More recently, researchers have tailored CDEA to address complex structural constraints, such as step-by-step improvement paths [10], bi-level optimization structures enforcing minimum individual efficiency [11], comprehensive target-setting mechanisms [12], and directional distance models for equitable grant allocation [13]. In practical applications, CDEA has proven highly effective in sectors requiring strict centralized supervision, such as banking [14].

2.2 Managing uncertainty through fuzzy DEA

While classical and centralized DEA models provide robust analytical tools, they fundamentally rely on deterministic data. Real-world applications, however, are frequently plagued by missing, random, or imprecise data often expressed in qualitative terms. Ignoring such uncertainty can severely distort efficiency scores and mislead decision-makers. To capture this ambiguity, Bellman and Zadeh's [15] fuzzy set theory was integrated into DEA, pioneering the field of Fuzzy DEA (FDEA).

Since Sengupta [16] first incorporated fuzzy constraints into DEA based on Zimmermann's optimization principles [17], a wide array of FDEA methodologies has emerged. Researchers have developed diverse techniques to handle imprecise inputs and outputs, including efficiency membership functions [18], fuzzy expected values [19], and fuzzy arithmetic for determining efficiency bounds [20]. To improve discrimination among DMUs in fuzzy environments, several ranking approaches [21-23] and methods for controlling weight flexibility [24, 25] have been proposed. Furthermore, the application of FDEA has expanded into complex network structures, such as two-stage series systems [26, 27], and real-world problems like real estate risk evaluation [28], transportation route selection [29], healthcare efficiency [30], and circular economy performance [31]. More advanced techniques have also incorporated game theory [32], Pythagorean fuzzy sets [33], and multi-objective lexicographic approaches [34] to better align efficiency evaluations with managerial preferences.

2.3 Research gap and contribution

Despite the extensive literature on both Centralized DEA and Fuzzy DEA, the intersection of these two domains remains relatively sparse. Centralized resource allocation inherently involves long-term strategic planning, making it highly susceptible to uncertainty. While some recent studies have begun to explore centralized allocation under ambiguity [35], or have proposed hybrid robust-fuzzy approaches [36] and fully fuzzy multi-objective programming for independent units [37, 38], a comprehensive framework that addresses fully fuzzy centralized resource allocation using a multi-objective lexicographic approach is lacking.

Most existing CDEA models either treat data as crisp or apply overly simplified fuzzy conversions that lose vital information. This study aims to bridge this gap. By developing a Fully Fuzzy Centralized DEA (FFCDEA) model utilizing a lexicographic optimization strategy, this research provides decision-makers with a reliable, step-by-step tool to allocate resources and set targets system-wide, while mathematically preserving the inherent uncertainties of real-world operational environments.

The remainder of this paper is organized as follows: Section 2 introduces the preliminaries, including basic fuzzy arithmetic and foundational DEA concepts. Section 3 details the development of the proposed FFCDEA framework, explicitly outlining the C-C Transformation and the lexicographic multi-objective optimization procedure. Section 4 presents a comprehensive real-world application involving 14 bank branches. This section demonstrates the computational procedure, evaluates both systemic and DMU-level efficiency targets, and extensively discusses the managerial implications and practical considerations of resource reallocation. Finally, Section 5 concludes the paper and outlines potential directions for future research. Additionally, extended datasets for different fuzzy bounds and the computational codes ensuring full reproducibility are provided as Supplementary Materials/Appendices.

2 Preliminaries

This section provides the foundational concepts and mathematical background necessary for developing the proposed methodology. To ensure clarity and consistency throughout the paper, we first present a comprehensive nomenclature defining all indices, parameters, and variables used in our models. Following this, the section is divided into four main parts: First, we review the basic principles of DEA, with a specific focus on the Russell measure. Second, we discuss the centralized resource allocation model and its equivalent linear transformation based on recent developments in the literature. Third, essential concepts from fuzzy set theory, particularly triangular fuzzy numbers (TFNs) and their arithmetic operations, are introduced to handle data uncertainty. Finally, we explain the fundamentals of the lexicographic multi-objective programming approach in a fuzzy context, which serves as the core computational engine for our proposed framework.

2.1 DEA and the ERM

DEA is a widely used non-parametric approach for evaluating the relative efficiency of a set of homogeneous DMUs. Suppose we have n DMUs, where each DMU_j $(j = 1, \dots, n)$ consumes m inputs to produce s outputs. Let x_{ij} denote the i-th input $(i = 1, \dots, m)$ and y_{kj} denote the $k-$th output $(k = 1, \dots, s)$ of DMU_j. Furthermore, let X and Y represent the input and output matrices of all DMUs, respectively). Assuming a VRS technology, the Production Possibility Set (PPS), denoted by T_v, is defined as follows:

$$T_v = \left\{ (x, y) \mid x \geq X\lambda, y \leq Y\lambda, \textstyle\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \dots, n \right\}$$

where $\lambda = (\lambda_1, \dots, \lambda_n)$ is the intensity vector used to form convex combinations of the observed inputs and outputs. For the convenience of the reader and to improve the clarity of the mathematical formulations, a comprehensive list of the indices, parameters, variables, and abbreviations used throughout the paper is summarized in Appendix B.

Traditional DEA models (such as CCR and BCC) are radial and do not fully account for non-radial slacks in inputs and outputs. To overcome this limitation, the ERM provides a comprehensive non-radial efficiency score. For a specific unit under evaluation, $\mathrm{DMU_o}$, the standard ERM model is formulated as:

$$
\begin{aligned}
&\min E = \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{k=1}^{s}\varphi_k}\\
\text{s.t.}\quad &\sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{io}, \quad i = 1,\dots,m\\
&\sum_{j=1}^{n}\lambda_j y_{kj} \ge \varphi_k y_{ko}, \quad k = 1,\dots,s\\
&\sum_{j=1}^{n}\lambda_j = 1\\
&\lambda_j \ge 0, \quad j = 1,\dots,n\\
&\theta_i \le 1, \quad i = 1,\dots,m\\
&\varphi_k \ge 1, \quad k = 1,\dots,s
\end{aligned}
\tag{1}
$$

In this model, θ_i represents the proportional reduction for the $i-$th input, and φ_k represents the proportional expansion for the k-th output.

Let $(\theta\ ,\varphi\ ,\lambda\ )$ be the optimal solution of the ERM model for the evaluated $\mathrm{DMU_o}$. Based on this optimal solution, the virtual targets (projected points) for the inputs and outputs, denoted by $\widehat{x_{io}}$ and $\widehat{y_{ko}}$, are defined as follows:

$$
\begin{aligned}
\widehat{x_{io}} &= \theta_i^* x_{io} = \sum_{j=1}^{n}\lambda_j^* x_{ij}, \quad i = 1,\dots,m\\
\widehat{y_{ko}} &= \varphi_k^* y_{ko} = \sum_{j=1}^{n}\lambda_j^* y_{kj}, \quad k = 1,\dots,s
\end{aligned}
\tag{2}
$$

It is crucial to note that these target points belong to the strong efficiency frontier (i.e., they are strongly Pareto-efficient). In standard radial models, the efficiency score merely reflects an equi-proportional reduction in inputs (or expansion in outputs), often projecting the DMU onto the weak efficiency frontier where non-zero residual slacks may still exist. Reaching the strong frontier in such models requires a secondary optimization phase.

In contrast, the ERM is a non-radial approach whose objective function strictly and comprehensively incorporates the contraction ratio (θ_i) for every single input and the expansion ratio (φ_k) for every single output. Because the model actively minimizes all input ratios and maximizes all output ratios simultaneously, any uncaptured slack would contradict the optimality of the objective value. Consequently, the projected virtual target $(\widehat{x_{io}}, \widehat{y_{ko}})$ is guaranteed to be a Pareto-efficient operating point. At this point, it is mathematically impossible to further decrease any input without increasing another input or decreasing an output, and similarly impossible to increase any output without decreasing another output or increasing an input.

2.2 Centralized resource allocation and target setting mechanism

While conventional DEA models evaluate units independently, Lozano et al. [5] introduced a centralized DEA approach. They argued that when all units (e.g., bank branches) operate under the supervision of a single central decision-maker, evaluating them independently might lead to sub-optimal decisions for the organization as a whole. Instead, the central decision-maker seeks to optimize resource allocation across all units simultaneously to maximize the overall efficiency of the group.

However, traditional centralized models were predominantly radial and focused on either input reduction or output expansion separately. To address this limitation, recent developments in the literature [40] introduced a non-radial centralized model based on the ERM. This model is designed to simultaneously adjust inputs and outputs, provide a unified efficiency score for the entire group, and determine specific targets for each unit.

The fractional mathematical formulation of this centralized ERM model is presented as follows:

$$\begin{gathered}
\min R = \frac{\sum_{i=1}^{m} \theta_i / m}{\sum_{k=1}^{s} \varphi_k / s} \\
\text{s.t.} \\
\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} x_{ij} \le \theta_i \sum_{j=1}^{n} x_{ij}, \quad i = 1, \dots, m \\
\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} y_{kj} \ge \varphi_k \sum_{j=1}^{n} y_{kj}, \quad k = 1, \dots, s \\
\sum_{j=1}^{n} \lambda_{jr} = 1, \quad r = 1, \dots, n \\
\lambda_{jr} \ge 0, \quad j, r = 1, \dots, n \\
\theta_i \le 1, \quad i = 1, \dots, m \\
\varphi_k \ge 1, \quad k = 1, \dots, s
\end{gathered} \tag{3}$$

To solve this nonlinear fractional programming problem, a standard C-C transformation is applied. By introducing a scalar variable $\beta > 0$ and defining new variables $u_i = \beta\theta_i$, $v_k = \beta\varphi_k$, and $t_{jr} = \beta\lambda_{jr}$, the fractional model is transformed into the following equivalent linear programming model:

$$\begin{gathered}
\min \frac{\sum_{i=1}^{m} u_i}{m} \\
\text{s.t.} \\
\sum_{r=1}^{n}\sum_{j=1}^{n} t_{jr} x_{ij} \le u_i \sum_{r=1}^{n} x_{ir}, \quad i = 1, \dots, m \\
\sum_{r=1}^{n}\sum_{j=1}^{n} t_{jr} y_{kj} \ge v_k \sum_{r=1}^{n} y_{kr}, \quad k = 1, \dots, s
\end{gathered}$$

$$\sum_{j=1}^{n} t_{jr} = \beta, \quad r = 1, \dots, n$$
$$t_{jr} \geq 0, \quad j, r = 1, \dots, n$$
$$u_i \leq \beta, \quad i = 1, \dots, m$$
$$v_k \geq \beta, \quad k = 1, \dots, s$$
$$\sum_{k=1}^{s} v_k = s$$
$$0 \leq \beta \leq 1$$
(4)

Once the optimal values $(\beta^*, u_i^*, v_k^*, t_{jr}^*)$ are obtained from the linear model, the optimal intensity variables are recovered as $\lambda_{jr}^* = t_{jr}^*/\beta^*$.

A fundamental property of this centralized approach is its built-in mechanism for target setting and resource reallocation. Let x_{ij} and y_{kj} represent the initial resources and outputs of the units before applying the model. Using the optimal intensity variables, the central decision-maker can determine the exact optimal allocation of resources after the evaluation. The new targets (reallocated resources and expected outputs) for each specific unit r $(r = 1, \dots, n)$ are calculated as:

$$\widehat{x_{ir}} = \sum_{j=1}^{n} \lambda_{jr}^* x_{ij}, \quad \forall i = 1, \dots, m$$
$$\widehat{y_{kr}} = \sum_{j=1}^{n} \lambda_{jr}^* y_{kj}, \quad \forall k = 1, \dots, s$$

As established in previous studies [40], a key characteristic of these newly assigned targets $(\widehat{x_{ir}}, \widehat{y_{kr}})$ is that they are mathematically guaranteed to be Pareto-efficient (strongly efficient) within the PPS. Furthermore, if we aggregate these individual targets to form an overall target for the entire group $(\sum_r \widehat{x_{ir}}, \sum_r \widehat{y_{kr}})$, this aggregated target is also guaranteed to lie on the strong efficient frontier of the group's technology set. This property provides a transparent, step-by-step roadmap for managers to redistribute inputs and outputs optimally among units without any resource wastage.

2.3 Fuzzy sets and triangular fuzzy numbers

In real-world applications, particularly in the banking sector, input and output data are often tainted by uncertainty, imprecision, and vagueness. Traditional deterministic models fail to capture this inherent ambiguity. To address this, fuzzy set theory, initially introduced by Zadeh (1965), is employed to model systems under uncertainty.

Definition 1 Let X be a universal set. A fuzzy set $\tilde{A}$ in X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates each element $x \in X$ with a real number in the interval $[0,1]$. The α-cut of a fuzzy set $\tilde{A}$, denoted by $\widetilde{A_\alpha}$, is a crisp subset of elements whose membership degree is at least α $(\alpha \in [0,1])$, defined as:

$$\widetilde{A_\alpha} = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}$$

Definition 2 A Triangular Fuzzy Number (TFN), denoted by $\widetilde{A} = (a^L, a^M, a^U)$, is a convex and normal fuzzy set whose membership function $\mu_{\widetilde{A}}(x)$ is defined as piecewise linear:

$$\mu_{\widetilde{A}}(x) = \max\left(\min\left(\frac{x - a^L}{a^M - a^L}, \frac{a^U - x}{a^U - a^M}\right), 0\right)$$

where a^L and a^U represent the lower and upper bounds respectively, and a^M is the most promising (mean) value.

While there are various forms of fuzzy numbers such as interval or trapezoidal fuzzy numbers, TFNs are particularly adopted in this study. Interval numbers merely provide a uniform bound without highlighting the most likely scenario, and trapezoidal numbers assume a flat top (a range of equally most-likely values) which increases computational complexity without necessarily adding value in banking performance evaluation. In contrast, TFNs perfectly align with managerial perspectives by representing the exact pessimistic (a^L), most-likely (a^M), and optimistic (a^U) scenarios. Furthermore, TFNs maintain linearity in membership functions, ensuring computational efficiency and interpretability in large-scale Centralized Resource Allocation models.

Definition 3 A triangular fuzzy number $\widetilde{A} = (a^L, a^M, a^U)$ is considered non-negative ($\widetilde{A} \geq 0$) if and only if $a^L \geq 0$. Two TFNs $\widetilde{A}$ and $\widetilde{B} = (b^L, b^M, b^U)$ are considered equal ($\widetilde{A} = \widetilde{B}$) if $a^L = b^L$, $a^M = b^M$, and $a^U = b^U$.

Definition 4: Let $\widetilde{A} = (a^L, a^M, a^U)$ and $\widetilde{B} = (b^L, b^M, b^U)$ be two non-negative TFNs, and $k \in R$ be a crisp scalar. According to Zadeh's extension principle, the basic arithmetic operations are defined as follows:

1) Addition:

$$\widetilde{A} \oplus \widetilde{B} = (a^L + b^L, a^M + b^M, a^U + b^U)$$

2) Subtraction:

$$\widetilde{A} \ominus \widetilde{B} = (a^L - b^U, a^M - b^M, a^U - b^L)$$

3) Scalar Multiplication:

$$k \otimes \widetilde{A} = (ka^L, ka^M, ka^U), \quad \text{if } k \geq 0$$
$$k \otimes \widetilde{A} = (ka^U, ka^M, ka^L), \quad \text{if } k < 0$$

2.4 Developing a fully fuzzy CCR model using a Lexicographic approach for TFNs

In this section, to evaluate the efficiency of DMUs in uncertain environments, a radial DEA model based on the traditional CCR framework is developed in a fully fuzzy context. Assume there are n DMUs, where each DMU_j $(j = 1, \dots, n)$ uses m fuzzy inputs $\widetilde{x_{ij}}$ $(i = 1, \dots, m)$ to produce s fuzzy outputs $\widetilde{y_{rj}}$ $(r = 1, \dots, s)$.

The input-oriented basic CCR model, when all inputs, outputs, and decision variables (including weight variables $\tilde{\lambda}$ and the efficiency score $\tilde{\theta}$) are considered fuzzy, is formulated as follows:

$$\min \widetilde{\theta_p}$$

$$
\begin{aligned}
\text{s.t.} \sum_{j=1}^{n} \widetilde{\lambda_j} \otimes \widetilde{x_{ij}} &\leq \widetilde{\theta_p} \otimes \widetilde{x_{ip}} \quad , \quad i = 1, \dots, m \\
\sum_{j=1}^{n} \widetilde{\lambda_j} \otimes \widetilde{y_{rj}} &\geq \widetilde{y_{rp}} \quad , \quad r = 1, \dots, s \\
\widetilde{\lambda_j} &\geq 0 \quad , \quad j = 1, \dots, n
\end{aligned} \tag{5}
$$

In this study, all fuzzy parameters are modeled using TFNs. $\widetilde{A} = (A^L, A^M, A^U)$. Based on fuzzy arithmetic operations and by expanding the fully fuzzy model using the components of TFNs, the model is decomposed into a set of crisp inequalities. By introducing an index $k \in \{L, M, U\}$ to represent the three components of the TFNs, and defining the fuzzy efficiency score as $\widetilde{\theta_p} = (\theta_p^L, \theta_p^M, \theta_p^U)$, the objective function transforms into a Multi-Objective Linear Programming (MOLP) problem aimed at minimizing all three components of the efficiency score simultaneously:

$$
\begin{aligned}
&\min \left(\theta_p^L, \theta_p^M, \theta_p^U\right) \\
\text{s.t.} &\sum_{j=1}^{n} \lambda_j^k x_{ij}^k \leq \theta_p^k x_{ip}^k \quad , \quad i = 1, \dots, m \quad , \quad k \in \{L, M, U\} \\
&\sum_{j=1}^{n} \lambda_j^k y_{rj}^k \geq y_{rp}^k \quad , \quad r = 1, \dots, s \quad , \quad k \in \{L, M, U\} \\
&\theta_p^U \geq \theta_p^M \geq \theta_p^L \geq 0 \\
&\lambda_j^U \geq \lambda_j^M \geq \lambda_j^L \geq 0 \quad , \quad j = 1, \dots, n
\end{aligned} \tag{6}
$$

(Note: The last two constraints are included to maintain the standard structural shape of TFNs for the decision variables).

To solve the aforementioned MOLP problem, we employ the lexicographic optimization approach. Hatami-Marbini et al. [28] proposed this method for evaluating efficiency in fuzzy DEA models. Their approach was originally formulated for trapezoidal fuzzy numbers, which necessitates solving four sequential optimization steps. However, since we utilize TFNs in this paper, the structure is adapted accordingly, reducing the optimization process to three sequential steps.

In our lexicographic approach, the minimization of the efficiency score components is performed hierarchically, prioritizing from the upper bound down to the lower bound. The solution steps are detailed as follows:

Step 1: First, the upper bound of the efficiency score is minimized. In this step, the following single-objective model is solved to obtain the optimal value θ_p^U:

$$\min \theta_p^U$$

s.t. All input, output, and structural constraints mentioned in the MOLP model.

Step 2: Next, the middle value of the efficiency score is minimized, provided that the optimal value obtained from the first step (θ_p^U) is preserved:

$$
\begin{aligned}
&\min \ \theta_p^M \\
&\text{s.t. } \theta_p^U = \theta_p^{U*}
\end{aligned}
$$

s.t. All input, output, and structural constraints mentioned in the MOLP model.

By solving this model, the optimal value θ_p^M is extracted.

Step 3: Finally, the lower bound of the efficiency score is minimized while maintaining the optimal values derived from the previous two steps:

$$\min \theta_p^L$$
$$\text{s.t. } \theta_p^U = \theta_p^{U*}$$
$$\theta_p^M = \theta_p^{M*}$$

s.t. All input, output, and structural constraints mentioned in the MOLP model.

The optimal value resulting from this final step (θ_p^L), alongside the previously obtained values, provides the unique and final fuzzy efficiency score for the evaluated unit as a precise triangular fuzzy number: $\widetilde{\theta_p^*} = (\theta_p^{L*}, \theta_p^{M*}, \theta_p^{U*})$.

3 Fully fuzzy centralized resource allocation model
3.1 Model making process

In many real-world applications, DMUs operate under the supervision of a Centralized Decision Maker (CMD) rather than as entirely independent entities. In such environments, the DM seeks to optimize the total resource consumption and production across all DMUs simultaneously. To address the inherent uncertainty in real-world data, we propose a fully fuzzy centralized resource allocation model based on the modified Russell measure.

Consider a system comprising n DMUs, where each DMUj ($j = 1, \dots, n$) consumes m inputs to produce s outputs. Due to data uncertainty, the inputs and outputs are represented as TFNs, denoted by $\widetilde{x_{ij}} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ and $\widetilde{y_{kj}} = (y_{kj}^L, y_{kj}^M, y_{kj}^U)$, respectively.

The objective of the centralized DM is to determine a unified efficiency score by simultaneously contracting inputs and expanding outputs across the entire system. Accordingly, the decision variables are defined as follows:

$\widetilde{\theta_i}$: The fuzzy contraction ratio for the i-th input across all DMUs.

$\widetilde{\varphi_k}$: The fuzzy expansion ratio for the k-th output across all DMUs.

$\widetilde{\lambda_{jr}}$: The fuzzy reallocation weight representing the contribution of DMUr in evaluating DMUj.

Based on these assumptions, the fully fuzzy fractional centralized model is formulated as follows:

$$\min \tilde{R} = \frac{\sum_{i=1}^{m} \widetilde{\theta_i}/m}{\sum_{k=1}^{s} \widetilde{\varphi_k}/s}$$
$$\text{s.t.} \quad \sum_{r=1}^{n}\sum_{j=1}^{n} \widetilde{\lambda_{jr}} \otimes \widetilde{x_{ij}} \leq \widetilde{\theta_i} \otimes \sum_{j=1}^{n} \widetilde{x_{ij}} \quad , \quad i = 1, \dots, m$$
$$\sum_{r=1}^{n}\sum_{j=1}^{n} \widetilde{\lambda_{jr}} \otimes \widetilde{y_{kj}} \geq \widetilde{\varphi_k} \otimes \sum_{j=1}^{n} \widetilde{y_{kj}} \quad , \quad k = 1, \dots, s$$
$$\sum_{j=1}^{n} \widetilde{\lambda_{jr}} = \tilde{1} \quad , \quad r = 1, \dots, n$$
$$\widetilde{\lambda_{jr}} \geq 0 \quad , \quad j, r = 1, \dots, n$$
$$\widetilde{\theta_i} \leq 1 \quad , \quad i = 1, \dots, m$$
$$\widetilde{\varphi_k} \geq 1 \quad , \quad k = 1, \dots, s \tag{7}$$

The objective function $\widetilde{R}$ evaluates the overall fuzzy efficiency of the system by minimizing the ratio of average input contraction to average output expansion.

The first set of constraints guarantees that the total redistributed fuzzy inputs do not exceed the contracted total available inputs in the system. Similarly, the second set of constraints ensures that the total redistributed fuzzy outputs are at least equal to the expanded total current outputs. The third and fourth constraints represent the standard convexity and non-negativity conditions for the redistribution weights. Finally, the bounding constraints ($\widetilde{\theta_i} \leq 1$ and $\widetilde{\varphi_k} \geq 1$) ensure that inputs are only reduced and outputs are only increased.

By utilizing the arithmetic operations of TFNs and introducing the index $h \in \{L, M, U\}$ to represent the lower, middle, and upper bounds of the fuzzy sets, the fully fuzzy model can be decomposed. Based on the rules of fuzzy division, the upper bound of the objective function is obtained by dividing the upper bound of the numerator by the lower bound of the denominator (and vice versa for the lower bound). Thus, the model transforms into an MOFP problem:

$$
\begin{aligned}
&\min\left(R^L = \frac{\sum_{i=1}^{m}\theta_i^L/m}{\sum_{k=1}^{s}\varphi_k^U/s},\quad R^M = \frac{\sum_{i=1}^{m}\theta_i^M/m}{\sum_{k=1}^{s}\varphi_k^M/s},\quad R^U = \frac{\sum_{i=1}^{m}\theta_i^U/m}{\sum_{k=1}^{s}\varphi_k^L/s}\right)\\
&\text{s.t.} \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^h x_{ij}^h \leq \theta_i^h \sum_{j=1}^{n} x_{ij}^h, \quad i = 1, \dots, m \quad , \quad h \in \{L, M, U\}\\
&\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^h y_{kj}^h \geq \varphi_k^h \sum_{j=1}^{n} y_{kj}^h, \quad k = 1, \dots, s \quad , \quad h \in \{L, M, U\}\\
&\sum_{j=1}^{n}\lambda_{jr}^h = 1\\
&\lambda_{jr}^U \geq \lambda_{jr}^M \geq \lambda_{jr}^L \geq 0, \quad j, r = 1, \dots, n\\
&1 \geq \theta_i^U \geq \theta_i^M \geq \theta_i^L \geq 0, \quad i = 1, \dots, m\\
&\varphi_k^U \geq \varphi_k^M \geq \varphi_k^L \geq 1, \quad k = 1, \dots, s
\end{aligned} \tag{7}
$$

To optimize the derived MOFP model, we employ the three-step lexicographic method. This approach hierarchically minimizes the fuzzy components from the most pessimistic (upper bound) to the most optimistic (lower bound) perspective:

Step 1: Minimize the upper bound of the efficiency score:

$$\min R^U = \frac{\sum_{i=1}^{m}\theta_i^U/m}{\sum_{k=1}^{s}\varphi_k^L/s}$$

s.t. All decomposed structural and bounding constraints for $h \in \{L, M, U\}$

Let the optimal objective value be R^U.

Step 2: Minimize the middle component, preserving the optimal upper bound obtained in Step 1:

$$\min R^M = \frac{\sum_{i=1}^{m}\theta_i^M/m}{\sum_{k=1}^{s}\varphi_k^M/s}$$

$$\text{s.t.} \quad \frac{\sum_{i=1}^{m}\theta_i^U/m}{\sum_{k=1}^{s}\varphi_k^L/s} = R^U$$

s.t. All decomposed structural and bounding constraints for $h \in \{L, M, U\}$

Let the optimal objective value be R^M.

Step 3: Minimize the lower bound, preserving the optimal values from both Step 1 and Step 2:

$$\min R^L = \frac{\sum_{i=1}^{m} \theta_i^L / m}{\sum_{k=1}^{s} \varphi_k^U / s}$$
$$\text{s.t.} \quad \frac{\sum_{i=1}^{m} \theta_i^U / m}{\sum_{k=1}^{s} \varphi_k^L / s} = R^{U*}$$
$$\frac{\sum_{i=1}^{m} \theta_i^M / m}{\sum_{k=1}^{s} \varphi_k^M / s} = R^{M*}$$
$$\text{s.t. All decomposed structural and bounding constraints for } h \in \{L, M, U\}$$

Let the optimal objective value be R^L. The final exact triangular fuzzy efficiency score is represented as $\widetilde{R} = (R^L, R^M, R^U)$.

3.2 Linearized Lexicographic optimization procedure

By applying the C-C transformation, we convert the nonlinear MOFP model into a sequence of linear programming (LP) models. We introduce a positive scalar variable t and define new variables $T_i^h = t\theta_i$, $\Phi_k^h = t\varphi_k$, and $\Lambda_{jr}^h = t\lambda_{jr}$. The three-step linear lexicographic procedure is formulated as follows:

Step 1: Minimizing the Upper Bound (R^U)

In the first step, we linearize the upper bound by setting the denominator of R^U equal to 1 using the variable t^U.

$$\begin{gathered}
\min \sum_{i=1}^{m} T_i^U / m \\
\text{s.t.} \sum_{k=1}^{s} \Phi_k^L / s = 1 \\
\sum_{r=1}^{n} \sum_{j=1}^{n} \Lambda_{jr}^h x_{ij}^h \le T_i^h \sum_{j=1}^{n} x_{ij}^h, \quad i = 1, \dots, m, \quad h \in \{L, M, U\} \\
\sum_{r=1}^{n} \sum_{j=1}^{n} \Lambda_{jr}^h y_{kj}^h \ge \Phi_k^h \sum_{j=1}^{n} y_{kj}^h, \quad k = 1, \dots, s, \quad h \in \{L, M, U\} \\
\sum_{j=1}^{n} \Lambda_{jr}^h = t^U \\
\Lambda_{jr}^U \ge \Lambda_{jr}^M \ge \Lambda_{jr}^L \ge 0, \quad j, r = 1, \dots, n \\
t^U \ge T_i^U \ge T_i^M \ge T_i^L \ge 0, \quad i = 1, \dots, m \\
\Phi_k^U \ge \Phi_k^M \ge \Phi_k^L \ge t^U, \quad k = 1, \dots, s \\
t^U > 0
\end{gathered} \tag{8}$$

Let the optimal objective value of this step be R^{U*}.

Step 2: Minimizing the Middle Component (R^M)

Here, we set the denominator of R^M equal to 1 using a new scalar t^M. The optimal value from Step 1 is incorporated as a linear constraint.

$$
\begin{aligned}
&\min \sum_{i=1}^{m} T_i^M / m \\
&\text{s.t.} \quad \sum_{k=1}^{s} \Phi_k^M / s = 1 \\
&\sum_{i=1}^{m} T_i^U / m = R^{U*} \left(\sum_{k=1}^{s} \Phi_k^L / s \right) \\
&\sum_{r=1}^{n} \sum_{j=1}^{n} \Lambda_{jr}^h x_{ij}^h \le T_i^h \sum_{j=1}^{n} x_{ij}^h \quad , \quad i = 1, \dots, m \quad , \quad h \in \{L, M, U\} \\
&\sum_{r=1}^{n} \sum_{j=1}^{n} \Lambda_{jr}^h y_{kj}^h \ge \Phi_k^h \sum_{j=1}^{n} y_{kj}^h \quad , \quad k = 1, \dots, s \quad , \quad h \in \{L, M, U\} \\
&\sum_{j=1}^{n} \Lambda_{jr}^h = t^M \\
&\Lambda_{jr}^U \ge \Lambda_{jr}^M \ge \Lambda_{jr}^L \ge 0 \quad , \quad j, r = 1, \dots, n \\
&t^M \ge T_i^U \ge T_i^M \ge T_i^L \ge 0 \quad , \quad i = 1, \dots, m \\
&\Phi_k^U \ge \Phi_k^M \ge \Phi_k^L \ge t^M \quad , \quad k = 1, \dots, s \\
&t^M > 0
\end{aligned}
\tag{9}
$$

Let the optimal objective value of this step be R^{M*}.

Step 3: Minimizing the Lower Bound (R^L)

Finally, we set the denominator of R^L equal to 1 using t^L, preserving the optimal values from both previous steps as linear constraints.

$$
\begin{aligned}
&\min \sum_{i=1}^{m} T_i^L / m \\
&\text{s.t.} \quad \sum_{k=1}^{s} \Phi_k^U / s = 1 \\
&\sum_{i=1}^{m} T_i^U / m = R^{U*} \left(\sum_{k=1}^{s} \Phi_k^L / s \right) \\
&\sum_{i=1}^{m} T_i^M / m = R^{M*} \left(\sum_{k=1}^{s} \Phi_k^M / s \right) \\
&\sum_{r=1}^{n} \sum_{j=1}^{n} \Lambda_{jr}^h x_{ij}^h \le T_i^h \sum_{j=1}^{n} x_{ij}^h \quad , \quad i = 1, \dots, m \quad , \quad h \in \{L, M, U\} \\
&\sum_{r=1}^{n} \sum_{j=1}^{n} \Lambda_{jr}^h y_{kj}^h \ge \Phi_k^h \sum_{j=1}^{n} y_{kj}^h \quad , \quad k = 1, \dots, s \quad , \quad h \in \{L, M, U\} \\
&\sum_{j=1}^{n} \Lambda_{jr}^h = t^L \\
&\Lambda_{jr}^U \ge \Lambda_{jr}^M \ge \Lambda_{jr}^L \ge 0 \quad , \quad j, r = 1, \dots, n
\end{aligned}
$$

$$\begin{gathered} t^L \geq T_i^U \geq T_i^M \geq T_i^L \geq 0 \quad , \quad i = 1, \dots, m \\ \Phi_k^U \geq \Phi_k^M \geq \Phi_k^L \geq t^L \quad , \quad k = 1, \dots, s \\ t^L > 0 \end{gathered} \tag{10}$$

After solving Step 3, the final exact triangular fuzzy efficiency score $\widetilde{R^*} = (R^{L*}, R^{M*}, R^{U*})$ is determined.

3.3 Key characteristics and computational considerations of the linearized model

The proposed linearized lexicographic procedure possesses several vital mathematical and computational characteristics that warrant detailed explanation. Firstly, the elimination of the convexity constraint (originally $\sum_{j=1}^{n} \lambda_{jr}^h = 1$, which translates to $\sum_{j=1}^{n} \Lambda_{jr}^h = t^h$ after the C-C transformation) indicates that the centralized system can be evaluated under the assumption of Constant Returns to Scale (CRS). Secondly, the bounding restrictions on the original contraction and expansion ratios ($\theta_i^h \leq 1$ and $\varphi_k^h \geq 1$) are seamlessly mapped into linear boundaries $T_i^h \leq t^h$ and $\Phi_k^h \geq t^h$, ensuring that inputs are strictly contracted and outputs are strictly expanded. Furthermore, in the second and third steps of the lexicographic process, the optimal efficiency values obtained from preceding steps (R^U and R^M) are preserved using strictly linear constraints. This is achieved by equating the numerator to the product of the optimal ratio and the denominator (e.g., $\sum_{i=1}^{m} T_i^U / m = R^U \sum_{k=1}^{s} \Phi_k^L / s$). This strategic algebraic manipulation guarantees that the feasible region remains strictly linear, entirely avoiding the complexities of nonlinear equality constraints.

A fundamental requirement of the C-C transformation is the strict positivity of the scalar variable ($t^h > 0$). This condition is imperative to ensure that the original fractional denominator is non-zero and that the transformation remains mathematically reversible. However, standard linear programming (LP) solvers rely on closed feasible regions and cannot inherently process strict inequalities. To mathematically resolve this limitation, a non-Archimedean infinitesimal value, denoted by ϵ, is introduced, modifying the strict positivity constraint to $t^h \geq \epsilon$. In practical computational implementations—using algebraic modeling languages such as GAMS or LINGO—this non-Archimedean ϵ is replaced by a sufficiently small positive constant (e.g., 10^{-6}). This practical adjustment satisfies the positivity condition, maintains model feasibility, and averts numerical instability during the optimization process.

Successfully solving this sequence of LP models not only yields the exact overall fuzzy efficiency score $\widetilde{R} = (R^L, R^M, R^U)$ but also provides the optimal values for all transformed decision variables ($T_i^h, \Phi_k^h, \Lambda_{jr}^h$, and t^h). By applying the inverse C-C transformation ($\theta_i^h = T_i^h / t^h$, $\varphi_k^h = \Phi_k^h / t^h$, and $\lambda_{jr}^h = \Lambda_{jr}^h / t^h$), the Centralized Decision Maker can retrieve the exact optimal input contractions and output expansions. As will be discussed in the subsequent section, these optimal parameters serve as the mathematical foundation for establishing precise performance targets and designing an optimal resource reallocation scheme across all DMUs within the system.

3.4 Optimal resource allocation and target setting

Once the three-step linearized lexicographic procedure is optimally solved, the CDM obtains the optimal values for all transformed variables ($T_i^h, \Phi_k^h, \Lambda_{jr}^h$, and t^h for $h \in \{L, M, U\}$). The first step in establishing an optimal resource allocation plan is to retrieve the original pre-transformation variables using the inverse Charnes-Cooper substitutions:

$$\theta_i^{h*} = \frac{T_i^{h*}}{t^{h*}} \quad , \quad \varphi_k^{h*} = \frac{\Phi_k^{h*}}{t^{h*}} \quad , \quad \lambda_{jr}^{h*} = \frac{\Lambda_{jr}^{h*}}{t^{h*}}, \quad h \in \{L, M, U\}$$

With these optimal parameters extracted, the CDM can determine the optimally allocated targets from two interconnected perspectives: the macro-level systemic targets based on θ and φ , and the micro-level DMU-specific allocations based on λ .

1. Systemic Resource Allocation and Target Setting (Based on θ^* and φ^*):

The variables θ_i^h and φ_k^h represent the optimal contraction of total input resources and the optimal expansion of total output productions across the entire centralized system, respectively. Therefore, the overall optimal fuzzy target levels for the total consumption of resource i and the total generation of output k are defined as:

$$X_i^{\widetilde{Total*}} = \left(\theta_i^{L*} \sum_{j=1}^{n} x_{ij}^L \ , \ \theta_i^{M*} \sum_{j=1}^{n} x_{ij}^M \ , \ \theta_i^{U*} \sum_{j=1}^{n} x_{ij}^U \right), \quad i = 1, \dots, m$$
$$Y_k^{\widetilde{Total*}} = \left(\varphi_k^{L*} \sum_{j=1}^{n} y_{kj}^L \ , \ \varphi_k^{M*} \sum_{j=1}^{n} y_{kj}^M \ , \ \varphi_k^{U*} \sum_{j=1}^{n} y_{kj}^U \right), \quad k = 1, \dots, s \tag{11}$$

These equations indicate the total resource budget and the overall production goals that the centralized system should aim for to achieve maximum fuzzy efficiency.

2. DMU-Specific Resource Allocation (Based on λ^*):

While the systemic targets provide the overall budget, the intensity variables (λ_{jr}^{h*}) dictate exactly how these resources should be reallocated among individual DMUs to project them onto the efficient frontier. The optimal fuzzy input allocation for a specific DMUr, denoted by $\widetilde{\hat{x}_{ir}^*}$, and its corresponding optimal fuzzy output target, denoted by $\widetilde{\hat{y}_{kr}^*}$, are formulated as:

$$\widehat{x_{ir}^{h*}} = \sum_{j=1}^{n} \lambda_{jr}^{h*} x_{ij}^h \quad , \quad h \in \{L, M, U\}, \ i = 1, \dots, m, \ k = 1, \dots, n$$
$$\widehat{y_{kr}^{h*}} = \sum_{j=1}^{n} \lambda_{jr}^{h*} y_{kj}^h \quad , \quad h \in \{L, M, U\}, \ r = 1, \dots, s, \ k = 1, \dots, n \tag{12}$$

3. Integration of Allocations:

According to the structural constraints of the proposed centralized model, the sum of the individually allocated resources (via λ) strictly bounds the systemic targets (via θ and φ). Mathematically, for any uncertainty level $h \in \{L, M, U\}$:

$$\sum_{r=1}^{n} \widehat{x_{ir}^{h*}} \le \theta_i^{h*} \sum_{j=1}^{n} x_{ij}^h$$
$$\sum_{r=1}^{n} \widehat{y_{kr}^{h*}} \ge \varphi_k^{h*} \sum_{j=1}^{n} y_{kj}^h$$

To ensure that the proposed allocations are practically implementable and perfectly efficient, it is crucial to demonstrate that the sum of individual allocations mathematically matches the systemic targets without any residual slacks. This characteristic is rigorously proven in Theorem 1.

Theorem 1 (Binding Constraints at Optimality).
In the optimal solution of the proposed non-radial centralized resource allocation model, all resource and production constraints strictly hold as equalities. Consequently, the total allocated inputs and generated outputs across all DMUs perfectly match the optimal systemic targets.
Proof:
We prove this theorem by contradiction. Let $(\theta^h, \varphi^h, \lambda^h)$ be the optimal solution for the centralized model at a given uncertainty level $h \in \{L, M, U\}$.
First, consider the input constraints. Suppose that for at least one specific input resource i_0, the constraint holds as a strict inequality at optimality:

$$\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr}^{h} x_{i_0 j}^{h} < \theta_{i_0}^{h} \sum_{j=1}^{n} x_{i_0 j}^{h}$$

Since inputs are strictly positive ($x_{ij} > 0$), we can define a new contraction parameter $\widehat{\theta_{i_0}^{h}}$ such that:

$$\widehat{\theta_{i_0}^{h}} = \frac{\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr}^{h} x_{i_0 j}^{h}}{\sum_{j=1}^{n} x_{i_0 j}^{h}} < \theta_{i_0}^{h}$$

By replacing $\theta_{i_0}^{h}$ with $\widehat{\theta_{i_0}^{h}}$, the feasible region is not violated. However, since the objective function of the fractional model (or its linearized lexicographic steps) strictly minimizes the average of θ_i^h in the numerator, substituting $\theta_{i_0}^{h}$ with a strictly smaller value $\widehat{\theta_{i_0}^{h}}$ yields a strictly better (lower) objective function value. This contradicts the assumption that $\theta_{i_0}^{h}$ was the optimal solution. Thus, the input constraints must hold as equalities.

Second, consider the output constraints. Suppose that for at least one specific output k_0, the inequality is strict:

$$\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr}^{h} y_{k_0 j}^{h} > \varphi_{k_0}^{h} \sum_{j=1}^{n} y_{k_0 j}^{h}$$

Similarly, we can define a new expansion parameter $\widehat{\varphi_{k_0}^{h}}$:

$$\widehat{\varphi_{k_0}^{h}} = \frac{\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr}^{h} y_{k_0 j}^{h}}{\sum_{j=1}^{n} y_{k_0 j}^{h}} > \varphi_{k_0}^{h}$$

Since the objective function aims to maximize the expansion variables φ_k^h (which act as the denominator in the original fractional model or the bounding constraints in the linearized form), increasing $\varphi_{k_0}^{h}$ to $\widehat{\varphi_{k_0}^{h}}$ strictly improves the overall efficiency score without violating any constraints. This again contradicts the optimality of $\varphi_{k_0}^{h}$. Therefore, at the optimal solution, both input and output inequalities must strictly bind as equalities:

$$\sum_{r=1}^{n} \widehat{x_{ir}^{h}} = \theta_i^h \sum_{j=1}^{n} x_{ij}^{h} \quad \text{and} \quad \sum_{r=1}^{n} \widehat{y_{kr}^{h}} = \varphi_k^h \sum_{j=1}^{n} y_{kj}^{h}$$

4. Managerial Implications:
Theorem 1 provides significant managerial insights for the CDM. In standard DEA models, the presence of slacks often implies that resources are wasted or targets are loosely set, which complicates the implementation of restructuring plans. However, the proposed non-radial centralized model mathematically guarantees a slack-free resource allocation scheme.
From a practical perspective, this means:

1. Zero Resource Waste: Every unit of input budget determined by the systemic contraction targets ($\widetilde{X_I^{Total}}$) is precisely and completely distributed among the operating DMUs. No resources are left unallocated or squandered.
2. Exact Production Matching: The sum of the production goals assigned to individual branches ($\widetilde{Y_k^{Total}}$) perfectly fulfills the overarching production targets expected by the headquarters.
3. Fairness and Feasibility: Since the macro-level budgets and micro-level quotas are mathematically synchronized, the CDM can confidently enforce these targets, knowing the proposed plan is globally feasible and logically projects the entire organizational network onto the fully fuzzy efficient frontier.

3.5 Executive algorithm for the proposed methodology

To synthesize the mathematical formulations developed in the previous sections into a clear, computable procedure, we present the following step-by-step algorithm. This algorithmic framework guides the CDM from raw fuzzy data collection to the final optimal allocation of resources and targets.

Algorithm: The Lexicographic centralized resource allocation procedure under fuzzy environment

Step 1: Initialization and Data Collection

- Identify the set of n DMUs operating under a centralized control system.
- Collect the input and output data as TFNs: $\widetilde{x_{IJ}} = \left(x_{ij}^L, x_{ij}^M, x_{ij}^U\right)$ for m inputs, and $\widetilde{y_{kJ}} = \left(y_{kj}^L, y_{kj}^M, y_{kj}^U\right)$ for s outputs.
- Define a sufficiently small strictly positive constant ϵ (e.g., 10^{-6}) to facilitate the C-C transformation.

Step 2: Phase I Optimization (Upper Bound Minimization)

- Construct the linearized centralized model (8) for the upper uncertainty level ($h = U$). In accordance with the lexicographic minimization logic for fuzzy numbers, the upper bound is minimized first to aggressively push the fuzzy set towards efficiency.
- Solve the linear programming problem to minimize the aggregated upper-bound input contractions.
- Output: Obt
- ain the optimal objective function value Z^{U*}, and the corresponding optimal transformed variables $T_i^{U*}, \Phi_k^{U*}, \Lambda_{jr}^{U*}$, and t^{U*}.

Step 3: Phase II Optimization (Middle Bound Minimization)

- Construct the linearized centralized model (9) for the middle uncertainty level ($h = M$).
- Incorporate the optimal objective value from the previous step (Z^U) as an exact equality constraint to preserve the structural integrity and the lexicographic priority of the fuzzy number.
- Solve the linear programming problem.
- Output: Obtain the optimal objective function value Z^M, and the optimal transformed variables $T_i^M, \Phi_k^M, \Lambda_{jr}^M$, and t^M.

Step 4: Phase III Optimization (Lower Bound Minimization)

- Construct the linearized centralized model (10) for the lower uncertainty level ($h = L$).
- Incorporate both Z^U and Z^M as exact equality constraints.
- Solve the final linear programming problem.
- Output: Obtain the optimal transformed variables $T_i^{L*}, \Phi_k^{L*}, \Lambda_{jr}^{L*}$, and t^{L*}.

Step 5: Inverse Transformation and Parameter Extraction

- For all uncertainty levels $h \in \{U, M, L\}$, apply the inverse C-C transformation to retrieve the original optimal parameters:
  - Optimal systemic contractions: $\theta_i^h = T_i^h/t^h$
  - Optimal systemic expansions: $\varphi_k^h = \Phi_k^h/t^h$
  - Optimal allocation intensities: $\lambda_{jr}^h = \Lambda_{jr}^h/t^h$

Step 6: Systemic Macro-Target Setting

- Utilize the extracted θ_i^h and φ_k^h to calculate the overall optimal fuzzy targets for the entire system by (11):
  - Total Systemic Input Budget: $\widetilde{X_\imath^{Total}}$
  - Total Systemic Output Goal: $\widetilde{Y_k^{Total}}$

Step 7: Micro-Level Resource Allocation

- Utilize the optimal intensity variables λ_{jr}^h to compute the exact, DMU-specific resource allocations ($\widetilde{\hat{x}_{\imath r}}$) and production targets ($\widetilde{\hat{y}_{kr}}$) for each individual branch r (Considering (12)). Implement the resulting optimal blueprint across the organizational network.

4 Application in bank branches

To quantitatively assess the applicability and efficiency of the proposed mathematical framework, the model is deployed on a real-world network of fourteen Refah Kargaran Bank branches. The banking sector, characterized by shared operational goals and a centralized management structure, serves as an ideal environment for Centralized DEA (CDEA).

4.1 Case description and data set

Our analytical scope hinges on six crucial performance metrics—comprising three inputs (I_1, I_2, I_3) and three outputs (O_1, O_2, O_3)—carefully selected to reflect the operational and financial dimensions of the branches. The inputs defining the operational context include the workforce size (I_1), the physical scale or space of each facility (I_2), and the average client demand (I_3). On the production side, the outputs used for evaluation consist of average institutional resources (O_1), average operational expenses (O_2), and the rate of activity/service provision (O_3).

Given the inherent uncertainties in the banking environment—such as fluctuating client demands, varying daily transaction volumes, and imprecise estimations of physical capacities—the data cannot be accurately captured as crisp values. Consequently, all input and output parameters are modeled as TFNs, denoted by (L, M, U). The empirical data set encompassing the fuzzy inputs and outputs for all fourteen DMUs is consolidated in Table 1 and Table 2, respectively.

Table 1 Fuzzy input data

DMU	I1_L	I1_M	I1_U	I2_L	I2_M	I2_U	I3_L	I3_M	I3_U
DMU1	4	8	14	121	135	216	1246	3115	5295
DMU2	7	11	15	428	535	963	10424	20848	35441
DMU3	11	13	20	433	542	758	2440	6102	8542
DMU4	4	8	15	206	295	472	2970	4244	6366
DMU5	5	7	9	156	392	548	1234	2469	3950
DMU6	7	13	19	501	627	877	4936	12341	22213
DMU7	3	5	7	74	124	173	9636	13766	19272
DMU8	4	6	11	96	240	408	2357	4715	8015
DMU9	6	7	13	288	320	448	7142	14284	21426
DMU10	2	4	7	132	165	231	1501	3753	5254
DMU11	3	5	9	279	466	838	2401	4802	9123
DMU12	4	6	9	496	552	772	1606	4015	6424
DMU13	4	7	12	246	274	520	584	731	1388
DMU14	4	6	10	164	411	657	4889	6112	9168

Table 2 Fuzzy output data

DMU	O1_L	O1_M	O1_U	O2_L	O2_M	O2_U	O3_L	O3_M	O3_U
DMU1	16682	41705	79239	112986	161409	274395	565	1131	1583
DMU2	42380	47089	65924	315679	450970	631358	5053	10107	14149
DMU3	49975	62469	118691	152073	380183	608292	2203	3673	6244
DMU4	34477	38308	57462	99933	249834	374751	1684	2406	4330
DMU5	17529	25042	37563	140929	176162	264243	1338	2231	3123
DMU6	92645	102939	154408	285274	356593	534889	5584	11169	17870
DMU7	21367	23742	45109	99253	165422	281217	2384	3406	6130
DMU8	30279	37849	56773	154097	192622	327457	2645	3779	7180
DMU9	22412	44825	71720	151056	251760	453168	542	1357	1899
DMU10	16070	17856	30355	90837	100930	151395	543	1359	2174
DMU11	12955	25911	41457	150116	166796	300232	2001	2859	4288
DMU12	24321	30402	42562	157179	174644	261966	2116	3024	5745
DMU13	21180	42361	76249	225430	250478	425812	0	0	0
DMU14	13152	26304	44716	172988	216235	410846	1979	2828	4807

4.2 Computational procedure and reproducibility

The proposed three-step lexicographic FFCDEA model requires a robust algorithmic execution, particularly due to the sequential dependency of the optimization phases ($U \rightarrow M \rightarrow L$). The mathematical formulations were transcribed into computational algorithms and solved using the Python programming language, leveraging the `PuLP` algebraic modeling library for linear programming.

To ensure full transparency, reproducibility, and to facilitate future adaptations by the scientific community, the complete source code, along with a concise execution manual, is made publicly available. Researchers can access these computational assets as Supplementary Material attached to this paper.

The execution of the lexicographic model yields a highly multidimensional dataset. For each of the 14 branches, the model generates optimal centralized targets $(\widehat{X}, \widehat{Y})$, structural

adjustment factors (θ , φ), and dual variables (λ) across three distinct fuzzy boundaries (Lower, Middle, Upper). Integrating this voluminous data directly into the main text would obscure the core analytical narrative. Therefore, a structured presentation strategy is adopted:
1. Macro-Level Analysis (Section 4.3): The systemic performance, representing the bank as a unified entity, is analyzed in the main text.
2. Micro-Level Highlights (Section 4.4): Key managerial insights and aggregated targets for selected branches (e.g., at the Middle bound M) are discussed to demonstrate intra-organizational resource reallocation.
3. Comprehensive Output (Appendix B): The exhaustive, finely granulated tables detailing all optimal values for every DMU across all variables and bounds are strategically placed in the Supplementary Appendices. This ensures the methodological rigor is fully documented without compromising the manuscript's readability.

4.3 Systemic performance evaluation

In the first phase of our analysis, the performance of the bank is evaluated from a macro perspective, treating the network of 14 branches as a single unified entity. The Centralized DEA model aims to optimize the total consumption of inputs and generation of outputs across the entire network. Table 3 presents the aggregate initial values alongside the optimal systemic targets computed at the most probable fuzzy bound (Middle bound, M).

Table 3 Systemic Initial Values vs. Optimal Targets (Middle Bound - M)

Variable	Description	Initial Total System	Optimal Target System	Adjustment Factor (θ^*, φ^*)	Required Change (%)
I_1	Workforce	106.00	106.00	$\theta^*_{I1} = 1.000$	0.00%
I_2	Physical Space	5078.00	4379.67	$\theta^*_{I2} = 0.862$	-13.8%
I_3	Client Demand	101297.00	101297.00	$\theta^*_{I3} = 1.000$	0.00%
O_1	Inst. Resources	566802.00	689126.72	$\varphi^*_{O1} = 1.216$	$+21.6\%$
O_2	Oper. Expenses	3294038.00	3294038.00	$\varphi^*_{O2} = 1.000$	0.00%
O_3	Activity Rate	49329.00	61039.66	$\varphi^*_{O3} = 1.237$	$+23.7\%$

Note: The overall systemic fuzzy efficiency at the Middle bound is $R^*_M = 0.8289$.

As depicted in Table 3, the overall systemic efficiency score at the middle bound (R_M) stands at approximately 0.8289, indicating that the banking network operates at roughly 82.9% of its maximum potential efficiency. To project the system onto the technically efficient frontier, significant structural adjustments are prescribed. While the total workforce (I_1) and client demand (I_3) remain perfectly scaled ($\theta = 1.00$), the system exhibits an excess in physical space (I_2), requiring a contraction of approximately 13.8% across the network.
Concurrently, on the output side, the centralized model dictates a substantial expansion. To justify the current level of inputs, the total institutional resources (O_1) and the activity rate (O_3) must be augmented by 21.6% and 23.7%, respectively. These systemic targets provide a strategic roadmap for top-level management to align the bank's total physical footprint with its production capabilities.

4.4 DMU-level target setting and resource allocation (Middle bound analysis)

While systemic targets define the macro-level strategic goals, the true utility of the Centralized DEA framework lies in its ability to dictate micro-level operational reallocations. Unlike standard DEA models that independently project underperforming DMUs onto the efficiency frontier (often ignoring the system's total resource capacity), the FFCDEA model dynamically reallocates total available resources among all 14 branches. This global optimization ensures that the system collectively achieves the systemic targets discussed in Section 4.3.

This centralized mechanism implies a compensatory relationship among the branches: while the network as a whole might contract an input (e.g., Physical Space, I_2), specific high-performing branches might simultaneously experience an expansion in that same input to absorb operations from downsizing peers.

To thoroughly examine this reallocation strategy, Table 4 presents the complete set of initial values and their corresponding optimal targets at the most probable fuzzy bound (Middle bound, M) for all 14 DMUs across the six inputs and outputs. (Note: Results for the pessimistic L and optimistic U bounds are comprehensively detailed in Appendix B).

Table 4 Initial vs. optimal targets for all 14 branches at middle bound (M)

DMU	I1_Init	I1_Target	I2_Init	I2_Target	I3_Init	I3_Target	O1_Init	O1_Target	O2_Init	O2_Target	O3_Init	O3_Target
DMU1	8	8.47	135	341.96	3115	13148.52	41705	58059.50	161409	248259.86	1131	6769.85
DMU2	11	6	535	240	20848	4715	47089	37849	450970	192622	10107	3779
DMU3	13	6	542	240	6102	4715	62469	37849	380183	192622	3673	3779
DMU4	8	6	295	240	4244	4715	38308	37849	249834	192622	2406	3779
DMU5	7	5	392	124	2469	13766	25042	23742	176162	165422	2231	3406
DMU6	13	6	627	240	12341	4715	102939	37849	356593	192622	11169	3779
DMU7	5	7	124	274	13766	731	23742	42361	165422	250478	3406	0
DMU8	6	10.33	240	489.78	4715	9790.16	37849	77667.36	192622	280655.11	3779	8255.2
DMU9	7	7	320	274	14284	731	44825	42361	251760	250478	1357	0
DMU10	4	5	165	124	3753	13766	17856	23742	100930	165422	1359	3406
DMU11	5	7	466	274	4802	731	25911	42361	166796	250478	2859	0
DMU12	6	13	552	627	4015	12341	30402	102939	174644	356593	3024	11169
DMU13	7	6.21	274	263.93	731	3044.88	42361	33870.71	250478	199171.03	0	1748.61
DMU14	6	13	411	627	6112	12341	26304	102939	216235	356593	2828	11169

Note: In the table headers, the _Init and _Target suffixes represent the initial observed values and the optimal projected values recommended by the model, respectively. Also, Decimal values represent continuous scale adjustments; fractional workforce targets (e.g., I_1 for DMU1 = 8.47) can be practically interpreted in terms of working hours or part-time staff allocations.

A deeper analysis of the results outlined in Table 4 reveals three distinct operational strategies prescribed by the optimization model: Expansion, Contraction, and Specialization/Consolidation.

1. Strategic Expansion (e.g., DMU1, DMU8, DMU12, DMU14):

Several branches are identified as highly capable hubs that should absorb excess demand and operational weight. For instance, DMU12 and DMU14 are designated to more than double their workforce (I_1 from 6 to 13. Similarly, DMU1 is instructed to expand its physical capacity (I_2) from 135 to 341.96 square meters and increase its activity rate (O_3) nearly six-fold (from 1,131 to 6,769.85). This indicates that the system benefits most when these specific nodes process a larger share of the banking network's total workflow.

2. Downsizing and Resource Extraction (e.g., DMU2, DMU3, DMU6):

Conversely, some historically oversized branches are slated for significant reduction. DMU6, which initially operated with 13 personnel and a vast physical space of 627 sq.m, is targeted to scale down to 6 personnel and 240 sq.m. The centralized model detects that the inputs consumed by these branches yield better marginal returns when reallocated to branches like DMU8 or DMU12.

3. Task Specialization and Consolidation (e.g., DMU7, DMU9, DMU11, DMU13):

Perhaps the most compelling finding of the CDEA model is its inclination towards structural reorganization rather than mere proportional scaling. For DMU7, DMU9, and DMU11, the optimal target for the activity rate/service provision (O_3) is reduced to precisely 0.00. In banking terms, this implies that certain specific services or localized activities should be completely halted at these branches and centralized elsewhere to eliminate redundancies. Interestingly, DMU13, which initially generated an O_3 value of 0, is now targeted to produce 1,748.61 units of this output, suggesting it should be newly assigned an operational role previously handled by other branches.

In summary, the optimal targets demonstrate that achieving systemic efficiency requires breaking away from homogeneous operations. The model directs management to tailor the size and functional scope of each branch according to its comparative advantage within the network.

4.5 Managerial implications and practical considerations

The application of the lexicographic FFCDEA model provides bank management with a rigorous blueprint for systemic optimization. However, translating these mathematical targets into real-world banking operations requires a nuanced approach, particularly regarding the inherent "liquidity" or transferability of different resources. Reviewers and practitioners might rightly question the physical feasibility of seamlessly reallocating certain inputs and outputs across the network.

To operationalize the findings derived in Section 4.4, management must categorize the prescribed targets into short-term tactical adjustments and long-term strategic initiatives:

1. Reallocation of Fluid Resources (Short-to-Medium Term):

Targets related to the workforce (I_1), operational expenses (O_2), and institutional resources (O_1) represent fluid assets that can be dynamically managed. For instance, the model's directive to reduce the workforce in DMU2 and DMU3 while doubling it in DMU12 and DMU14 can be pragmatically executed through staff redeployment programs, altering hiring freezes, or offering geographic transfer incentives. Similarly, adjusting operational budgets (O_2) is entirely within the immediate purview of the central financial committee.

2. Management of Fixed Assets and Physical Space (Long-Term Strategy):

The most critical practical challenge in implementing CDEA targets lies in fixed assets, specifically the physical space (I_2). Unlike human capital, physical square footage cannot be physically detached from one branch and appended to another. When the model dictates a contraction in physical space for a branch (e.g., DMU6 shrinking from 627 to 240 sq.m), it strategically signals an over-investment in real estate relative to the branch's productive output. In practice, management should interpret these targets as guidelines for a Corporate Real Estate Portfolio Strategy. The excess capacity can be addressed by:

Subleasing or Repurposing: Unused physical space in oversized branches can be leased out to third parties or repurposed into centralized, non-client-facing administrative hubs (e.g., IT data centers, regional archive rooms) that do not count towards the branch's direct operational inputs.

Relocation and Capital Expenditure: For branches requiring massive spatial expansion (e.g., DMU1 and DMU8), the targets mathematically justify the capital expenditure required to relocate the branch to a larger facility within the same commercial district, or to acquire adjacent properties.

3. Redirecting Client Demand (I_3) and Activity Specialization (O_3):

Client demand (I_3) is tied to demographic and regional factors. To shift demand away from downsized branches and towards expanded operational hubs, the central management must employ zoning policies. This involves restructuring the geographic coverage areas of specific branches, redirecting corporate client accounts to designated hubs (like DMU12 and DMU14), and employing digital banking incentives to decouple demand from physical footfall. Furthermore, as observed in the model's recommendation to reduce the activity rate (O_3) to zero in DMU7, DMU9, and DMU11, management is advised to establish Specialized Service Hubs. Instead of every branch offering every service (e.g., foreign exchange, large-scale corporate loans), redundant services should be phased out in smaller branches and centralized in the expanded units.

Ultimately, the FFCDEA targets should not be viewed as an overnight restructuring mandate, but rather as an evidence-based roadmap. It provides the central authority of the Refah Kargaran Bank with the precise mathematical justification needed to guide multi-year strategic planning, property acquisition, and corporate restructuring under uncertain conditions.

5 Conclusion and future research directions

5.1. Conclusion

This study addressed the complex challenge of resource allocation and target setting in organizational networks operating under high uncertainty. We proposed a Fully Fuzzy Centralized DEA model under the VRS assumption. Unlike traditional DEA models that evaluate units independently, our centralized approach optimized the performance of the entire system (macro-level) while simultaneously determining the most efficient targets for individual DMUs (micro-level). To solve the proposed fully fuzzy model, a rigorous three-stage lexicographic approach combined with the C-C transformation was employed, ensuring computationally tractable and mathematically sound solutions.

The practical applicability of the proposed approach was demonstrated through a real-world numerical example involving 14 bank branches. By modeling inputs and outputs as TFNs, the study successfully captured the inherent ambiguity in banking operations, such as fluctuating customer demands and unquantifiable service qualities. The findings revealed that systemic efficiency can be achieved through strategic resource reallocation—specifically, by downsizing underutilized inputs (e.g., physical infrastructure) and expanding operational outputs. Furthermore, the analysis provided actionable managerial insights by categorizing branches into candidates for expansion, downsizing, or consolidation, while taking into account the practical limitations of resource liquidity (e.g., the inflexibility of physical spaces).

5.2. Future research directions

While the proposed FFCDEA model provides a robust framework for performance evaluation under uncertainty, several promising avenues for future research can be identified:
1. Network and Dynamic DEA: The current model treats each DMU as a "black box." Future studies could extend this approach to a Network DEA framework to examine the internal operational stages of bank branches (e.g., deposit generation vs. loan allocation). Additionally, incorporating Dynamic DEA would allow for the evaluation of centralized target setting over multiple time periods.
2. Inclusion of Undesirable Outputs: In the banking sector, operations often generate undesirable outputs, such as Non-Performing Loans (NPLs) or customer complaints. Integrating undesirable outputs into the fully fuzzy centralized model would provide a more comprehensive and realistic assessment of bank performance.
3. Alternative Uncertainty Approaches: While this study utilized TFNs, future research could explore other advanced uncertainty modeling techniques, such as Type-2 Fuzzy Sets, Intuitionistic Fuzzy Sets, or Robust Optimization methodologies, to compare their effectiveness in handling highly volatile data.
4. Weight Restrictions and Preferences: Incorporating weight restrictions or decision-maker preferences (e.g., using the Analytic Hierarchy Process or Delphi method) into the objective function could align the systemic targets more closely with the strategic priorities of the central management.

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Appendix A: Notation and Symbols

This appendix summarizes the indices, parameters, variables, and abbreviations used throughout the paper. Providing this notation table facilitates the understanding of the mathematical formulations presented in the modeling section.

Symbol /Abbreviation	Description
	Abbreviation
DEA	Data Envelopment Analysis
DMU	Decision-Making Unit
CDEA	Centralized Data Envelopment Analysis
FDEA	Fuzzy Data Envelopment Analysis
ERM	Enhanced Russell Measure
MOFP	Multi-Objective Fractional Programming
LP	Linear Programming
C-C	Charnes-Cooper transformation
CRS	Constant Returns to Scale
CDM	Centralized Decision Maker
	Index
i	Index for inputs $i = 1 \dots m$
k	Index for outputs $k = 1 \dots s$
j	Index for Decision-Making Units $j = 1 \dots n$
r	Index for the target DMU in resource allocation models
p	Index for the evaluated DMU in the ERM model
f	Index for triangular fuzzy number components (Lower Middle Upper bounds)
	Parameter / Variable
$\widehat{x\hat{y}}$	Virtual targets (projected points) for inputs and outputs on the strong frontier based on ERM
u	Positive scalar variable introduced in the C-C transformation
ε	Non-Archimedean infinitesimal value for strict positivity constraints
$\tilde{\Theta}$	Optimal fuzzy contraction factor for total input resources across the centralized system
$\tilde{\Phi}$	Optimal fuzzy expansion factor for total output productions across the centralized system
$X_{\imath}^{\widetilde{sys\ target}}$	Overall optimal fuzzy target level for the total consumption of resource i
$Y_{k}^{\widetilde{sys\ target}}$	Overall optimal fuzzy target level for the total generation of output k
λ_{jr}	Intensity variable dictating the reallocation weight of DMU j for DMU r
$x_{\imath r}^{\widetilde{target}}$	Optimal fuzzy input allocation target for a specific DMU r
$y_{kr}^{\widetilde{target}}$	Optimal fuzzy output target for a specific DMU r

Appendix B: Lower and Upper Bound Results of the FFCDEA Model

This appendix provides supplementary data and computational results for the lower (L) and upper (U) bounds of the triangular fuzzy numbers used in the proposed FFCDEA model under the Variable Returns to Scale (VRS) assumption.

While Section 4 focuses primarily on the most plausible middle level (M) to facilitate clear managerial insights and practical analysis, Tables B.1 and B.2 present the complete initial and target values for all inputs (I_1, I_2, I_3) and outputs (O_1, O_2, O_3) across the 14 Decision Making Units (DMUs) at the L and U levels, respectively. Providing these extreme bounds offers a comprehensive view of the uncertainty margins and the full spectrum of optimal resource allocation scenarios for each bank branch.

Table B.1 Initial and optimal target values of inputs and outputs for the 14 bank branches at the lower bound

DMU	I1_Init_L	I1_Target_L	I2_Init_L	I2_Target_L	I3_Init_L	I3_Target_L	O1_Init_L	O1_Target_L	O2_Init_L	O2_Target_L	O3_Init_L	O3_Target_L
DMU1	4	4.73	121	259.03	1246	7599.40	16682	52253.05	112986	179859.27	565	3770.62
DMU2	7	4	428	96	10424	2357	42380	30279	315679	154097	5053	2645
DMU3	11	4	433	96	2440	2357	49975	30279	152073	154097	2203	2645
DMU4	4	4	206	96	2970	2357	34477	30279	99933	154097	1684	2645
DMU5	5	3	156	74	1234	9636	17529	21367	140929	99253	1338	2384
DMU6	7	4	501	96	4936	2357	92645	30279	285274	154097	5584	2645
DMU7	3	4	74	246	9636	584	21367	21180	99253	225430	2384	0
DMU8	4	5.51	96	391.40	2357	3915.72	30279	69900.43	154097	227521.66	2645	4086.71
DMU9	6	4	288	246	7142	584	22412	21180	151056	225430	542	0
DMU10	2	3	132	74	1501	9636	16070	21367	90837	99253	543	2384
DMU11	3	4	279	246	2401	584	12955	21180	150116	225430	2001	0
DMU12	4	7	496	501	1606	4936	24321	92645	157179	285274	2116	5584
DMU13	4	3.78	246	176.89	584	1526.87	21180	22352.60	225430	170109.22	0	1062.07
DMU14	4	7	164	501	4889	4936	13152	92645	172988	285274	1979	5584

Table B.2 Initial and optimal target values of inputs and outputs for the 14 bank branches at the upper bound.

DMU	I1_Init_U	I1_Target_U	I2_Init_U	I2_Target_U	I3_Init_U	I3_Target_U	O1_Init_U	O1_Target_U	O2_Init_U	O2_Target_U	O3_Init_U	O3_Target_U
DMU1	14	12.20	216	478.06	5295	20546.39	79239	92470.24	274395	391137.68	1583	11217.16
DMU2	15	11	963	408	35441	8015	65924	56773	631358	327457	14149	7180
DMU3	20	11	758	408	8542	8015	118691	56773	608292	327457	6244	7180
DMU4	15	11	472	408	6366	8015	57462	56773	374751	327457	4330	7180
DMU5	9	7	548	173	3950	19272	37563	45109	264243	281217	3123	6130
DMU6	19	11	877	408	22213	8015	154408	56773	534889	327457	17870	7180
DMU7	7	12	173	520	19272	1388	45109	76249	281217	425812	6130	0
DMU8	11	15.44	408	685.12	8015	17175.78	56773	117561.36	327457	420982.31	7180	13207.92
DMU9	13	12	448	520	21426	1388	71720	76249	453168	425812	1899	0
DMU10	7	7	231	173	5254	19272	30355	45109	151395	281217	2174	6130
DMU11	9	12	838	520	9123	1388	41457	76249	300232	425812	4288	0
DMU12	9	19	772	877	6424	22213	42562	154408	261966	534889	5745	17870
DMU13	12	10.36	520	444.69	1388	4960.83	76249	56354.60	425812	326599.40	0	2974.13
DMU14	10	19	657	877	9168	22213	44716	154408	410846	534889	4807	17870