International Journal of Applied Operational Research
Vol. 13, No. 4, pp. 89-99, Autumn 2025
Journal homepage: ijorlu.liau.ac.ir

A developed P-median model for landfill site selection: A case study in Mazandaran province

S. Mansouri, S.H. Nasseri*

Received: 7 February 2025; **Accepted:** 8 May 2025

Abstract The increasing generation of municipal solid waste in recent decades has become one of the major challenges of urban management, emphasizing the necessity of redesigning and improving solid waste management networks. Proper siting of landfill facilities and their associated components plays a decisive role in reducing environmental impacts, improving operational efficiency, and enhancing the sustainability of waste management systems. The aim of this study is to present an analytical–optimization framework for evaluating the spatial suitability of landfill sites and redesigning the solid waste management network in Mazandaran province. In this study, the spatial suitability of different areas for landfill siting in Mazandaran province was first evaluated using the fuzzy analytic hierarchy process within a geographic information system environment. Subsequently, by developing a capacitated p-median model, the redesign of the solid waste management network was addressed as a location–allocation problem. To deal with existing uncertainties and to prevent infeasibility of the model, the coverage radius constraint was formulated in a flexible fuzzy manner based on the decision-maker's acceptance level. The results indicate that the proposed hybrid approach is capable of simultaneously considering spatial constraints, facility capacity limitations, and operational uncertainties, and can provide a coherent and practically applicable structure for solid waste management networks in Mazandaran province. This framework can be employed as an analytical decision-support tool for urban and regional solid waste management planning.

Keywords: Location Problem, Fuzzy Mathematical Programming, P-Median Problem, Fuzzy Analytic Hierarchy Process, Geographic Information System (GIS).

1 Introduction

Mazandaran province, due to its significant floating population, high tourism activity, and unique climatic conditions, is experiencing a substantial increase in municipal solid waste generation. Facility location and resource allocation constitute major challenges in this domain, directly affecting operational costs and service quality. From an operations research perspective, this

* **Corresponding Author.** (✉)
E-mail: nhadi57@gmail.com (S.H. Nasseri)

S. Mansouri
Department Mathematics, University of Mazandaran, Babolsar, Iran.

S.H. Nasseri
Department Mathematics, University of Mazandaran, Babolsar, Iran.

problem can be formulated as a location optimization problem. In this study, a hybrid approach was employed to address these challenges. In the first stage, a fuzzy AHP method within a GIS environment was applied for multi-criteria weighting and spatial analysis, identifying suitable candidate locations based on environmental, economic, and logistical criteria. In the second stage, the network was designed using a P-median-based mathematical model, in which coverage radius constraints were formulated flexibly to enhance model feasibility and adaptability. This integrated methodology enables the design of an efficient, flexible, and realistic waste management network for Mazandaran Province. Tadić et al. (2023) investigated the p-median problem incorporating user preferences, demonstrating its applicability in complex and uncertain decision environments [1]. Similarly, Tamir (2023) proposed a mean–variance value at risk criterion for solving p-median problems with type-2 Fuzzy weights, highlighting the capability of fuzzy models to handle uncertainty in input data [2]. Subsequent studies have focused on the development of fuzzy and hybrid models. For instance, Atta et al. (2022) and Lucchese et al. (2023) applied metaheuristic algorithms to solve capacitated maximal covering problems with fuzzy coverage areas, showing that integrating fuzzy sets with intelligent search heuristics improves solution quality [3,4]. Koçoğlu (2023) developed a fuzzy hierarchical location model providing high flexibility and reliability for network design decisions [5]. Within the context of municipal solid waste management, fuzzy, hybrid, and advanced p-median models have proven effective. Ghaffari-far et al. (2023) proposed a vehicle location model under fuzzy flexible conditions for waste management, simultaneously optimizing site locations and resource allocation [6]. Babaee Tirkolaee et al. (2021) introduced a sustainable fuzzy multi-trip location model for medical waste, accommodating uncertainty in scheduling and capacity [7]. Other studies demonstrated the applicability of fuzzy modeling for hazardous and urban waste networks. Raeisi & Jafarzadeh Ghoushchi (2022) developed a multi-objective fuzzy model considering economic, environmental, and operational objectives concurrently [8], while Rathinabhagya & Merline Vinotha (2025) applied fuzzy modeling to optimize municipal waste collection systems with the goal of reducing greenhouse gas emissions [9]. Further research has investigated fuzzy-based optimization in facility location and resource allocation under uncertain information. Temur (2024) employed volumetric fuzzy sets for site selection, demonstrating integration with geospatial data [10]. Ghahremani-Nahr et al. (2023) developed a fuzzy hierarchical model offering high flexibility and reliability in decision-making [11]. Amini et al. (2020) presented a bi-objective transportation-location model optimizing both operational cost and network efficiency [12]. Recent international studies have also advanced fuzzy and hybrid approaches in transportation and waste networks. Malekdar & Shirazi (2025) designed a fuzzy hub network for air cargo systems, improving flexibility and performance [13]. Guo et al. (2023) applied swap-based deep reinforcement learning to facility location problems, enhancing both solution accuracy and computational efficiency [14]. Furthermore, a fuzzy multi-objective model has been proposed for reverse urban waste collection networks, simultaneously optimizing economic, environmental, and operational objectives [15]. Overall, evidence indicates that integrating advanced p-median, fuzzy, hybrid approaches, and multi-criteria decision-making models provides a comprehensive analytical framework for municipal solid waste network design. This approach not only reduces operational costs and increases service coverage but also supports robust decision-making under complex urban uncertainties, making it particularly suitable for Mazandaran Province with its high waste generation and unique urban conditions.

2 P-median location models

The P-median model is one of the classic location models in operations research, developed to determine the optimal placement of facilities within a network. The model aims to select a fixed number of facilities such that the total service cost to demand points is minimized. It has been widely applied in urban planning and public service problems, including landfill site selection, distribution centers, and service facilities. Classic p-median problems aim to determine the locations of ppp facilities within a network such that the total service cost is minimized. The service cost is defined as the cost of providing services from demand node i to its nearest established facility. This cost typically depends on factors such as the distance between demand node i and the serving facility, as well as the magnitude of demand at that node.

2.1 Assumption of the classic median problem

The median problems are intended to find the location of P facility on the network, so that the total cost can be at minisum. The cost means the cost of providing services from node i to the nearest node established there. This cost depends on factors like distance between the node of i and servicing node and volume of demands of in node. The Classic Median Problem has some assumption as follows:

- Linear relationship between cost and distance
- Good facilitated
- Infinite time horizon
- Infinite facility capacity
- Don’t have an initial setup cost
- Exogenous problem
- Same facilities
- Stationary facilities
- Constant node’s demand
- Discrete problem

In this problem:

$$x_{ij} = \begin{cases} 1 & \text{If the demand of node } i \text{ is served by a facility established at node } j \\ 0 & \text{ow} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{If a facility is established at node } j \\ 0 & \text{ow} \end{cases}$$

d_{ij} : The distance between demand node i and candidate node j for facility establishment

P : The number of facilities to be established

h_i : Demand of i node
n : Number of nodes
This model was proposed by ReVelle [16] and Swain:

$$\begin{aligned}
&\min \sum_i \sum_j h_i d_{ij} X_{ij} \quad i,j=1,2,...,n. && (1)\\
&s.t. \ \sum_j X_{ij} = 1 \qquad \forall i, && (2)\\
&\sum_j y_j = p, && (3)\\
&X_{ij} \le Y_j \qquad \forall i,j, && (4)\\
&X_{ij}, Y_j \in \{0,1\} \qquad \forall i,j. && (5)
\end{aligned} \tag{1}$$

This problem aims to minimize the total cost required to satisfy the demands of all nodes, which is commonly represented as the minimization of the total demand–distance product. Constraint (2) ensures that all demands are fully satisfied and that each demand node is assigned to exactly one facility (demand assignment constraint). Constraint (3) enforces the inner structure of the problem by guaranteeing that exactly P facilities are established. Constraint (4) indicates that only opened facilities are allowed to serve demand nodes. Therefore, the demand of node i can be assigned to node j , (x_{ij}) only if a facility is Located at node j , $(y_j = 1)$.

2.2 Capacitated facility location problem with multiple facilities (CPLPM)

In this part, we introduce the capacitated plant location problem (CPLP) with multiple facilities in the same site (CPLPM). This problem represents a special case of the classical Capacitated Facility Location Problem (CPLP), in which multiple facilities are allowed to be established at the same location. The CPLPM has various applications in public service facility location problems. Although the CPLPM can be modeled and solved as a standard CPLP, such an approach usually leads to poor computational performance. The CPLP is a classical combinatorial optimization problem [17]. This location problem is of significant importance for both public and private organizations, as it aims to determine a set of capacitated facilities (such as warehouses, factories, distribution centers, transfer stations, and others) in a way that minimizes the total facility opening and transportation costs. Unlike many other facility location problems, the establishment of multiple facilities at a single location is permitted. The CPLP is strongly NP-hard [18] and has been extensively studied in the fields of clustering and location theory. In This problem:

U : The set of potential facilities
V : The set of customers
d_j : The demand of customer j , $(j \in V)$ where $d_j \ge 0$
q_i : The capacity of facility i , $(i \in U)$ where $q_i \ge 0$
c_{ij} :The cost of supplying all the demand of customer j , $(j \in V)$ from facility i , $(i \in U)$
f_i : The fixed cost associated with opening facility i , $(i \in U)$ where $f_i \ge 0$
p : The desired number of open facilities (also referred to as medians)
y_i : A binary decision variable, which

$$y_i = \begin{cases} 1 & \text{If a facility is established at node } i \\ 0 & \text{Ow} \end{cases}$$

And another binary variable:

$$x_{ij} = \begin{cases} 1 & \text{If the demand of node } i \text{ is served by a facility established at node } j \\ 0 & \text{Ow} \end{cases}$$

Then CPLP can be formulated as a integer linear programming problem as follows:

$$\begin{aligned}
&(CPLP)\ Min \sum_{i \in U} \sum_{j \in V} c_{ij} x_i + \sum_{i \in U} f_i y_i && (1) \\
&s.t. \quad \sum_{i \in U} x_{ij} = 1, \quad j \in V && (2) \\
&\sum_{j \in V} d_j x_{ij} \le q_i y_i, \quad i \in U && (3) \\
&\sum_{i \in U} y_i = p, && (4) \\
&x_{ij} \in \{0,1\}, \quad i \in U, j \in V && (5) \\
&y_i \in \{0,1\} \quad i \in U && (6)
\end{aligned} \qquad (2)$$

The objective function (1) represents the minimization of the total cost. Constraint (2) ensures that the demand of each customer is fully satisfied. Constraint (3) establishes the relationship between the decision variables x_{ij} and y_i, indicating that no customer can be served by a closed (inactive) facility and that the total demand served by open (active) facilities does not exceed their capacities. Constraint (4) enforces that the number of open facilities equals p. Finally, constraints (5) and (6) guarantees the binary variables of the decision variable corresponding to facility establishment.

2.3 Capacitated P-median problem (CPMP)

If no fixed costs are associated with potential facilities, the capacitated facility location problem (CPLP) can be regarded as the capacitated p-median problem (CPMP) [19]. Where $N = \{1,...,n\}$ is the index set of entities to allocate and also of possible medians with $x_{ij} = 1$ If entity i is allocated to median j, and $x_{ij} = 0$ otherwise. The CPMP is formulated as follows:

$$Min \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} \qquad (1)$$

$$s.t. \quad \sum_{i \in N} x_{ij} = 1, \quad i \in N \qquad (2)$$

$$\sum_{j \in N} x_{jj} = P, \qquad (3)$$

(3)

$$\sum_{i \in N} d_i x_{jj} \le q_i x_{jj}, \quad j \in N \qquad (4)$$

$$x_{ij} \in \{0,1\}, \quad i \in N, \quad j \in N \qquad (5)$$

Constraints (2) and (3) ensure that each unit is assigned to exactly one median. Constraint (4) requires that the total capacity of all medians is not exceeded, and constraint (5) enforces the validity of the decision variables.

In this study, the problem of landfill site selection in Mazandaran Province was addressed through a two-stage hybrid approach. In the first stage, relevant criteria for landfill site selection were weighted using a Fuzzy Analytic Hierarchy Process (FAHP) based on expert opinions, and spatial analysis was conducted in a GIS environment. This process resulted in the identification of 26 candidate regions as initial options for landfill establishment. The spatial distribution of these candidate regions and the corresponding suitability map for landfill site selection in Mazandaran Province are illustrated in Figure 1:

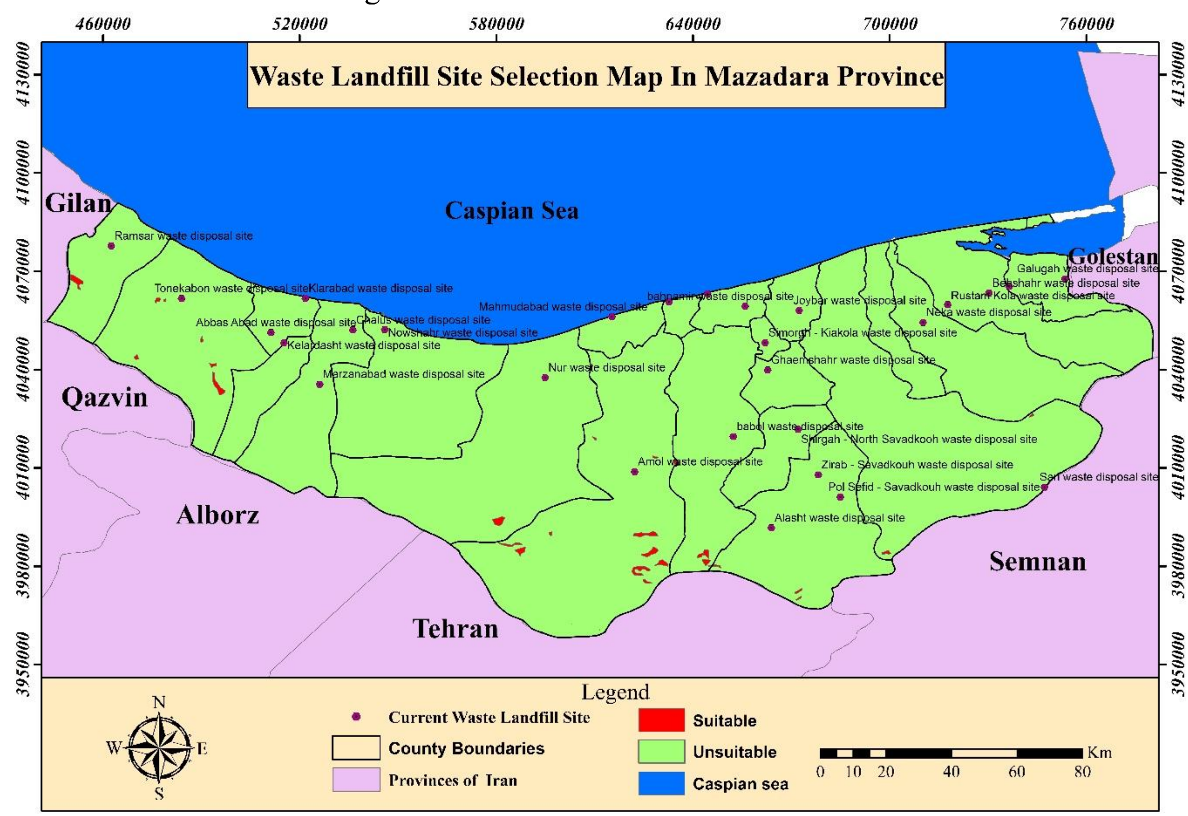

Fig. 1 Waste Landfill Site Selection Map in Mazandaran Province

In the second stage, using these identified regions, a capacitated P-median optimization model was developed to determine the final landfill locations, aiming to satisfy the total demand of the province while minimizing the combined costs of construction and transportation. In this study, detailed information regarding the data and computational procedures is omitted, and only the final results of the model are discussed. In the next section, the development of the P-median model and the results obtained from this stage are presented.

3 Mathematical modeling of landfill sites in Mazandaran province: Development of a P-median model

In the second stage of the study, a capacitated P-median model was employed to determine the optimal locations for landfill sites. The candidate sites identified through the spatial analysis in the first stage were considered as potential options, and the following model was developed to select the optimal locations with the objective of minimizing the total cost while reducing the number of sites. The model presented here is an extension of the P-median formulation introduced in the previous section.

Definition of Sets:

$I = \{1,...,22\}$: Counties

$J = \{1,...,26\}$: Candidate Sites

$K = \{1,...,7\}$: Potential Counties in Which the Candidate Sites Are Located

Parameters:

h_i : Amount of waste generated in city i (demand)

cap_j : Capacity of candidate site j for waste collection

d_{ij} : Distance between county i and candidate site j

f_j : Setup cost of candidate site j

c_{ij} : Cost of collecting waste from county i to candidate site j

$r_{\max}$: Maximum coverage radius

p : The number of candidate sites allowed to be established

y_{j_k} : Site j in the candidate counties k where the potential landfill sites are located

Decision Variables:

$$x_{ij} = \begin{cases} 1 & \text{if waste from county } i \text{ is assigned to candidate site } j \\ 0 & \text{Otherwise} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{if candidate site } j \text{ is established} \\ 0 & \text{Otherwise} \end{cases}$$

Based on the defined sets, parameters, and decision variables, the mathematical model is formulated as follows:

$$
\begin{aligned}
&\min \sum_{i} \sum_{j} c_{ij} x_{ij} + \sum_{j} f_j y_j && (1)\\
&s.t. \sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I && (2)\\
&\sum_{i \in I} h_i x_{ij} \le cap_j y_j && (3)\\
&\sum_{j \in J} y_j = p && (4)\\
&\sum_{j_k \in J} y_{j_k} = 1 && (5)\\
&\sum_{j} d_{ij} x_{ij} \le r_{\max} && (6)\\
&x_{ij} \le y_j \qquad \forall i \in I, \forall j \in J && (7)\\
&x_{ij} \in \{0,1\}, y_j \in \{0,1\} && (8)
\end{aligned}
\tag{4}
$$

The objective function (1) represents the minimization of the total cost. Constraint (2) ensures that the demand of each customer is fully satisfied. Constraint (3) establishes the relationship between the decision variables x_{ij} and y_j, indicating that no customer can be served by a closed (inactive) facility and that the total demand served by open (active) facilities does not exceed their capacities. Constraint (4) enforces that the number of open facilities equals p. Constraint (5) ensures that exactly one main site is established in each of the seven counties with potential landfill sites. Constraint (6) ensures that Maximum coverage radius of the sites. Constraint (7) indicating that no customer can be served by a closed (inactive) facility. Finally, constraint (8) guarantee the binary variables of the decision variable corresponding to facility establishment.

The coverage radius in the model was treated as Fuzzy and Flexible to account for system tolerances. This fuzzy approach allows the model to be formulated with greater flexibility, yielding more practical and operationally realistic optimal solutions. Therefore, the next model is formulated as follows:

$$
\begin{aligned}
&\min \sum_{i} \sum_{j} c_{ij} x_{ij} + \sum_{j} f_j y_j\\
&s.t. \sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I\\
&\sum_{i \in I} h_i x_{ij} \le cap_j y_j\\
&\sum_{j \in J} y_j = p\\
&\sum_{j_k \in J} y_{j_k} = 1\\
&\sum_{j} d_{ij} x_{ij} \le r_{\max} + \lambda(1-\gamma_i)\\
&x_{ij} \le y_j \qquad \forall i \in I, \forall j \in J\\
&x_{ij} \in \{0,1\}, y_j \in \{0,1\}\\
&\gamma_i \in [0,1]
\end{aligned}
\tag{5}
$$

where λ is a predefined tolerance parameter and γ_i is the flexibility parameter.

Based on the results obtained from the first stage of fuzzy AHP in a GIS environment, 26 geographic patches were identified as potential areas for landfill site establishment, distributed across seven counties of the province. Subsequently, the developed P-median mathematical model was formulated to determine the optimal locations, reduce the number of sites, and minimize construction and transportation costs, and it was solved using LINGO software. The model results indicated that seven sites were selected as the primary landfill sites with deposition capacities ranging from 200 to 700 tons, and five of the existing sites were designated as temporary transfer stations with a maximum capacity of 100 tons per day. It should be noted that there are currently 28 existing landfill sites in the province, most of which were found to be environmentally unsuitable; however, considering the municipal solid waste generation demand and the province's road network, some of the existing sites were utilized as temporary transfer stations.

Table 1. Objective function values of the above problem for different values of γ_i

γ_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Objective function value	140688300000	140688300000	140688300000	140688300000	140688300000	140688300000	141232600000	141232600000	141232600000	151294100000	151294100000

The above table shows that the objective function value remains relatively constant for different γ values, indicating the stability and robustness of the solution against the uncertainty in the coverage radius. The fuzzy model was used not only to achieve numerical improvements but also to examine the stability of the decisions. The observed constancy of the objective function value also confirms the effectiveness of the fuzzy–flexible model in supporting stable decision-making. As γ approaches one, the feasible solution space becomes smaller and the objective function value worsens, reflecting the increased stringency of the constraints.

4 Innovations, Constraints and Coprarative Analysis of the Proposed Model

The classical P-median model operates under several simplifying assumptions that often hinder its applicability to real-world scenarios. Key among these are the assumptions of an infinite time horizon, unlimited facility capacity, and the absence of initial setup costs. These assumptions are often impractical and suboptimal when applied to real-world operational challenges. In contrast, our proposed model addresses these assumptions by incorporating more realistic assumptions: Finite Time Horizon: The lifespan of waste landfill sites was considered with a finite time horizon of 20 years, aligning with current industry standards. Finite Facility Capacity: A finite capacity was imposed on the sites, with capacity estimations derived using established formulas .Initial Setup Costs: The model explicitly includes initial investment costs associated with the establishment of these sites, reflecting a more accurate economic perspective. By relaxing these restrictive assumptions, the proposed model offers a more pragmatic and operationally relevant framework for facility location problems, particularly in the context of waste management. In Model 4, two additional constraints, namely Constraints 4 and 5, were incorporated into the problem formulation. The problem was then examined and solved using a hybrid two-stage methodology. In the first stage, the Fuzzy Analytic Hierarchy Process (FAHP) was employed within a Geographic Information System (GIS) framework. In the second stage, the P-median model was further developed and solved to determine the optimal facility locations.This methodology is highly realistic and fully consistent with environmental standards. Specifically, the first stage enabled the identification of potential sites based on environmental suitability criteria, whereas the second stage facilitated site selection among these environmentally compliant alternatives on the basis of expert judgment. Accordingly, the proposed approach offers a practical and environmentally responsible framework for facility location decision-making. The main distinction between this approach and similar studies lies in the extent to which real-world conditions are reflected in the proposed model.

5 Conclusion and Future Research

Landfill site selection is a complex problem involving environmental, spatial, and economic considerations. In many regions, landfills have been established without comprehensive analysis, leading to environmental issues and high operational costs. This study demonstrates that a hybrid approach combining FAHP–GIS analysis with a developed capacitated P-median model provides an effective and practical framework for landfill site selection. The results indicate that a large portion of Mazandaran Province is unsuitable for landfill development, with only a limited number of potential sites identified. By solving the developed P-median model, it was found that the optimal network consists of seven high-capacity main sites and five lower-capacity transfer stations. This configuration minimizes construction and transportation costs while satisfying the spatial constraints of the current waste management system. The hybrid approach not only offers practical guidance for designing efficient waste management networks but also demonstrates that combining multi-criteria analysis with optimization models enables decision-making that is realistic, implementable, and adaptable to environmental and spatial constraints. This framework can serve as a reference for other regions with similar environmental and demographic conditions. For future research, integrating the proposed model with dynamic temporal data, population growth scenarios, and changes in waste generation patterns is recommended.

Acknowledgment

The authors sincerely thank the distinguished reviewers in advance for their professional insights, which will significantly enhance the content and structure of the revised versions of this manuscript.

References

1. Tadić, S., Krstić, M., Stević, Ž., & Veljović, M. (2023). Locating collection and delivery points using the p median Location problem. Logistics, 7(1), 10.
2. Tamir, A. (2023). Mean variance value at risk criterion for solving a p median location problem on networks with type 2 intuitionistic fuzzy weights. Journal of Computational and Applied Mathematics, 437, 115481.
3. Atta, S., Mahapatra, S., & Mukhopadhyay, S. (2022). Solving a new variant of the capacitated maximal covering location problem with fuzzy coverage area using metaheuristic approaches. Computers & Industrial Engineering, 170, 108315.
4. Lucchese, C., Rossi, G., Smaldone, G., & Vairo, G. (2023). A hybrid population based algorithm for fuzzy capacitated maximal covering location problem. Computers & Industrial Engineering, 177, 108982.
5. Koçoğlu, F. Ö. (2023). Development of a novel fuzzy hierarchical location routing optimization model considering reliability. Logistics, 7(3), 64.
6. Ghaffari far, F., Nasseri, S. H., & Tavakkoli Moghaddam, R. (2023). A vehicle location routing model for waste management problem under fuzzy flexible conditions. Iranian Journal of Operations Research, 14(1), 80–103.
7. 7 Babaee Tirkolaee, E., Abbasian, P., & Weber, G. W. (2021). Sustainable fuzzy multi trip location routing problem for medical waste management during the COVID 19 outbreak. Science of the Total Environment, 756, 143607.
8. 8 Raeisi, D., & Jafarzadeh Ghoushchi, S. (2022). A robust fuzzy multi objective location routing problem for hazardous waste under uncertain conditions. Applied Intelligence, 52(12), 13435–13455.
9. 9 Rathinabhagya, P., & Merline Vinotha, J. (2025). Fuzzy vehicle routing problem for a municipal solid waste management system with greenhouse gas emission. The Scientific Temper, 16(04), 4077–4084.
10. 10 Temur, H. T. (2024). Optimal locating by integrating volumetric fuzzy sets and geographic coordinate system. Computers & Operations Research, 161, 106377.
11. Ghahremani-Nahr, J., Nozari, H., Rahmaty, M., Zeraati Foukolaei, P., & Sherejsharifi, A. (2023). Development of a novel fuzzy hierarchical location-routing optimization model considering reliability.Coputers& Industrial Engineering, 177,108982.
12. Amini, A., Tavakkoli Moghaddam, R., & Ebrahimnejad, S. (2020). A bi objective transportation location arc routing problem. Transportation Letters, 12(9), 623–637.
13. Malekdar, A. M., & Shirazi, M. A. (2025). Fuzzy location and allocation hub network design for air cargo transportation. arXiv preprint arXiv:2506.12064.
14. Guo, W., Xu, Y., & Jin, Y. (2023). Swap based deep reinforcement learning for facility location problems in networks. arXiv preprint arXiv:2312.15658.
15. Hashemi, S. E. (2021). A fuzzy multi objective optimization model for sustainable reverse logistics network design of municipal waste collecting. Journal of Cleaner Production, 318, 128577.
16. ReVelle, C., & Swain, R. (1970). Central facilities location. Geographical Analysis, 2, 30–42.
17. Bramel, J., & Simchi Levi, D. (1998). The logic of logistics. Springer Verlag.
18. Mirchandani, P. B., & Francis, R. L. (1990). Discrete location theory. Wiley.
19. Lorena, L., & Senne, E. (2004). A column generation approach to capacitated p median problems. Operations Research, 31(5), 863–876.