International Journal of Applied Operational Research
Vol. 3, No. 4, pp. 1-6, Autumn 2013

Journal homepage: www.ijorlu.ir

Two Stage Flow Shop Scheduling Problem Including Transportation Time and Weightage of Jobs with Branch and Bound Method

D. Gupta*, P. Singla, Sh. Bala

Received: 27 March 2013 ; **Accepted:** 14 August 2013

Abstract This paper presents an algorithm with the help of branch and bound approach for the two stage flow shop scheduling problem including transportation time and weightage of jobs; processing times are associated with their respective probabilities. Our main objective is to find the optimal/ near optimal sequence of jobs in order to minimize the total elapsed time. An algorithm is clarified with the help of a numerical example.

Keywords Flow Shop Scheduling, Branch and Bound Method, Elapsed Time, Transportation Time, Weightage of Jobs.

1 Introduction

In the context of manufacturing, scheduling is fundamentally related to the problem of finding a successive assignment of limited resources to a number of jobs which is optimal in terms of certain performance measures. On many occasions in manufacturing environments, a set of processes is needed to be serially performed in several stages before a job is completed. Such systems are referred to as flow shop environments. In a flow shop system, a set of n different jobs are needed to be processed on a sequential set of m machines. That is, each job consists of m operations where each operation must be performed on a different machine for an amount of processing time. Each machine can handle only one job at a time, and the operation of a machine on a job usually cannot be pre-empted. The research into flow shop problems has drawn a great attention in the last decades with the aim to decrease the cost and to increase the effectiveness of industrial production. Johnson [1] gave a procedure to obtain the optimal sequence for n-jobs, two – three machines flow shop scheduling problem with an objective to minimize the makespan. The work was developed by Ignall and Scharge [2], Brown and Lomnicki [3], Bagga [4], Smith, et at [5], Gupta, [6], Maggu and Das [7], Yoshida and Hitomi [8], Singh, [9], Chander Sekharan [10], Anup [11], Gupta Deepak [12], Lomnicki, [13], Chandermouli [14] etc. by considering the various parameters.

* **Corresponding Author.** (✉)
E-mail: guptadeepak2003@yahoo.co.in (D. Gupta)

D. Gupta
Prof. & Head, Department of Mathematics, Maharishi Markandeshwar University, Mullana, Ambala (India)

P. Singla
Research Scholar, Department of Mathematics, Maharishi Markandeshwar University, Mullana, Ambala (India)

Sh. Bala
Research Scholar, Department of Mathematics, Maharishi Markandeshwar University, Mullana, Ambala (India)

[Downloaded from ijaor.com on 2025-10-23]

The weight of job shows the relative priority over some other jobs in a schedule of jobs. Higher the weight a job has, the more important it becomes for processing in comparison with other jobs in the operating schedule. The scheduling problems with weights arise when inventory costs for jobs are involved. Further the scheduling problem which does not involve "weight" of job is called "simple or the unweighted scheduling problem", whereas the scheduling problem involving "weight" of jobs is referred to as "weighted scheduling problem".

There are practical scheduling situations when certain times are required by jobs for transportation from one machine to another machine. This situation can be visualized when the machines on which jobs are to be processed are planted at different places, and these jobs require additional times in the forms of loading-time of jobs, moving time of jobs and then unloading time of jobs. The sum of all these times is known as transportation time of jobs. This paper studies the two stage flow shop scheduling problem introducing the weightage of jobs and transportation time. This makes the problem wider and more practical in process/ production industry.

Assumptions:

1. No passing is allowed.
2. Each operation once started must be performed till completion.
3. Jobs are independent of each other.
4. A job is entity, i.e. no job may be processed by more than one machine at a time.

Notations:

We are given n jobs to be processed on a three stage flowshop scheduling problem, and we have used the following notations:

a_i	:	Processing time for i^{th} job on machine A
b_i	:	Processing time for i^{th} job on machine B
p_i	:	Probability associated to the processing time a_i.
q_i	:	Probability associated to the processing time b_i.
A_i	:	Expected Processing time for i^{th} job on machine A
B_i	:	Expected Processing time for i^{th} job on machine B.
t_i	:	Transportation time from machine A to machine B.
C_{ij}	:	Completion time for job i^{th} on machines A and B
S_0	:	Optimal sequence
J_r	:	Partial schedule of r scheduled jobs
$J_{r'}$	:	The set of remaining (n-r) free jobs

2 Mathematical Development

Consider n jobs say i=1, 2, 3 … n are to be processed on two machines A & B in the order AB. Let a_i and b_i be the processing times of i^{th} job on machine A and machine B with probability p_i and q_i respectively. A_i and B_i be the expected processing times of i^{th} job on each machine A & B respectively. Let t_i be the transportation time of i^{th} job from machine A to machine B, and Let w_i be the weightage of i^{th} job.

[Downloaded from ijaor.com on 2025-10-23]

The mathematical model of the problem in matrix form can be stated as:

Table 1 The mathematical model of the problem in matrix form

job	Machine A		t_i	Machine B		weight
i	a_i	p_i		b_i	q_i	w_i
1	a_1	p_1	t_1	b_1	q_1	w_1
2	a_2	p_2	t_2	b_2	q_2	w_2
3	a_3	p_3	t_3	b_3	q_3	w_3
-	-	-	-	-	-	-
-	-	-	-	-	-	-
n	a_n	p_n	t_n	b_n	q_n	w_n

Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time, using branch and bound technique.

3 Algorithm

Step 1: Calculate
(i) $A_i = a_i \times p_i$ (ii) $B_i = b_i \times q_i$

Step 2: (i) $G_i = A_i + t_i$ and (ii) $H_i = B_i + t_i$

Step 3: Compute Minimum (G_i, H_i)

1) If Min (G_i, H_i) = G_i then define $G_i' = G_i + w_i$ and $H_i' = H_i$
2) If Min (G_i, H_i) = H_i then define $G_i' = G_i$ and $H_i' = H_i + w_i$

Step 4: Define a new reduced problem with G_i'' and H_i'' where

$$G_i'' = G_i' / w_i \quad \text{and} \quad H_i'' = H_i' / w_i$$

Step 5: Calculate

(i) $l_1 = t(J_r,1) + \sum_{i\in J_r'} G_i'' + \min_{i\in J_r'}(H_i'')$

(ii) $l_2 = t(J_r,2) + \sum_{i\in j_r'} H_i''$

Step 6: Calculate $l = \max(l_1, l_2)$

Evaluate l for the n classes of permutations, i.e., for these starting with 1,2,…n respectively. Explore the lowest lower bound vertex for (n-1) subclasses and again concentrate on the lowest label vertex. Thus, we get the optimal/near optimal sequence.

Step 7: Prepare in-out table for the optimal sequence obtained in step 6 and get the minimum

[Downloaded from ijaor.com on 2025-10-23]

total elapsed time.

4 The Numerical Illustration

Consider 5 jobs 2 machine flow shop problem whose processing time of the jobs on each machine, Transportation time t_i and weights of jobs w_i are given in Table 2.

Table 2 Processing time, transportation time, and weights of jobs of the problem

job	Machine A		t_i	Machine B		weight
i	a_i	p_i		b_i	q_i	w_i
1	50	0.3	4	105	0.2	2
2	180	0.1	3	170	0.1	1
3	80	0.2	5	75	0.2	3
4	70	0.3	2	70	0.2	4
5	190	0.1	6	60	0.3	2

Our objective is to obtain an optimal schedule for above said problem in order to minimize the total elapsed time.

Solution: As per Step 1: Expected processing times are as in Table 3:

Table 3 Expected processing times of step 1

job	Machine A	t_i	Machine B	weight
i	A_i		B_i	w_i
1	15	4	21	2
2	18	3	17	1
3	16	5	15	3
4	21	2	14	4
5	19	6	18	2

As per Step 2: Reduced problem is in Table 4:

Table 4 The reduced form of the problem

job	Machine A	Machine B	weight
i	G_i	H_i	w_i
1	19	25	2
2	21	20	1
3	21	20	3
4	23	16	4
5	25	24	2

As per Step 3 &4: Expected Weighted times are as in Table 5:

[Downloaded from ijaor.com on 2025-10-23]

Table 5 Expected weighted times of steps 3 and 4

job	Machine A	Machine B
i	G''_i	H''_i
1	10.5	12.5
2	21	21
3	7	7.6
4	5.75	5
5	12.5	13

As per Step 5 & 6: The lower bounds are as in Table 6:

Table 6 The lower bounds of steps 5 and 6

Node	LB(Jr)
1	69.5
2	80.09
3	66.09
4	64.84
5	71.5
41	70.34
42	80.8
43	69.25
45	72.34
431	69.75
432	80.25
435	71.75
4312	78.25
4315	77.75

Thus, the optimal sequence is 4-3-1-5-2

As per Step 7: The flow time table for the optimal sequence is as follows:

Table 7 The flow time table for the optimal sequence

job	Machine A	t_i	Machine B	weight
i	In – Out		In - Out	w_i
4	0 -21	2	23 – 37	4
3	21 – 37	5	42 – 57	3
1	37 – 52	4	57 – 78	2
5	52 – 71	6	78 – 96	2
2	71 – 89	3	96 – 113	1

Hence, for the optimal sequence 4-3-1-5-2, the total elapsed time is 113 units.

[Downloaded from ijaor.com on 2025-10-23]

References

1. Johnson, S. M., (1954). Optimal Two and Three Stage Prouction Schedules with Setup Times Include. Naval Research Logistics Quarterly, 1(1), 61-68.
2. Ignall, E., Schrage, L., (1965). Application of the branch-and-bound technique to some flowshop scheduling problem. Operations Research, 13,400-412.
3. Brown, A. P. G., Lomnicki, Z. A., (1966). Some applications of the branch and bound algorithm to the machine scheduling problem. Operational Research Quarterly, 17, 173-182.
4. Bagga, P. C., (1969). Sequencing in a rental situations. Jr. of Canadian Operation Research Society, 7, 152-153.
5. Smith, W. E., (1956). Various Optimizers for single stage production. Naval Research Logistic, 2, 59-66.
6. Gupta, D., (2011). Application of Branch and Bound Technique for $n \times 3$ Flow Shop Scheduling. In Which Processing Time Associated With Their Respective Probabilities, Mathematical Modelling and Theory, 2(1), 31-36.
7. Maggu, D., (1981). On $n \times 2$ sequencing problem with transportation time of jobs. Pure and Applied Mathematika Sciences, 12-16.
8. Yoshida, T., Hitomi, K., (1979). Optimal two stage production schedule with set up time separated. AIIE Transaction, II, 261-263.
9. Singh, T. P., (1994). $n \times 2$ flow shop scheduling with random processing and transportation time under group restriction on jobs. Journal of Information Sciences, 5(2).
10. Shekharan, C., Chanderi, D., (1992). An efficient heuristic approach to the scheduling of jobs in a flow shop, European Journal of Operation Research, 61(3), 318-325.
11. Anup, (2002). On two machine flow shop problem in which processing time assumes probabilities and there exists equivalent for an ordered job block. JISSOR, XXIII(1-4), 41-44.
12. Gupta, J. N. D., (1975). Optimal Schedule for specially structured flowshop, Naval Research Logistic, 22(2), 255-269.
13. Lomnicki, Z. A., (1965). A branch-and-bound algorithm for the exact solution of the three-machine scheduling problem, Operational Research Quarterly, 16, 89-100.
14. Chandramouli, A. B., (2005). Heuristic approach for n jobs, 3 machine flow shop scheduling problem involving transportation time, breakdown time and weights of jobs. Mathematical and Computational Applications, 10, 301-305.

[Downloaded from ijaor.com on 2025-10-23]